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Cosmic String in theory of General Relativity

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COSMIC STRING IN THEORY OF GENERAL RELATIVITY

BY

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ABSTRACT

This thesis is organized as follows.
Chapter I contains a brief discussion of the production of various topological defects in cosmological phase transitions in the early universe. The three kinds of topological defects associated with spontaneous symmetry breaking: domain wall, cosmic string and magnetic monopole. The existence and stability of these defects is dictated by topological considerations. This chapter also includes a discussion on the standard cosmology and cosmological phase transitions.

In the chapter II we review some physical properties of the cosmic string. Pair of cosmic strings intercommuting at two points can form closed loops. A closed loop will oscillate, gradually loosing energy , until it disappear. Only significant energy loss mechanism is gravitational radiation. Gravitational radiation is indeed dominant. There is no local gravitational field due to the cosmic string. The cosmic string acts as a gravitational lens for both light rays and particles.

Chapter III addresses the topic of cosmic string evolution and structure formation.

In the chapter IV we give a brief account of the Plebanski space-time. The Plebanski space-time includes many interesting space-times which are not black hole space-times but are important from the physical point of view.

In the chapter V we study the equilibrium configurations of a cosmic string described by the Nambu-action in the NUT-Kerr-Newman space-time which includes as special cases the Kerr-Newman black hole space-time as well as NUT space-time which is considered as cosmological model. In this study it is interesting to note that one can obtain parallel results for Kerr-Newman black hole as well as for NUT space-time.

Finally, in the chapter VI we study the equilibrium configurations of a cosmic string described by the Nambu-action in curved space-time such as the Kerr-Newman-Kasuya space-time which is the Kerr-Newman space-time involved with extra magnetic monopole charge. In this study it is interesting to note that the physical results remain the same whether or not the magnetic monopole exist in nature.

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INTRODUCTION

Recent attempts to incorporate the grand unified theories of particle physics into the general relativistic models of the early evolution of the universe have predicted the possible existence of enormously long objects called cosmic strings. Cosmic strings have attracted a lot of interest. A brief review of the cosmic strings has been made in the chapter I, II and III of this thesis.

Different people showed interest in cosmic strings in different ways. Frolov et al.[1] studied the possible configuration of a cosmic string in the gravitational field of a rotating charged Kerr-Newman black hole. The main theme of this thesis is to extend the study of the equilibrium configuration of a cosmic string in the gravitational field of a Kerr-Newman black hole [1] to others space-times which are not black hole space-times but have common feature with the black hole space-times that they have horizons. The motivation of this extension came from the fact that the different results of black hole physics (such as superradiance phenomenon, Hawking radiation, etc.) can be extended to other space-times [2-16] which are not black hole space-times but have common feature with the black hole that have horizons.

Keeping this objective of extension in different space-times having horizons in mind we have shown how one can obtain a large number of space-times having horizons as special class of a more general space-times. These space-times include the black hole spacetimes which are asymptotically flat as well as asymptotically de sitter. Besides these black hole spacetimes, the special classes include also other space-times like NUT-Kerr-Newman and Kerr-Newman-Kasuya space-times.

In the chapter V we studied the equilibrium configuration of a cosmic string in NUT-Kerr-Newman space-time [17]. The results obtained in this chapter can be specialized for the Kerr-Newman black hole [1] as well as for the NUT space-times. It will be interesting to note that the NUT space-time is considered as homogeneous anisotropic model of the universe [18].

In chapter VI we studied the equilibrium configuration of a cosmic string in the Kerr-Newman-Kasuya space-time [19] with an extra magnetic monopole charge. The monopole hypothesis was given by Dirac relatively long ago. Recently development of gauge theories have shed new light on monopole hypothesis.

We have demonstrated in the chapters V and VI of this thesis how one can extend the study of equilibrium configuration of a cosmic string in the gravitational field of a rotating black hole space-time [1] to other space-times like NUT-Kerr-Newman [chapter V] and Kerr-Newman-Kasuya [chapter VI] space-times which are not black hole space-times but have common feature with the black hole space-times that they have horizons.

We belive that the extension, as made in the space-times like NUT-Kerr-Newman and Kerr-Newman-Kasuya space-times can be done also in other special class of space-times having horizons (mentioned in the chapter IV). In support of our this claim, we add a discussion at the end of this thesis.

CHAPTER - I

THE PRODUCTION OF TOPOLOGICAL DEFECTS

1.1 INTRODUCTION

At the beginning the universe was very hot and dense. It began to expand and cool down through several phase transitions. These phase transitions in the early universe can produce some macroscopic topological defects: domain walls, cosmic strings and monopoles. Domain walls are one-dimensional defects, Cosmic strings are two-dimensional defects. Point-like defects are called magnetic monopoles. These defects are governed by the topology of the manifold M which was first discussed by Kibble [20, 21]. Firstly, if the manifold M has two or more disconnected pieces corresponding to spontaneous breaking of discrete symmetry, then domain walls can exist [22]. If the manifold M contains unshrinkable loops, cosmic strings can appear. Finally, if the manifold M contains closed two-dimensional surfaces that cannot be shrunk to a point, then monopoles can exist. Hybrid topological defects such as domain walls bounded by strings [23-25] and monopoles connected by strings [26, 27] can also be produced by the model with a sequence of phase transitions. These hybrid defects can arise in more complicated symmetry breaking patterns [28]. In this chapter we briefly describe the production of various topological defects in cosmological phase transitions in the early universe. Before discussing the production of these defects we will review the standard cosmology and cosmological phase transitions.

1.2 STANDARD COSMOLOGY

The history of the universe broadly divides into three stages according to current thinking. These stages are called the early universe, the adolescent universe and the late universe. The early universe was dominated by the radiation, and the adolescent universe was dominated by the matter. In the absence of vacuum energy, the late universe will continue to be dominated by the matter. The universe is assumed to be homogeneous and isotropic. The kinematics of a universe is described by the famous Robertson-Walker space-time metric which can be written in the form

$$ds^2 = dt^2 - R^2(t)\{\frac{dr^2}{1-kr^2} + r^2 d\vartheta^2 + r^2 \sin^2\vartheta d\phi^2\} \quad \text{.......................(1.1)}$$

Where R(t) be the radius of a spherical volume expanding with the general expansion of the universe. The dynamics of the expanding universe only appeared implicitly in the time dependence of the scale factor R(t). The 0-0 component of the Einstein equation,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} + \lambda g_{\mu\nu} \quad \text{...(1.2)}$$

gives the so-called Friedmann equation

$$\frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = \frac{8}{3}\pi G\rho + \lambda \quad \text{...(1.3)}$$

where ρ is the energy density and k and λ are constants. Neglecting cosmological constant λ,which is indeed experimental value to a good approximation at least in the present phase of the universe [29]. The above equation can be rewritten in the form of an energy conservation equation for a particle on the surface of our sphere:

$$\frac{1}{2}\dot{R}^2 - \frac{G\rho\left(\frac{4}{3}\pi R^3\right)}{R} = -\frac{1}{2}k \quad \text{............................ ....(1.4)}$$

We observe that $k > 0$ corresponds to a bound orbit. This is the case for a closed universe that will eventually reverse its expansion and contract to a new singularity. Similarly, the unbound case $k \leq 0$ corresponds to an open universe that will continue to expand forever. At early times of the universe ρ increases as $R \rightarrow 0$ so that in this case k relatively unimportant. Therefore the equation (1.4) becomes

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8}{3}\pi G\rho \quad \text{...(1.5)}$$

Hence for $k = 0$, the universe is very nearly flat.

For a perfect fluid characterized by a time dependent energy density $\rho(t)$ and pressure p(t), the stress energy tensor can be written as

$$T^{\mu}_{\nu} = \mathrm{diag}(\rho, -p, -p, -p) \quad \text{...(1.6)}$$

The $\mu = 0$ component of the conservation of stress energy $(T^{\mu\nu}_{;\nu} = 0)$ gives the energy conservation law which can be written as

$$d(\rho R^3) = -p\,d(R^3) \quad \text{.. ....(1.7)}$$

This is the first law of thermodynamics in the familiar form.

In the very early universe also assuming the temperature T much larger than masses of bosons and fermions, here the matter may be treated as an ideal relativistic gas undergoing adiabatic expansion. Then the density is given by

$$\rho = \frac{\pi^2}{30} N(T) T^4 \quad \text{...(1.8)}$$

where $N(T) = N_b(T) + \frac{7}{8} N_f(T)$, $N_b(T)$ and $N_f(T)$ are the numbers of distinct helicity states for bosons and fermions respectively. The relative factor $\frac{7}{8}$ accounts for the difference in Fermi and Bose statistics.

During the expansion of the universe the entropy per comoving volume element remains constant and we can write

$$d(SR^3) = 0 \quad \text{...(1.9)}$$

where

$$S = \frac{2\pi^2}{45} N(T)T^3 \qquad (1.10)$$

is the entropy density.

If the temperature T drops below the mass threshold, $N(T) = \text{constant}$ and R(T)T= constant.

The equation (1.5) and (1.7) give

$$R(T) \propto T^{-1} \propto t^{\frac{1}{2}} \qquad (1.11)$$

In this chapter we use the planck mass $m_p = 1.2 \times 10^{19}\,\text{GeV}$ and planck time $t_p = 5.3 \times 10^{-44}\,\text{sec}$ in terms of which the time-temperature relation may be written as

$$tT^2 = \frac{0.3}{\sqrt{N}} m_p \qquad (1.12)$$

During the early radiation-dominated epoch, $p = \frac{\rho}{3}$. When the universe becomes matter-dominated , $p = 0$ and we acquire

$$R(T) \propto t^{\frac{2}{3}} \quad \text{...(1.13)}$$

The age of the universe can be measured by using the expansion rate of the universe. The expansion rate of the universe is determined by the Hubble parameter $H = \frac{\dot{R}}{R}$, Where dot is the time derivative. The Hubble parameter is not constant and in general varies as t^{-1}. At the present epoch the value of H denoted by H_0 is called the Hubble constant.

The Friedmann equation can be written as

$$\frac{k}{H^2R^2} = \frac{\rho}{\frac{3H^2}{8\pi G}} - 1 \equiv \Omega - 1 \quad \text{...(1.14)}$$

where Ω is the ratio of the density ρ to the critical density ρ_c.

The critical density is

$$\rho_c = \frac{3H^2}{8\pi G} = 2 \times 10^{-29} h^2 gcm^{-3} \quad \text{...(1.15)}$$

where h is the dimensionless parameter and is equal to

$$\frac{H}{100\text{kmsec}^{-1}\text{Mpc}^{-1}} \quad \text{.......(1.16)}$$

The universe is closed, open and flat according to $\Omega > 1, \Omega < 1$ and $\Omega = 1$, respectively. The inflationary cosmological model [30] predicts that $\Omega = 1$ with very high accuracy. The present age of the universe can be found as

$$t_{pres} = \frac{2}{3}H^{-1} = 2 \times 10^{17} h^{-1}\text{sec} \quad \text{.......(1.17)}$$

The range of plausible values of h and Ω are

$$0.5 \le h \le 1 \quad \text{.......(1.18)}$$

$$0.1 \le \Omega \le 1 \quad \text{.......(1.19)}$$

In fact, nucleosynthesis provides the most precise determination of the baryon density. Nucleosynthesis considerations demand that the baryons density, ρ_B, should be $\rho_B \le 0.1\rho_c$ and then $\Omega_B \le 0.1$. This leads to conclude that if $\Omega \to 1$, the universe must be dominated by particles other than baryons.

As a first simple application, let us establish the cosmological red-shift formula

$$1 + z = \frac{R(t_{pres})}{R(t)} \qquad (1.20)$$

which is convenient to use for discussing the recent evolution of the universe. The cosmological red-shift is really an expansion effect rather than a velocity effect.

For $\Omega = 1$,

$$t = \frac{2}{3} H^{-1} (1 + z)^{\frac{-3}{2}} \qquad (1.21)$$

This implies that

$$(1 + z) \propto t^{\frac{-2}{3}} \qquad (1.22)$$

Since the photon and neutrino species are decoupled, their entropies are separately conserved; hence the present energy density and entropy density are

$$\rho_r = 8.09 \times 10^{-34}\, gcm^{-3} \qquad (1.23)$$

$$S = 2970 cm^{-3} \qquad (1.24)$$

assuming that the present temperature of the photon gas is $T = 2.75^0 k$.

The total density of radiation including photon and N_ν species of massless neutrons is

$$\rho_r = (1 + 0.23N_\nu)\rho_\gamma \quad (1.25)$$

At $t > t_{eq}$, $\frac{\rho_r}{\rho}$ decreases as (1+z). It then follows that the red-shift of equal matter and radiation energy densities is given by

$$1 + z_{eq} = 2 \times 10^4 \pi h^2 \quad (1.26)$$

The density of the universe at t_{eq} is

$$\rho_{eq} = 3.2 \times 10^{-16}(\pi h^2)^4 gcm^{-3} \quad (1.27)$$

The most important epoch in cosmic history of the universe is the decoupling of matter and radiation when protons and electrons combine to form hydrogen atoms. In this case

$$z_{dec} \cong 1300 \quad (1.28)$$

$$t_{dec} \cong 5 \times 10^{11}(\pi h^2)^{-\frac{1}{2}} \sec \quad (1.29)$$

The standard cosmology is completely an achievement which is comparable to the standard model of low-energy particle physics, the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge theory of the strong, weak, and electromagnetic interactions.

On very large scales, the universe is very smooth and there is no physical explanation for this in the standard cosmology. This is the first untidy fact about the standard cosmology and is frequently referred to as the horizon problem. In the context of unified gauge theories there are relics which are grossly overproduced early in the history of the universe and contribute to the present energy density. There is no mechanism in the standard cosmology to get rid the universe of these relics.

Inflation is an attractive candidate for solving the problems mentioned above. It had an epoch when vacuum energy was the dominant component of the energy density of the universe. Inflation is cosmologically attractive because it proposes the possibility of performing the present state of the universe. Topological defects which produced before inflation are inflated away and should be interested only in the defects produced after or near the end of inflation. The phase transition was never completed and most of the universe continued to inflate forever [31, 32].

1.3 COSMOLOGICAL PHASE TRANSITION

The introduction of the grand unified theories into the hot big bang models predicts cosmological phase transitions at the separation energies. The phase transition is a thermal process and should study the formation of defects using statistical field theory. Cosmological phase transitions are very similar to phase transitions in more familiar solids and liquids, like vapor turning into water and then ice. When water is freezed then it produces a crystal of ice with some defects such as dislocation or vacancies. Similarly, cosmological phase transitions can produce some topological defects. A symmetry-breaking phase transition can be first or second order, in general. In a simple model, the phase transition from the symmetric phase to the broken phase is second order. Let us consider a field theory with a symmetry group G and Highs field Φ with a potential of self-intersection $V(\Phi)$.

To illustrate some of the features of finite temperature effects we consider

$$G = U(1) \qquad (1.30)$$

and

$$V(\Phi) = \frac{1}{2}\lambda(\Phi^{+}\Phi - \sigma^{2})^{2} \qquad (1.31)$$

where Φ is a complex scalar field and λ is the Higgs coupling constant. The symmetry of phase transformation $\Phi \to e^{i\alpha}\Phi$ is a symmetry group U(1). The minimum values of the potential are at non zero values of Φ, therefore the symmetry is spontaneous broken and Φ tends to earns a non zero vacuum expectation value

$$\langle\Phi\rangle = \sigma e^{i\vartheta} \qquad (1.32)$$

This implies that

$$|\langle\Phi\rangle|^{2} = \sigma^{2} \qquad (1.33)$$

This equation only gives the magnitude of $\langle\Phi\rangle$ and does not fix its direction. The situation is similar to that of a completely isotropic ferromagnetic cooled through its curie point. It must earn non zero magnetization of an arbitrary direction. This magnetization is determined in practice by any small external field or, in the absence of such fields by the random fluctuations. We have in fact, a degenerate set of vacuum states. In the general case, the set of vacua forms a quotient space M. Let us assume that the original group is G and its unbroken subgroup is H, in the symbolic notation $G \to H$. All elements of G, which make the VEV$\langle\Phi\rangle$ unaltered, belong to H. The manifold of the equivalent vacuum states M, then, become the quotient space $\frac{G}{H}$.

By adding temperature-dependent terms, we can write the effective potential for Φ at finite temperatures in the form

$$V_T(\Phi) = B\,T^2\Phi^+\Phi + V(\Phi) \quad \text{.........(1.34)}$$

where $B > 0$ is a dimensionless constant which is a combination of the Higgs-coupling constant λ and other coupling of the field Φ.

From equations (1.31) and (1.34) we get the effective mass of the field Φ at temperature T is

$$m^2(T) = BT^2 - \lambda\sigma^2 \quad \text{.........(1.35)}$$

Then, for $T = T_c$, we have

$$m^2(T) = 0 \qquad (1.36)$$

where

$$T_c = \left(\frac{\lambda}{B}\right)^{\frac{1}{2}} \sigma \qquad (1.37)$$

is the critical temperature of the phase transition from the symmetric phase to the broken-symmetry phase. When $\lambda << 1$, we have $T_c \sim \sigma$. While for $T > T_c$, $m^2(T)$ is positive. The minimum value of $m^2(T)$ is positive. At $\Phi = 0$ the value of $V(\Phi)$ is minimum and then $\langle\Phi\rangle = 0$ and we are in the symmetric phase.

There is no barrier at the critical temperature for a second-order phase transition and the phase transition occurs smoothly. More complicated models can guide to first-order phase transitions, where the symmetric phase remains metastable at $T < T_c$ and the phase transition proceeds through the nucleation of bubbles. For a first-order transition there is a potential barrier separating the minima. In a cosmological phase transition Φ evolves from the symmetric minimum $\Phi = 0$ (high temperature) to the symmetry-breaking minimum $\Phi = \sigma$ (low temperature).

Since a uniform Higgs field is energetically preferred, much of the initial random variation of $\langle\Phi\rangle$ will rapidly disappear in the course of further evolution. For energetic reasons $\langle \Phi \rangle$ will tend towards spatial uniformity unless prevented from so doing by trapped defects of some kind.

1.4 DOMAIN WALLS

If a discrete symmetry is broken, domain walls can appear. For a real scalar field, the Lagrangian which undergoes spontaneous symmetry breaking is given by

$$L = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{4}\lambda(\phi^2 - \sigma^2)^2 \quad \text{.....................................(1.38)}$$

where ϕ is a real scalar field. The reflection symmetry $\phi \rightarrow -\phi$ present in the Lagrangian is broken by the vacuum expectation values $\langle \phi \rangle = \pm \sigma$. Let us assume that all of space is in the same ground state and that space is divided into two regions. The vacuum expectation value of ϕ is.$\langle \phi \rangle = +\sigma$ in one region of space and $\langle \phi \rangle = -\sigma$ in the other region of space. When we go from a region with $\langle \phi \rangle = +\sigma$ to a region with $\langle \phi \rangle = -\sigma$ then we should inevitably pass through $\langle \phi \rangle = 0$ and this implies that there must be a region where $\phi = 0$, that is a region of false vacuum. This region between the two vacua is called a domain wall. Hill, Schramn and Fry [33] remark that a very-low-energy phase transition produced very light domain walls which might be the seed fluctuation of a large-scale structure. Domain walls are two-dimensional topological defects which might also be able to account for the large-scale streaming motion [34].

The wall is given by the solution of the equation of motion for ϕ

$$-\frac{\partial^2\phi}{\partial z^2}+\lambda\phi(\phi^2-\sigma^2)=0 \quad\text{.....(1.39)}$$

The solution of the equation of motion with boundary conditions, at $z = -\propto$, $\phi = -\sigma$ and at $z = +\propto$, $\phi = +\sigma$ is

$$\phi(z)=\sigma\tanh\left(\frac{z}{\Delta}\right). \quad\text{.....(1.40)}$$

where

$$\Delta=\left(\frac{\lambda}{2}\right)^{-\frac{1}{2}}\sigma^{-1} \quad\text{.....(1.41)}$$

is the thickness of the wall. The thickness of the wall which is finite, but non-zero is easy to understand. We can calculate the stress-energy tensor, to estimate the surface energy density of the wall η for a scalar field.

$$T_{\mu}^{\nu}=f(z)\mathrm{diag}(1,1,1,0) \quad\text{.....(1.42)}$$

where

$$f(z) = \frac{\lambda}{2}\sigma^4 \cosh^{-1}\frac{z}{\Delta} \qquad (1.43)$$

The surface energy density associated with the wall is given by

$$\eta = \int T_0^0 dz = \frac{2\sqrt{2}}{3}\lambda^{\frac{1}{2}}\sigma^3 \qquad (1.44)$$

This is identical to the integrated, transverse components of the stress, and is accurately equal to the surface tensor in the wall. Since $\phi(z)$ is a scalar field independent of x, y, t and having the same invariance, the stress-energy tensor T_μ^ν is invariant with respect to Lorentz boosts in the xy-plane. This suggests only about transverse motion of the wall; motion in tangential direction is unobservable. Of course this applies only to plane walls, but macroscopic walls with curvature radii $R >> \Delta$ can locally be considered as flat.

The domain walls are inherently relativistic and their gravitational effects are inherently non-Newtonian. An infinite domain wall repulse the test particles. Two infinite domain walls repulse one another [35, 36]. An infinite domain wall will be formed during a domain wall forming phase transition. The infinite domain wall will move under its own tension and try to straighten out. After the phase transition, the motion of the wall is damped by friction but as the plasma gets diluted by Hubble expansion, the drag decreases and eventually the motion of the domain wall is effectively undamped by friction. The single domain wall in the universe cannot disappear since it is protected by topology and would be present in the universe if it were ever produced.

1.5 COSMIC STRINGS

The abelian Higgs model illustrates the cosmic strings which are much more palatable to a cosmologist than domain walls. If the model contains a U(1) gauge field A_μ , and a complex Higgs field Φ and carries U(1) charge e, then the Lagrangian of this model is given by

$$L = D_\mu \Phi D^\mu \Phi^+ - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \lambda(\Phi^+ \Phi - \frac{\sigma^2}{2})^2 \quad \text{.........................}(1.45)$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$D_\mu \Phi = \partial_\mu \Phi - ieA_\mu \Phi$$

and e is the gauge coupling.

First of all, Nielson and Olesen [37] discussed the string solution to the equations of motion for the Lagrangian in this model. The Higgs field at large distances from an infinite string in cylindrical coordinates (r, ϑ, z) has the form

$$\Phi = \left(\frac{\sigma}{\sqrt{2}}\right)e^{in\vartheta} \qquad (1.46)$$

where n is an integer and ϑ is the polar angle in the xy-plane and the gauge field is

$$A_\mu = \frac{1}{ie}\partial_\mu\left[\ln\left(\frac{\sqrt{2}\Phi}{\sigma}\right)\right] \qquad (1.47)$$

Since at large distances from the string $F_{\mu\nu} = 0$ and $D_\mu\Phi = 0$, so that the energy density vanishes outside the string core.

There is no general solution to the coupled equations of motion for Φ and A_μ. Using the stokes theorem we find

$$\int \bar{B}\cdot d\bar{s} = \oint \bar{A}\cdot d\bar{l} \qquad (1.48)$$

where $\bar{B} = \nabla \times \bar{A}$ is the magnetic field associated with the U(1) gauge field. Therefore the total magnetic flux within the string is $\frac{2\pi n}{e}$.

Consider the complex field $\Phi = \frac{1}{\sqrt{2}}(\phi + i\phi_1)$. If the vacuum expectation value (VEV) is chosen to lie in the real direction, then the potential can be written in the form

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - \sigma^2)^2 \quad \text{.........(1.49)}$$

where

$$\langle|\Phi|\rangle = \frac{\langle\phi\rangle}{\sqrt{2}} \quad \text{.........(1.50)}$$

The phase of the VEV defined by

$$\langle\phi\rangle = \frac{\sigma e^{i\vartheta}}{2} \quad \text{.........(1.51)}$$

where ϑ varies on the position. Since $\langle\phi\rangle$ is a single valued , the charge of ϑ around any closed path in space must be an integer multiple of 2π Viz. $\Delta\vartheta = 2\pi n$, n is an integer. Consider a closed path $\Delta\vartheta = 2\pi$ and the closed path is shrink to a point. Here $\Delta\vartheta$ cannot change continuously from $\Delta\vartheta = 2\pi$ to $\Delta\vartheta = 0$.

As a result, we must encounter at least one point within the closed path where the phase ϑ is undefined. There is at least one tube of false vacuum inside any closed path which has $\Delta\vartheta \neq 0$. Such tubes of false vacuum can have no ends and must either be closed or infinite in length. These tubes of false vacuum contain a characteristic transverse dimension very much smaller than their length, so they can be behaved as one-dimensional material thing and are called cosmic strings.

The energy-momentum tensor associated with a long thin, straight cosmic string lying along the z-axis has the form

$$T_{\mu}^{\nu} = \mu\delta(x)\delta(y)\mathrm{diag}(1,0,0,1) \quad \text{...(1.52)}$$

where μ is the mass of the string per unit length. This shows that the string tension is equal to the mass per unit length.

Cosmic strings discussed above are called gauge cosmic strings. There are also cosmic strings associated with the spontaneous symmetry breaking of a global U(1) symmetry. Such cosmic string are called global cosmic strings. The cosmological properties of global cosmic strings are very similar to gauge cosmic strings but the physical properties of gauge and global cosmic strings are somewhat different. Global cosmic strings are strongly coupled to massless scalar Goldstone bosons. The loops of these cosmic strings lose all their energy after about 20 oscillations [38, 39]. There are no unwanted particles like these in the case of gauge cosmic strings.

The Lagrangian of the model which contains a U(1) global field is given by equation (1.45) with set A_μ equal to zero:

$$L = \partial_\mu \Phi^+ \partial^\mu \Phi - \lambda\left(\Phi^+\Phi - \frac{\sigma^2}{2}\right)^2 \quad \text{.........(1.53)}$$

The phase of Φ alters by 2π around the string, the radius of the core is $\Delta = \sqrt{2}\lambda^{-\frac{1}{2}}\sigma^{-1}$ and outside the phase Φ is given by

$$\Phi = \frac{\sigma}{\sqrt{2}} e^{i\vartheta} \quad \text{.........(1.54)}$$

The mass per unit length of the global cosmic string is

$$\mu = \pi\sigma^2 \mathrm{In}\frac{R}{\Delta} \quad \text{.........(1.55)}$$

where R is the cut-off radius.

Their energy per unit length is logarithimically divergent. Two parallel strings with opposite sense of $\Delta\vartheta$ are attracted to one another with a force per unit length

$$F = \frac{\partial\mu}{\partial R} = \frac{\pi\sigma^2}{R} \qquad \text{.........(1.56)}$$

Here the role of the cut-off radius is played by the distance between the strings. The concept of this forces is interchangeable to the interaction of strings with a long-range Goldstone field ϑ. Of course, the lagrangian (1.53) with a complex field Φ describes both the Goldstone field and the cosmic string. The force per unit length due to tension in curved strings is

$$F \sim \frac{\mu}{R} \qquad \text{.........(1.57)}$$

This force is greater than the interaction force (1.56) by a large factor

$$\text{In}\left(\frac{R}{\Delta}\right) \sim 100. \qquad \text{..(1.58)}$$

This remarks that the dynamics of global cosmic strings is dominated by tension.

Wittin [40] has shown that some spontaneously broken gauge theories will generate cosmic strings which are superconductors. The charge carries on such cosmic strings can be either fermions or bosons and the critical currents can be as large as $10^{20}\,A$. The mass per unit length associated with the electromagnetic field for the expected large currents can be of the same order as μ, where μ is the mass per unit length of the string. Superconducting cosmic strings [40] can have very dramatic cosmological signatures.

1.6 MONOPOLES

Monopoles are zero-dimensional point-like topological defects which arise in gauge theory that undergoes spontaneously symmetry breaking. For this theory, the Lagrangian is given by

$$L = \frac{1}{2} D_\mu \Phi^a D^\mu \Phi^a - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \frac{1}{8}\lambda(\Phi^a\Phi^a - \sigma^2)^2 \quad \text{................}(1.59)$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - e \in_{abc} A^b_\mu A^c_\nu$$

$$D_\mu \Phi^a = \partial_\mu \Phi^a - e \in_{abc} A^b_\mu \Phi^c$$

Monopoles exists if the vacuum manifold associated with the symmetry breaking pattern $G \rightarrow H$, contains unshrinkable surface, that is

$$\Pi_2(M) = I \quad \text{..}(1.60)$$

where $\Pi_2(M)$ is the homotopy group classifying unshrinkable surface in M and I is the trivial group.

To analyze the topological defects the following theorem from the homotopy group is very useful. Let us assume that the group G is broken to a subgroup H:G $\rightarrow$ H.

Theorem: If $\Pi_n(G) = \Pi_{n-1}(G) = I$, then

$$\Pi_n(M) = \Pi_{n-1}(H) \quad \text{...(1.61)}$$

where the equality sign " = " indicate isomorphism..

Let us assume that the group G is such that $\Pi_2(G) = \Pi_1(G) = I$, then, applying the above theorem for $n = 2$ we get

$$\Pi_2(M) = \Pi_1(H) \quad \text{...(1.62)}$$

From (1.61) and (1.62) we get the condition for the formation of monopoles:

$$\Pi_1(H) \neq I \quad \text{...(1.63)}$$

For example, monopoles will be formed at a phase transition

$$G \rightarrow K \times U(1) \qquad \text{...(1.64)}$$

The important thing abut this result is that monopoles must appear if H contains at least one U(1) factor. We know that a U(1) factor must first appear at some stage in the sequence of symmetry breaking from G down to $SU(3)_c \otimes U(1)_{EM}$. So that formation of monopoles in the early universe can not be avoided.

CHAPTER - II

PHYSICAL PROPERTIES OF COSMIC STRINGS

2.1 INTRODUCTION

In the previous chapter we have discussed the production of cosmic strings. In this chapter we shall describe their physical properties. The key bit of physics crucial to the evolution of a string network is intercommutation. Through this process, long string cut up into smaller segments and /or loops, thereby regulating the energy of the string network. Another prediction of the cosmic string scenario is a stochastic gravitational wave background. This is produced by oscillating loops that lived and decayed at various epochs through the emission of gravitational radiation. The gravitational field around an infinitely extending straight cosmic string has a special property. An idealized static cosmic string, which has an infinitesimal thickness, cannot create a Newtonian gravitational potential around it [21, 41]. There are non-Newtonian gravitational effects associated with the cosmic strings. The discussions of these properties of cosmic strings are given below.

2.2 INTERCOMMUTATION

The process that plays a crucial role in the evolution of a string network is intercommutation. When two strings intersect, they reconnect, or intercommute [42, 43]. Pair of strings intercommuting at two points can form closed loops. Loops can also be formed by self-intersection of individual strings. A closed loop of string oscillating under the action of its tension, may intersect itself and break into two smaller loops whose lifetime would be shorter. The fragments may further break, but this process is finite. After several rounds of fragmentation, a family of non-intersecting daughter loops is leftbehind. This process is important, since loops eventually radiate away their energy and save the universe from string domination. Based upon numerical experiments it appears that the probability for intercommutation to occur is nearly unity [42- 44]. Recently Shellard has done analysis regarding the intercommutation [42] for the case of global strings of the model

$$L = \partial_\mu \Phi^+ \partial^\mu \Phi - \frac{\lambda}{2}\left(\Phi^+\Phi - \sigma^2\right) \qquad \text{.................................(2.1)}$$

He considered two strings at right angles moving towards one another with relative velocity up to 0.5c. Here, c is the speed of light. Consider, in all cases the result is that the strings can intercommute. It may be that intercommutation is angle and model-dependent. These results suggest that intersecting strings intercommute with high probability. Through the process of intercommutation, long string can be cut up into smaller segments and / or loops. The intercommutation of intersecting strings leads to the continual chopping up long strings into smaller loops. The loop formation and their fragmentation into smaller loops are of much interest to the string scenario for galaxy formation. A description of the energy distribution of loops has been given within the framework of the statistical mechanics of string in ref. [45- 48]. If a loop self-intersects, it breaks into two loops. These loops are called daughter loops. We assume that the daughter loops have roughly equal masses. These loops extend from nearly horizon size downward. The loops decay into elementary particles when their size becomes comparable to the string thickness $\Delta \sim \sigma^{-1}$.

If a loop self-intersects, it breaks into two daughter loops. In this case most of the loop-energy goes into kinetic energy of daughter loops. After n rounds of fragmentation of a loop of mass M, we have 2^n loops. The energy of these loops is

$$E_n \sim \frac{M}{2^n} \qquad (2.2)$$

and rest mass

$$M_0 \sim (1-F)^n E_n \qquad (2.3)$$

Where F is the fraction of loop energy that goes into kinetic energy of daughter loops.

When $M_0 \sim \sigma$, The loops decay and the number of steps required is

$$n_* \sim \frac{\ln\left(\frac{M}{\sigma}\right)}{\ln\left(\frac{2}{1-F}\right)} \quad \text{.......................................(2.4)}$$

When the fraction of loop energy is not too small, then the smallest loop and the resulting elementary particles become very relativistic:

$$\gamma_n \sim \frac{E_n}{M_0} \sim (1\text{-}F)^{-n} \quad \text{.......................................(2.5)}$$

The period of oscillation of the loop in their respective rest frame is

$$t_n^{(0)} \sim \frac{M_0}{2\mu} \quad \text{.......................................(2.6)}$$

But in the frame of the initial loop the period of oscillation is

$$t_n \sim \gamma_n t_n^{(0)} \sim 2^{-n} \frac{M}{2\mu} \quad \text{.......................................(2.7)}$$

Therefore the time scale of the whole decay process is

$$t \sim \sum_{n=0}^{n_*} t_n \sim \frac{M}{\mu}. \quad \text{.......................................(2.8)}$$

Clearly non-intersecting oscillation loops can disappear only by emitting some short of massless radiation.

2.3 GRAVITATIONAL RADIATION

Oscillating loops of cosmic strings that lived and decayed at various epochs produce a stochastic gravitational waves. We can look forward to the regular observation of gravitational waves before the end of this century. The millisecond pulsar is the best way to look for such stochastic gravitational waves. This is a very accurate clock which sends pulses to the Earth at acutely timed intervals. If there are gravitational waves, they will make any sound into the timing of the pulses. The gravitational radiation is the dominant energy loss mechanism for non-intersecting oscillating loops. The emission of gravitational radiation from oscillating loops provides a mechanism for energy loss. This energy loss mechanism proves to be important for the viability of the string hypothesis. The discovery of gravitational radiation is one of the outstanding goals of modern experimental physics and observational astronomy. Closed loops can be formed by self-intersection of a cosmic string. This process of the production of loops and their subsequent decay may be effective enough to prevent the strings from dominating the universe [49- 52].

The equation of motion for a thin string (thickness much less than radius of curvature) are derived form

$$I = -\mu \int d^2 l \left[-\det\left(g_{\alpha\beta} \frac{\partial x^\alpha}{\partial l^a} \frac{\partial x^\beta}{\partial l^b} \right) \right]^{\frac{1}{2}} \quad \text{.................................(2.9)}$$

where μ is the mass of the string per unit length $g_{\alpha\beta}(\alpha,\beta,= 1,2,3,4)$ is an external gravitational field and l^a denotes the world-sheet co-ordinates (a, b = 0, 1; $l^0 = \tau$, $l^1 = \sigma$).The integral in equation (2.9) is just the surface area of the world-sheet described by the string.

The trajectory of the string is described by a vector function $\bar{x}(\sigma,t)$. Here, t is clock time, σ is proportional to position along the string from a fixed point. Hence the equation of motion for the trajectory of the string takes the form of a wave equation

$$\frac{\partial^2 \bar{x}}{\partial t^2} - \frac{\partial^2 \bar{x}}{\partial \sigma^2} = 0 \quad \text{.................................(2.10a)}$$

with

$$\left(\frac{\partial \bar{x}}{\partial t}\right)^2 + \left(\frac{\partial \bar{x}}{\partial \sigma}\right)^2 = 1 \text{ and } \frac{\partial \bar{x}}{\partial t} \cdot \frac{\partial \bar{x}}{\partial \sigma} = 0 \quad \text{.......................(2.10b)}$$

Of course the general solution of equation (2.10a) is

$$\bar{x}(\sigma, t) = \frac{1}{2}\left[\bar{a}(\sigma - t) + \bar{b}(\sigma + t)\right] \quad \text{..................................(2.11)}$$

and equation (2.10b) give the following constraints for the otherwise arbitrary functions $\bar{a}$ and $\bar{b}$:

$$\left(\frac{\partial \bar{a}}{\partial \sigma}\right)^2 = \left(\frac{\partial \bar{b}}{\partial \sigma}\right)^2 = 1 \quad \text{...(2.12)}$$

The motion of a closed loop of invariant length L is described by a solution of the form (2.11), (2.12) where $\bar{a}(\sigma)$ and $\bar{b}(\sigma)$ are periodic functions with period $L = \frac{M}{\mu}$ and M is the mass of the loop:

$$\bar{a}(\sigma + L) = \bar{a}(\sigma); \bar{b}(\sigma + L) = \bar{b}(\sigma) \quad \text{..............................(2.13)}$$

From equation (2.11), it is clear that the motion of the loop must also be periodic in time with the same period. We have seen in the preceding section, self-intersecting loops can intercommute and break into smaller pieces. If all loop trajectories intersect themselves at some point during the period, then the loop will rapidly decay into a cascade of smaller and small loops. When the size of the loop is much smaller than the horizon, effects of expansion can be neglected and we have regular oscillating loop. The motion of a small closed loop is specially simple. As a result, a closed loop of characteristic radius R oscillates relativistically under the action of its tension. As it oscillates, it also produces gravitational radiation and radiates away all its energy on a time-scale

$$\tau \sim R(\Gamma G\mu)^{-1} \qquad (2.14)$$

Where the coefficient Γ is a numerical factor of order 10^2 for certain family of string loops. This numerical factor has been determined by a computer calculation [53].

So that a loop of cosmic string will under go about $10^{-2}(g\mu)^{-1}$ oscillations before it die outs.

The intercommutation of intersecting of pair of strings at two points can form closed loops. Closed loops may thus be formed by self-intersection of individual strings. These processes are important, since loops eventually radiate away their energy and save the universe from string domination. The string network consists of both infinite strings and closed loops of string. Many authors [54, 55] have studied the evolution of cosmological string networks by numerical simulation. Numerical simulations show that after the string network is produced, it rapidly approaches a so-called "scaling solution".

A scaling solution obtained in a radiation-dominated universe with string energy density

$$\rho_s \cong \frac{30\mu}{t^2} \qquad \text{.......(2.15)}$$

Therefore $\rho_s \propto R^{-4}$ and the ratio of string energy density to radiation density remains constant:

$$\frac{\rho_s}{\rho_r} \cong 30\frac{32\pi}{3}G\mu \qquad \text{.......(2.16)}$$

The number density of loops at time t with energy E to E+dE is

$$n_l(E,t)dE \cong 0.3\left(\frac{\mu t}{E}\right)^{\frac{3}{2}}\frac{dE}{Et^3} \qquad \text{.......(2.17)}$$

So that the total energy density in loops is

$$\rho_l = \int E n_l dE \cong 0.6\mu^{\frac{3}{2}}t^{-\frac{3}{2}}E_{min}^{-\frac{1}{2}} \qquad \text{.......(2.18)}$$

Hence we may conclude that

$$\rho_l \propto t^{-\frac{3}{2}} \propto R^{-3}$$

and that ρ_l diverges as $E_{min}^{-\frac{1}{2}}$.

Before giving radiation, a loop undergoes oscillations of order $(\Gamma G\mu)^{-1}$

Therefore at time t, the smallest loop has a characteristic size

$R_{min} \sim \Gamma G\mu t$ and energy $E_{min} \sim \Gamma G\mu^2 t$.

2.4 COSMIC STRING GRAVITY

In this section, we shall discuss the gravitational field of cosmic strings. The gravitational field of cosmic strings is very different from that of Newtonian strings. There are non-Newtonian gravitational effects associated with the cosmic strings. A Newtonian string would produce a gravitational field that is inversely proportional to the distance from it. On the other hand, a cosmic string produces no local gravitational field but, instead, eats up space from around itself. The ratio of the circumference of a circle around the cosmic string to its radius is less than 2π by an amount proportional to its mass per unit length. This gives the most uncommon result that a test particle moving in the "gravitational field" of a cosmic string never follows a closed trajectory and always ends up at infinity.

When the energy-momentum tensor has the form $T^{\mu}_{\nu} = \mathrm{diag}(\rho,-p_1,-p_2,-p_3)$,the correct Newtonian limit of Poisson's equation is

$$\nabla^2\Phi = 4\pi G(\rho + p_1 + p_2 + p_3) \qquad (2.19)$$

where Φ is the gravitational potential, G is the Newtonian gravitational constant, ρ is the energy density and p_i are three eigenvalues of pressure. For non-relativistic matter $p_i << \rho$ then (2.19) becomes

$$\nabla^2\Phi = 4\pi G\rho \qquad (2.20)$$

But, in the case of an infinite straight cosmic strings in the z direction, $p_3 = -\rho$ and $p_1 = p_2 = 0$. Then Poisson's equation becomes

$$\nabla^2\Phi = 0 \qquad (2.21)$$

This equation is called Laplace equation. The Laplace equation suggests that infinite straight cosmic strings produce no gravitational force on surrounding matter.

Vilenkin has solved Einstein's equations for the metric outside an infinite straight cosmic string assuming that the parameter $G\mu$ is small [41]. J. R. Gott discussed large values of $G\mu$ and the internal metric of the cosmic string [56]. Outside the cosmic string core the metric in terms of the cylindrical co-ordinates (r, ϕ, z) is

$$ds^2 = dt^2 - dz^2 - dr^2 - (1 - 4G\mu)^2 r^2 d\phi^2 \quad (2.22)$$

where μ is the mass per unit length of the string which extends along the z axis from $-\infty$ to ∞. This metric is just the flat space metric, except for the factor $(1 - 4G\mu)^2$. The extra factor can be absorbed by defining a new angular variable as

$$\phi' = (1 - 4G\mu)\phi \quad (2.23)$$

Then the metric (2.20) becomes the flat space Minkowski metric, but then the range of the flat space angular variable ϕ' is only $0 \le \phi' \le 2\pi(1 - 4G\mu)$. In this case, it can be said that the space has a conical singularity at the location of the idealized cosmic string. The equation (2.20) describes a conical space which becomes a flat space with wedge of angular width $\Delta = 8\pi G\mu$ removed. The identified gravitational effects of the cosmic string depend upon the dimensionless quantity $G\mu$. The quantity $G\mu$ is a very important parameter characterising all cosmological effects of cosmic strings. The Newton's gravitational constant $G = m_p^{-2}$ where $m_p \sim 10^{19}$ GeV is the Planck mass and using $\mu \sim \sigma^2$ we can write

$$G\mu = \left(\frac{\sigma}{m_p}\right)^2 \quad \text{.....(2.24)}$$

As long as the symmetry breaking scale $\sigma << m_p$, $G\mu$ is a small number. For grand unification strings with $\sigma \sim 10^{16}$GeV, then $G\mu \sim 10^{-6}$ and the angular width Δ is a few arc seconds. In the co-ordinates (t, z, r, ϕ') the geodesics are just straight lines. Here we see that a particle initially at rest relative to the string will remains at rest and will not experience any gravitational attraction.

Although the metric (2.22) is locally flat, its global properties are different from that of Minkowski space. The conical nature of space around a straight cosmic string can give rise to some interesting properties. Let us consider a light source, say, quasar Q. Light rays from the quasar to an observer O are in Fig.1. It is clear shown from the figure that rays from the quasar Q intersect behind the string and the observer looks two images of the same quasar.

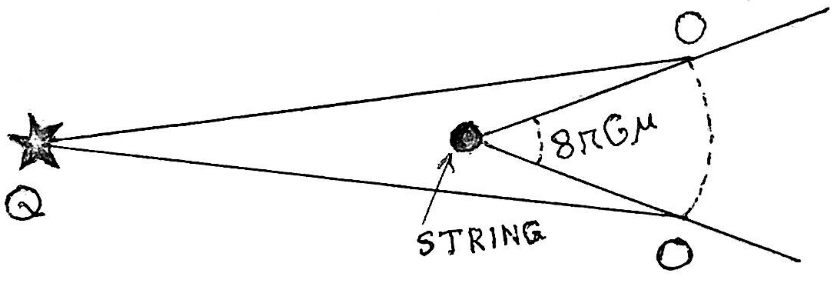

Fig. 1. Observer sees a double image of Quasar Q.

Cosmic strings can act as gravitational lenses, producing double images of distant galaxies and clusters. if the distances of the string from the quasar and the observer are l and d respectively then the angular separation between the two images is determined by

$$\sin\left(\frac{\delta\alpha}{2}\right) = \sin\left\{\left(\frac{\Delta\vartheta}{2}\right)\left(\frac{l}{d+l}\right)\right\}$$

$$\text{or } \delta\alpha = \Delta\vartheta\left(\frac{l}{d+l}\right)$$

$$\text{or } \delta\alpha = 8\pi G\mu\left(\frac{l}{d+l}\right)$$

Here the third equation is a small-angle approximation. Gravitational Lensing by a string is similar to Aharonov-Bohm effect [57]. Space-time curvature is limited to the string core, but its effect is "felt" by the photons propagating in flat space-time region around it.

CHAPTER - III

COSMIC STRING EVOLUTION AND STRUCTURE FORMATION

3.1 INTRODUCTION

In this chapter we would like to review cosmic string evolution and structure formation . The evolution of the network of cosmic strings is governed by string tension, Hubble expansion, intercommuting and gravitational radiation. The evolution of cosmic string networks has been studied by numerical simulation [54, 55]. A network of cosmic strings moving at a relativistic speed may generate a characteristic pattern of anisotropy in the temperature of the radiation. The upper limit of anisotropy places constraints on $G\mu$. After the universe becomes matter dominated, matter can begin to clump, and structure starts to form. Cosmic strings also should clump and participate in the formation of structure. Cosmic strings may provide the seeds for structure formation if $G\mu \sim 10^{-6}$ or so. A cosmic string dominated early universe will then be able to describe the present observed large scale structure of galaxies. Form time to time closed loop may intersect itself and break into two smaller loops. Oscillating closed loops can serve as point-like seeds for structure formation. The sheets that from in the wakes of long straight cosmic strings play an important role in structure formation.

3.2 COSMIC STRING EVOLUTION

Cosmological phase transition in the early universe can produce cosmic strings. These cosmic strings form a stochastic network permeating the entire universe. The initial network consists of both infinite cosmic strings (about 80% by length) and closed loops (about 20% by length). The loops will collapse, radiate energy and die out but the infinite strings will survive as they are protected by topology. The intercommutation of intersecting string segments leads to the continual chopping up of long strings into smaller loops. These loops oscillate and eventually decay into gravitational waves. The gradual loss of energy from strings into forms of radiation plays an important role in the evolution of the string network.. This is a very complicated system. Numerical simulation are the only reliable way to study it. As follows from numerical simulations, cosmic strings are belived to be formed with the shape of Brownian curves.

Cosmic stings have the shape of random walks of step ~ ξ with a typical distance between the neighboring string segments. Here, ξ is the correlation length of $\langle\Phi\rangle$. The simulation have the Abelian Higgs model (1.45), a spontaneously broken U(1) gauge theory. After the phase transition Φ develops a $\mathrm{VEV}\langle\Phi\rangle = \frac{\sigma e^{i\vartheta}}{\sqrt{2}}$, where ϑ varies on the scale of the correlation length ξ. There are few long strings extending across the lattice and there is a large number of small closed loops. A picture of a lattice is similar to the cubic volume. At the vertices of a cubic lattice the phase ϑ is randomly assigned. For simplicity the phase at the vertices is allowed to take only three values $\vartheta = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$. The size of the cubic lattice is identified with ξ. When a string passes through the face of cubic lattice, then ϑ changes by 2π around the face. It can be verified that all strings are either closed or end at the boundaries of the lattice.

To analyze the cosmological evolution of cosmic strings, it is important to know the frictional force experienced by moving strings due to their interaction with particles. The frictional force per unit length of a moving cosmic string with velocity v [58] is

$$F_s \sim \frac{N_s T^3 v}{\ln^2(T\Delta)} \quad \text{...(3.1)}$$

where N_s is the number of light particles interacting with the fields of the cosmic string, T is the temperature of the universe and Δ is the thickness of the string. At very early times the motion of cosmic strings is heavily damped by the frictional force (3.1). For roughly estimate omitting numerical and logarithmic factors in equation (3.1), we get the frictional force

$$F_s \sim T^3 v \qquad (3.2)$$

Tension in convoluted strings give a force per unit length

$$f \sim \frac{\mu}{R} \qquad (3.3)$$

where μ be the linear mass density and R be the local curvature radius of the cosmic string. As a result, the velocity of the cosmic string is obtained by the balance between tension and friction $f \sim F_s$:

$$v \sim \frac{\mu}{T^3 R} \qquad (3.4)$$

At time t, the typical curvature radius of strings is

$$R(t) \sim vt \qquad (3.5)$$

If we substitute this in equation (3.4) and use $T^4 \sim \rho \sim \frac{1}{Gt^2}$ then we write

$$R(t) \sim (G\mu)^{\frac{1}{2}} \left(\frac{t}{t_p} \right)^{\frac{1}{4}} t \quad \text{.......................................(3.6)}$$

where $t_p = 10^{-43}$ sec. is the plank time.

The frictional force is more important than the Hubble expansion as the matter density. However, with time, the matter density gets redshifted and the Hubble expansion dominates the frictional force. The Hubble expansion drag become comparable to that due to friction at the time $t_* \sim (G\mu)^{-2} t_p$

When $t > t_*$, then the characteristic scale of the string is $R(t) \sim t$. Therefore the force of tension is

$$f \sim \frac{\mu}{t} \quad \text{.......................................(3.7)}$$

and the frictional force is

$$F_s \sim \left(\frac{\mu}{t} \right) \left(\frac{t}{t_*} \right)^{-\frac{1}{2}} \quad \text{.......................................(3.8)}$$

So that for $t >> t_*$, the frictional force can be ignored [59]. For grand unification strings with $\sigma \sim 10^{16}$ GeV we get $G\mu \sim 10^{-6}$ and $t_* \sim 10^{-31}$ sec.

In the course of expansion of the universe the strings becomes less and straight on scales smaller than the horizon and they are being conformally stretched on scales greater than the horizon. The effects of expansion on large Brownian loops in a radiation-dominated universe are unimportant.

Cosmic strings move under their own tension and try to straighten out. This motion is damped due to the frictional force of the ambient matter [58,60] and is also slowed due to the Hubble expansion. Consider the string segment has an effective cross section ξ_0 per unit length, it will acquire a retarding force of order $\xi_0\,\rho v$, where v is the velocity of the cosmic string segment through a medium of relativistic particles or radiation and ρ is the energy density. As a result, we observe that for the cosmic string velocity the typical dissipation time is [20]

$$t_d \sim \frac{\mu}{\xi_0 \rho} \sim \frac{h\sigma^3}{NT^4} \quad \text{...(3.9)}$$

where N is the total number of distinct helicity states of low-mass particles and h is the Higgs coupling constant.

If initially this string cross section with a local curvature radius R is at rest then its initial acceleration $\sim \frac{\mu}{\mu R} = \frac{1}{R}$.

When the medium is compact $t_d << R$, then the string will earn a limiting velocity $\sim \frac{t_d}{R}$. In this case the loop of string will straightened out in a time of order $\frac{R^2}{t_d}$.

Not too long after the phase transition, the system of strings includes a scale invariant regime of evolution. The statistical properties of the string network in the regime do not alter with time. The only a fact that alters is the overall scale, which is set by the horizon scale t. The simulations indicate that the long strings have a significant small-scale structure on scales much smaller than t. The typical scale L of this structure is comparable to the size of the smallest loops. But the resolution of the simulations and their limited dynamical range do not accept a reliable determination of L. It is expected that the scale L is to satisfy the inequality

$$L >> \Gamma G \mu t \quad \text{...(3.10)}$$

where the coefficient Γ is numerically found to be order 100 for certain family of string loops [53].

Let us consider that initially a random tangle of strings exists. We may estimate the length scale L by spatial variations in $\langle\Phi\rangle$ so that the correlation length ξ at the Ginzburg temperature to be initially of order

$$L \sim \xi_G \sim \frac{1}{h^2 T_c} \quad \text{...(3.11)}$$

where T_c is the critical temperature.

The long strings are not smooth but have a lot of irregularities. The scale of the irregularities is guessed to be the same as the size of the loops $\sim \Gamma G\mu t$ at any time t. These irregularities are called kinks. The tension of the string will act on small kinks to straighten out. Sometimes this process will lead to a collision of a string with another string. These strings intersect and exchange partners, thus yielding new sharp kinks which straighten out in turn. Suddenly, small loops may shrink to a point and die out.

From (3.9) and (3.11) we observe that initially t_d << L. This validates the assumption made earlier. It is reasonable that the time scale for growth of L is $\frac{L}{t_d}$, i. e.,

$$\frac{1}{L}\frac{dL}{dt} \sim \frac{t_d}{L^2} \qquad \text{...(3.12)}$$

At the beginning, L grows like $t^{\frac{1}{2}}$ increasing the ratio $\frac{L}{t_d}$ rapidly. Covering a long time, from (3.9) we see that $t_d \propto t^2$ whence (3.12) yields $L \propto t^{\frac{3}{2}}$. Thus, eventually will catch up with L. When both t_d and L are of the same order as the age of the universe t; then it is not hard to verify that this will happen ; in fact when [20]

$$t_d \sim L \sim t \sim \frac{m_p^2}{h\sigma^3}$$

For strings which were present at the grand unification transition, we get $t \cong 10^{-27}$ sec, therefore this stage is reached long before the Weinberg-Salam transition. However, t depends sensitively on σ. When $\sigma \sim 10^{12}$ GeV then we get $t \cong 10^7$ sec. The Weinberg-Salam transition itself presumably does not produce cosmic strings, but if there were an intermediate transition not too far above it. This intermediate transition produce cosmic strings. The late stage may be relevant to theories of galaxy formation.

The scale size L of the tangle of strings grows until it is of the same order of magnitude as the distance t to the causal horizon. Then L cannot grow faster than t, but t_d continues to grow, therefore strings move with relativistic speeds.

J. H. Jeans discussed the basic mechanism of gravitational condensation. He showed that there is a minimum length scale demanded for a density fluctuations to grow [61] in any gravitational system. This length is called Jeans length and is defined as

$$\lambda_j \cong \frac{c_s}{\sqrt{G\rho}} \cong c_s t \qquad (3.13)$$

where c_s is the sound velocity. During the radiation-dominated era and before electron-proton recombination $c_s \cong \frac{1}{\sqrt{3}}$ Hence $c_s t$ is a proper fraction of the radius of the universe. Accordingly only very large-scale perturbations could start to grow in amplitude. Perturbations do not grow while they are outside the causal horizon but they can grow essentially linearly before coming inside the causal horizon. Until the Jeans scale becomes too large, perturbation-generating process continues for only a short time.

After the recombination era, quite occasionally c_s falls to the value

$$c_s \cong \left(\frac{5T}{3m_H}\right)^{\frac{1}{2}} \quad \text{...(3.14)}$$

typical of hot hydrogen gas.

During the plasma era, photon scattering maintains isothermal conditions. The adiabatic perturbation on a galactic scale or less swiftly disappear, but any isothermal density fluctuation-will remain. There is no effective damping mechanism, and so the small-scale density fluctuations are not erased. The density fluctuations may have triggered the formation of galaxies. The string scenario of galaxy formation is discussed in the next section.

3.3 STRUCTURE FORMATION

A long-standing cosmological mystery is the origin of structure in the universe. When the universe become matter-dominated then the formation of structure began. The knowledge of the present distribution of matter in the universe is crucial to understanding the origin of structure in the universe. In recent years, it has developed to testing the detailed scenarios of structure formation. The formation of structure is often simply referred to as galaxy formation. On small scales the density inside a galaxy is about 10^5 times the average density of the universe, and that inside a cluster of galaxies is about 10^2 to 10^3 times the average density of the universe. Of course on very large scales (greater than 100 Mpc), the universe is smooth, as evidenced by the isotropy of the CMBR, the isotropy of the X-ray background, and number counts of radio sources. Lappernt, Geller and Huchra [62] prove that galaxies are distributed on sheets and filaments with a typical scale $\sim 25h^{-1}\mathrm{Mpc}$. In general, this structure has evolved from small density fluctuations in the early universe, but the nature of the seeds fluctuations is unknown. There is strong observational evidence that the bulk of the material in the universe today should be non-baryonic. We have a very long list of candidate relics whose present energy density can produce closure density. A natural candidate for role of dark matter would be neutrino (say γ_μ) with a small mass ~ 30 eV. Neutrino dark matter is called hot dark matter because it remains relativistic until a very later epoch. In a hot dark matter model the formation of galaxies is necessarily a rather complicated process, involving the hydrodynamics and thermodynamics of shocked material. Cold dark matter is non-relativistic when all cosmological intersecting scales enter the horizon. It is very successful scenario for structure formation. It is motivated and is able to reproduce most of the feature of the observed universe. Several groups [63, 64] have numerical simulated structure formation in a neutrino-dominated universe. The neutrinos being so weakly interacting do not collide with one another or the baryons. Fluctuations in the neutrino density are washed out as neutrinos simply stream out of overdensed regions. The neutrinos cannot waste their gravitational energy, and therefore cannot collapse into lightly bound objects. Of course some slowly moving neutrinos may subsequently be captured by the baryon-dominated galaxies. At later times if the neutrino velocities are significant red-shifted by Hubble expansion, then these fluctuations begin to grow. As a result, galaxies can form only very late at $z \sim 1$, by fragmentation of super-cluster-size objects. Cosmic strings can act as seeds for structure formation in several ways.

(i) Wakes formed behind rapidly moving long strings can help to explain the structure formation.

(ii) Slowly moving wiggly strings accrete filamentary structure by their gravitational potentials $\Phi \sim -G\delta\mu \ln r$

where $\delta\mu$ is the extra mass per unit length due to the wiggles [65,66].

(iii) Closed loops would tend to oscillate and collapse rapidly. The rapid motions of oscillating and collapsing loops may have triggered the formation of galaxies.

The first two mechanism naturally lead to galaxy distribution along sheets and filaments. Since the size of the loops are very small, hence the last mechanism is probably unimportant. The difference between sheets and filaments is in string velocity v. Sheets are produced by the first-moving strings and filaments are produce by the slow moving strings. Cosmic string characterized by $G\mu \sim 10^{-6}$ or so provides a potentially viable means of seeding structure formation in the universe. Cosmic string may play an important role in structure formation. This possibility has spurred a very active area of research in recent years [67,68].

Cosmic strings produce density fluctuations which are not in the form of waves with random phases. This can explain the observed deviations from the Gaussian behaviour. Effect of cosmic strings which arise from the conical structure of the space-time around a string are string wakes. The wake formed behind a straight cosmic string has the shape of a wedge with an opening angle ~$8\pi G\mu$. Cosmic string loops or flattened structures formed in the wakes of cosmic strings can possibly serve as seeds to initiate structure formation in the universe. Consider a cosmic string wake moving through the universe with velocity v_s. This wake formed by a string segment of length L at time t_i. Hence the initial dimensions of the wake are

$$t_i \times v_s t_s \qquad (3.15)$$

The typical distance to nearest wakes formed at the same time is~t_i. As the universe expands, the length and width of the wake grow like the scale factor $R(t) \sim t^{\frac{2}{3}}$, while the total mass of the wake grows by gravitational instability like $M \propto R(t)$. Therefore the widths of the wake at the present time are

$$t_i z_i \times v_s t_i z_i \qquad (3.16)$$

where z_i is the redsheft at t_i. In a universe which dominated by light neutrinos, wake perturbations are damped by neutrino free streaming on co-moving scales smaller than $\lambda_\nu(t) \sim v_\nu(t)t$ where $v_\nu(t) \cong v_{eq}\left(\frac{t_{eq}}{t}\right)^{\frac{2}{3}}$ is the rms velocity of neutrinos and $v_{eq} \approx 0.2$. The time of matter-radiation equality is the initial epoch for structure formation. Wakes formed well before the time of matter-radiation equality are completely washed out. After the time of matter-radiation wakes formed eventually collapse. The nonlinear structure formation is delayed until the time when the transverse dimension of the overlapping streams of matter in the wake becomes greater than $\lambda_\nu(t)$. Here, $\lambda_\nu(t)$ plays the role of the neutrino Jeans length.

The characteristic scale of the large scale structure in this scenario is

$$t_{eq} z_{eq} \sim 10h^{-2}\ \mathrm{Mpc} \qquad (3.17)$$

With $h = 0.5$ it is comparable to the scale-suggested by observations [62] (~25h^{-1} Mpc). The surface density of the neutrino wakes produce subsequently decrease but decrease is only $\propto t_i^{-\frac{1}{3}}$. The large scale wake could be prominent simply because it rearranges the small scale structure into sheets and filaments.

In a universe dominated by neutrinos, baryonic wakes start collapsing after baryons decouple from radiation, $t > t_{dec}$. However, the growth of these wakes is strongly suppressed, since baryons constitute only a small fraction of the total density. Baryonic wakes could explain the existence of quasars of large redshefts. The scale of baryonic wakes is

$$t_{dec} z_{dec} \sim 50 h^{-1}\,\text{Mpc}. \quad (3.18)$$

It is comparable to the largest-scale structure observed in the universe.

A novel outcome of the cosmic string which predicts the generation of primordial magnetic fields is vorticity [65, 69, 70]. After decoupling of matter and radiation, the relativistic motion of strings persuades vorticity in the baryonic fluid. Then the vorticity leads to the generation of primordial magnetic fields. The existence of vorticity in the baryonic fluid flow indicates protons and electrons are in vortical motion. But we also have ambient photons and neutral particles which interact with the protons and electrons. There is an electric current. The resulting electric current produce a magnetic field. This seed field can be further amplified by turbulence in the wake and by a galactic dynamo.

The density fluctuations due to strings are balanced by the corresponding variations in matter and radiation density on scales greater than the horizon. On such scales the cosmic string scenario of structure formation assumes that the universe is initially homogeneous and isotropic. Such initial conditions can be explained if we assume that there was period inflation before the string formation.

CHAPTER - IV

A GENERAL CLASS OF SPACETIMES HAVING HORIZON

Plebanski [71] studied a class of solutions of Einstein-Maxwell equations. In Boyer co-ordinates (p, σ, q, τ) these solutions are given by

$$ds^2 = \frac{p^2+q^2}{x}dp^2 + \frac{x}{p^2+q^2}(d\tau + q^2 d\sigma)^2 + \frac{p^2+q^2}{y}dq^2 - \frac{y}{p^2+q^2}(d\tau - p^2 d\sigma)^2 \quad \text{.......(4.1a)}$$

where

$$x = x(p) = b - g^2 + 2np - \in p^2 - \left(\frac{\lambda}{3}\right)p^4 \quad \text{.......(4.1b)}$$

$$y = y(q) = b + e^2 - 2Mq + \in q^2 - \left(\frac{\lambda}{3}\right)q^4 \quad \text{.......(4.1c)}$$

with electric potential

$$A_\mu dx^\mu = \frac{eq}{p^2+q^2}\left(d\tau - p^2 d\sigma\right) \quad \text{.......(4.2)}$$

Besides the cosmological constant λ, the metric (4.1) includes six parameters b, e, g, M, n, ε. Under the proper co-ordinate transformation along with the suitable adjustment of the kinetical parameters b and ε, the metric (4.1) gives many physically interesting solutions of Einstein or Einstein-Maxwell equations. The surfaces y = 0 at which the metric exhibits apparent singularity have been interpreted as horizon.

4.1 PROPERTIES OF THE PLEBANSKI SPACE-TIME

The electromagnetic field associated with the metric is given by

$$W = -d\left\{\frac{e+ig}{q+ip}(d\tau - ipq\,d\sigma)\right\} \quad \text{.........(4.3)}$$

The components of the Weyl tensor $C^{(a)}$ are

$$C^{(1)} = C^{(2)} = C^{(4)} = C^{(5)} = 0$$

$$C^{(3)} = \frac{1}{6}\frac{\ddot{x}+\ddot{y}}{p^2+q^2} - \frac{1}{p^2+q^2}\left\{\frac{\dot{y}+i\dot{x}}{q+ip} - \frac{2(y-x)}{(q+ip)^2}\right\} \quad \text{.........(4.4)}$$

The components of Ricci tensor R_{ab} are

$$R_{11}=R_{22}=R_{13}=R_{33}=R_{14}=R_{44}=R_{23}=R_{24}=0$$

$$R_{12}=\frac{1}{2}\frac{\ddot{x}}{p^2+q^2}-\frac{1}{p^2+q^2}\left\{(p\dot{x}-q\dot{y})-(x-y)\right\}\qquad(4.5)$$

$$R_{34}=\frac{1}{2}\frac{\ddot{y}}{p^2+q^2}+\frac{1}{p^2+q^2}\left\{(p\dot{x}-q\dot{y})-(x-y)\right\}$$

The scalar curvature R is

$$R=\frac{\ddot{x}+\ddot{y}}{p^2+q^2}=2(R_{12}+R_{34})\qquad(4.6)$$

where here and above dots denote differentiation with respect to the argument.

Now we turn to the equation (4.4). This equation can be simplified further for $C^{(3)}$. Using equations (4.1b) and (4.1c) we obtain from (4.4)

$$C^{(3)}=\frac{-2}{(q^2+p^2)(q+ip)^2}\left[Mq+np-e^2-g^2-i(Mp-nq)\right]\qquad(4.7)$$

Consequently if any of the constants M, n, e, g is not zero, the metric (4.1) is of Petrov type D. In the case $C^{(3)}=0$, the metric is conformally flat. In addition to

$C^{(a)} = 0$; if also $R_{ab} = 0$, then the metric becomes flat. The metric is asymptotically de Sitter when $\lambda \neq 0$ but asymptotically flat when $\lambda = 0$.

A large number of solutions can be obtained by contracting the metric (4.1) by appropriate limiting procedures.

4.2 CONTRACTIONS OF THE P LEBANSKI SPACE-TIME

(a) If we perform a simple co-ordinate transformation from $x^{\mu} = (p, \sigma, q, \tau)$ to $x'^{\mu} = (p', \sigma', q', \tau')$ by defining

$$p = p_o + \epsilon_o\, p', \sigma = \frac{\sigma'}{\epsilon_o}, q = q', \tau = \tau' + \frac{(p_o^2 \sigma')}{\epsilon_o} \quad \text{.............................(4.8)}$$

where p_o is an arbitrary constant and ϵ_o denotes the contraction parameter, the metric (4.1) becomes

$$ds^2 = \frac{\{(p_o + \epsilon_o\, p')^2 + q'^2\}}{\epsilon_o^{-2}\, x(p_o + \epsilon_o\, p')} dp'^2$$

$$+\frac{\epsilon_0^{-2}\, x(p_0+\epsilon_0\, p')}{\{(p_0+\epsilon_0\, p')^2+q'^2\}}\{\epsilon_0\, d\tau'+(p_0^2+q'^2)d\sigma'\}^2$$

$$+\frac{1}{\{(p_0+\epsilon_0\, p')^2+q'^2\}^{-1} y(q')}dq'^2$$

$$-\{(p_0+\epsilon_0\, p')^2+q'^2\}^{-1} y(q')\{d\tau'-(2p_0p'+\epsilon_0\, p'^2)d\sigma'\}^2 \quad \text{.....................(4.9a)}$$

where

$$\epsilon_0^{-2}\, x(p_0+\epsilon_0\, p')$$

$$=\epsilon_0^{-2}\left\{b-g^2+2n(p_0+\epsilon_0\, p')-\epsilon(p_0+\epsilon_0\, p')^2-\left(\frac{\lambda}{3}\right)(p_0+\epsilon_0\, p')^4\right\} \quad \text{............(4.9b)}$$

$$\{(p_0+\epsilon_0\, p')^2+q'^2\}^{-1} y(q')$$

$$=\{(p_0+\epsilon_0\, p')^2+q'\}^{-1}(b+e^2-2Mq'+\epsilon\, q'^2-\frac{\lambda}{3}q'^4) \quad \text{..........(4.9c)}$$

Simplifying we can write equation (4.9) in the form

$$ds^2=\frac{\{(p_0+\epsilon_0\, p')^2+q'^2\}}{x_1}dp'^2+\frac{x_1}{\{(p_0+\epsilon_0\, p')^2+q'^2\}}\{\epsilon_0\, d\tau'+(p_0^2+q'^2)d\sigma'\}^2$$

$$+\frac{dq'^2}{y_1}-y_1\{d\tau'-(2p_0p'+\epsilon_0\, p'^2)d\sigma'\}^2 \quad \text{.................................(4.10a)}$$

where

$$x_1 = \epsilon_0^{-2}\, x(p_0 + \epsilon_0\, p') = \alpha_0 + 2\beta_0 p' - \gamma_0 p'^2 - \frac{4}{3}\lambda p_0\, \epsilon_0\, p'^3 - \frac{\lambda}{3}\epsilon_0^2\, p'^4 \quad \text{.........(4.10b)}$$

$$y_1 = \left\{(p_0 + \epsilon_0\, p')^2 + q'^2\right\}^{-1} y(q')$$

$$= \left\{(p_0 + \epsilon_0\, p')^2 + q'^2\right\}(\alpha_0\, \epsilon_0^2 - 2\beta_0 p_0\, \epsilon_0 - \gamma_0 p_0^2 + g^2 + \lambda p_0^4$$

$$+ e^2 - 2Mq' + \epsilon\, q'^2 - \frac{\lambda}{3} q'^4) \quad \text{...............(4.10c)}$$

$$\alpha_0 = \epsilon_0^{-2}\left(b - g^2 + 2np_0 - \epsilon\, p_0 - \frac{\lambda}{3} p_0^4\right) \text{................................(4.10d)}$$

$$\beta_0 = \epsilon_0^{-2}\left(n - \epsilon\, p_0 - \frac{2\lambda}{3} p_0^3\right) \text{...(4.10e)}$$

$$\gamma_0 = \epsilon + 2\lambda p_0^2 \text{...(4.10f)}$$

with α_0, β_0 and γ_0 being constants independent on ϵ_0.

Now if $\epsilon_0 \longrightarrow 0$, then we have from (4.10)

$$ds^2 = \left(p_0^2 + q'^2\right)\left(\frac{dp'^2}{x_1} + x_1 d\sigma'^2\right) + \frac{dq'^2}{y_1} - y_1(d\tau' - 2p_o p' d\sigma')^2 \quad \text{......(4.11a)}$$

where

$$x_1 = \alpha_o + 2\beta_o p' - \gamma_o p'^2 \quad \text{..(4.11b)}$$

$$y_1 = \left(p_0^2 + q'^2\right)\left(g^2 - \gamma_o p_o^2 + \lambda p_0^4 + e^2 - 2Mq' + \in q'^2 - \frac{\lambda}{3} q'^4\right) \text{...........(4.11c)}$$

The metric (4.11) can ultimately be put in the form

$$ds^2 = \left(p_o^2 + q'^2\right)\left(\frac{dp'^2}{x_1} + x_1 d\sigma'^2\right) + \frac{dq'^2}{y_1} - y_1(d\tau' - 2p_o p' d\sigma')^2 \quad \text{.......(4.12a)}$$

where

$$x_1 = \alpha_o + 2\beta_o p' - \gamma_o p'^2 \text{..(4.12b)}$$

$$y_1 = \gamma_o - \frac{\lambda}{3}(q'^2 + 5p_o^2) - 2\,\mathrm{Re}\left(\frac{M + in_o}{q' + ip_o}\right) + \left|\frac{e + ig}{q' + ip_o}\right|^2 \quad \text{.............(4.12c)}$$

$$n_o = \gamma_o p_o - \frac{4\lambda}{3} p_o^3 \quad \text{..(4.12d)}$$

The contracted solution given by equation (4.12) in the co-ordinates $x'^\mu = (p', \sigma', q', \tau')$ is the generalised NUT solution.

(b) If we use the co-ordinate transformation from $x^{\mu} = (p, \sigma, q, \tau)$ to $x'^{\mu} = (p', \sigma', q', \tau')$ defined by

$$p = p', \sigma = \frac{\sigma'}{\epsilon_0}, q = q_0 + \epsilon_0 q', \tau = \tau' - \frac{(q_0^2 \sigma')}{\epsilon_0} \quad \text{.........................(4.13)}$$

where q_0 is arbitrary constant and ϵ_0 is the contraction parameter.

Under the co-ordinate transformation (4.13) the metric (4.1) reduce to the form

$$ds^2 = \frac{1}{\left\{p'^2 + \left(q_0 + \epsilon_0 q'\right)^2\right\}^{-1} x(p')} dp'^2$$

$$+ \left\{p'^2 + \left(q_0 + \epsilon_0 q'\right)^2\right\}^{-1} x(p') \left\{d\tau' + \left(2q_0 q' + \epsilon_0 q'^2\right) d\sigma'\right\}^2$$

$$+ \frac{\left\{p'^2 + \left(q_0 + \epsilon_0 q'\right)^2\right\}}{\epsilon_0^{-2} \, y\left(q_0 + \epsilon_0 q'\right)} dq'^2$$

$$- \frac{\epsilon_0^{-2} \, y\left(q_0 + \epsilon_0 q'\right)}{\left\{p'^2 + \left(q_0 + \epsilon_0 q'\right)^2\right\}} \left\{\epsilon_0 \, d\tau' - \left(q_0^2 + p'^2\right) d\sigma'\right\}^2 \quad \text{.....................................(4.14a)}$$

where

$$\left\{p'^2+(q_o+\epsilon_o q')^2\right\}^{-1} x(p')$$

$$=\left\{p'^2+(q_o+\epsilon_o q')^2\right\}^{-1}\left(b-g^2+2np'-\epsilon p'^2-\frac{\lambda}{3}p'^4\right) \quad \text{(4.14b)}$$

$$\epsilon_o^{-2}\, y(q_o+\epsilon_o q')$$

$$=\epsilon_o^{-2}\left\{b+e^2-2M(q_o+\epsilon_o q')+\epsilon(q_o+\epsilon_o q')^2-\frac{\lambda}{3}(q_o+\epsilon_o q')^4\right\} \quad \text{(4.14c)}$$

Simplifying we can write equation (4.14) in the form

$$ds^2=\frac{dp'^2}{x_2}+x_2\left\{d\tau'+(2q_o q'+\epsilon_o q'^2)d\sigma'\right\}^2$$

$$+\frac{p'^2+(q_o+\epsilon_o q')^2}{y_2}dq'^2$$

$$-\frac{y_2}{p'^2+(q_o+\epsilon_o q')^2}\left\{\epsilon_o\, d\tau'-(q_o^2+p'^2)d\sigma'\right\}^2 \quad \text{(4.15a)}$$

where

$$y_2=\epsilon_o^{-2}\, y(q_o+\epsilon_o q')$$

$$=\alpha_1+2\beta_1 q'+\gamma_1 q'^2-\frac{4}{3}\lambda q_0 q'\,\epsilon_o-\frac{1}{3}\lambda q'^4\,\epsilon_o^2 \quad \text{(4.15b)}$$

$$x_2=\left\{p'^2+(q_o+\epsilon_o q')^2\right\}^{-1} x(p')$$

$$= \left\{p'^2 + \left(q_o + \epsilon_o\, q'\right)^2\right\}^{-1} \{\alpha_1 \epsilon_o^2 - 2\beta_1 q_o \epsilon_o + \gamma_o q_o^2 + \lambda q_0^4\}$$

$$-e^2 - g^2 + 2np' - \epsilon\, p'^2 - \frac{1}{3}\lambda p'^4\} \qquad \text{(4.15c)}$$

$$\alpha_1 = \epsilon_0^{-2}\left(b + e^2 - 2Mq_0 + \epsilon\, q_0^2 - \frac{1}{3}\lambda q_0^4\right) \qquad \text{(4.15d)}$$

$$\beta_1 = \epsilon_0^{-1}\left(\epsilon\, q_0 - M - \frac{2}{3}\lambda q_0^3\right) \qquad \text{(4.15e)}$$

$$\gamma_1 = \epsilon - 2\lambda q_o^2 \qquad \text{(4.15f)}$$

with α_1, β_1 and γ_1 being constants independent on ϵ_0.

Now if $\epsilon_o \to 0$, then from (4.15) we have

$$ds^2 = \frac{dp'^2}{x_2} + x_2\left(d\tau' + 2q_o q' d\sigma'\right)^2$$

$$+\left(q_o^2 + p'^2\right)\left(\frac{dq'^2}{y_2} - y_0 d\sigma'^2\right) \qquad \text{(4.16a)}$$

$$y_2 = \alpha_1 + 2\beta_1 q' + \gamma_1 q'^2 \qquad \text{(4.16b)}$$

$$x_2 = \left(p'^2 + q_0^2\right)^{-1}\left\{\gamma_0 q_0^2 + \lambda q_0^4 - e^2 - g^2 + 2np' + \in p'^2 - \frac{1}{3}\lambda p'^4\right\} \quad \text{.........(4.16c)}$$

The metric (4.16) can ultimately be put into the form

$$ds^2 = \frac{dp'^2}{x_2} + x_2\left(d\tau' + 2q_0 q' d\sigma'\right)^2$$

$$+\left(q_0^2 + p'^2\right)\left(\frac{dq'^2}{y_2} - y_2 d\sigma'^2\right) \quad \text{...(4.17a)}$$

where

$$y_2 = \alpha_1 + 2\beta_1 q' + \gamma_1 q'^2 \quad \text{..(4.17b)}$$

$$x_2 = -\gamma_1 - \frac{\lambda}{3}\left(p'^2 + 5q_0^2\right) + 2\,\mathrm{Re}\left(\frac{m_0 + in}{q_0 + ip'}\right) + \left|\frac{e + ig}{q_0 + ip'}\right|^2 \quad \text{..............(4.17c)}$$

$$m_0 = \gamma_1 q_0 + \frac{4}{3}\lambda q_0^3 \quad \text{...(4.17d)}$$

This contracted solution given by (4.17) is the generalised anti-NUT solution.

4.3 CANONICAL FORMS OF THE CONTRACTED SOLUTIONS

(a) Let us consider the generalized NUT solution given by (4.12) and restrict the parameter γ_0 associated with it to the discrete values

$\gamma_0 = 1, 0, -1$

Case I : $\gamma_0 = 1$

If we put $\alpha_0 = 1, \beta_0 = 0$ and consider co-ordinate transformation from $x'^{\mu} = (p', \sigma', q', \tau')$ to $x''^{\mu} = (\theta, \phi, q', t')$ defined by

$$p' = \cos\theta, \sigma' = \phi, q' = q', \tau' = t' \qquad \text{.....(4.18)}$$

then the metric given by (4.12) takes the form

$$ds^2 = \left(p_0^2 + q'^2\right)\left(d\vartheta^2 + \sin^2\vartheta d\phi^2\right) + \frac{dq'^2}{y_1} - y_1\left(dt' - 2p_0\cos\vartheta d\phi\right)^2 \qquad \text{.......(4.19a)}$$

where

$$y_1 = 1 - \frac{\lambda}{3}\left(q'^2 + 5p_0^2\right) - 2\,\mathrm{Re}\left(\frac{M + in_0}{q' + ip_0}\right) + \left|\frac{e + ig}{q' + ip_0}\right|^2 \qquad \text{..............(4.19b)}$$

$$n_0 = p_0 - \frac{4}{3}\lambda p_0^3 \qquad \text{.....(4.19c)}$$

Case II : $\gamma = 0$

If we set $\alpha_0 = 1$, $\beta_0 = 0$ and introduce new co-ordinates

$p' = \vartheta \cos\phi, \sigma' = \vartheta \sin\phi,\ \tau' = t' + p_0 p' \sigma'$

then the metric given by (4.12) reduces to the form

$$ds^2 = \left(p_0^2 + q'^2\right)\left(d\vartheta^2 + \vartheta^2 d\phi^2\right) + \frac{dq'^2}{y_1} - y_1\left(dt' + p_0 \vartheta^2 d\phi\ \right)^2 \qquad (4.21a)$$

where

$$y_1 = -\frac{\lambda}{3}\left(q'^2 + 5p_0^2\right) - 2\,\mathrm{Re}\left(\frac{M + in_0}{q' + ip_0}\right) + \left|\frac{e + ig}{q' + ip_0}\right|^2 \qquad (4.21b)$$

$$n_0 = -\frac{4}{3}\lambda p_0^3 \qquad (4.21c)$$

Case III : $\gamma_0 = -1$

If we set $\alpha_0 = -1, \beta_0 = 0$ and introduce new co-ordinates

$$p' = \cosh\vartheta, \alpha' = \phi, \tau' = t' \qquad (4.22)$$

then we have (4.12) as

$$ds^2 = \left(p_0^2 + q'^2\right)\left(d\vartheta^2 + \sinh^2\vartheta d\phi^2\right) + \frac{dq'^2}{y_1} - y_1\left(dt' - 2p_0 \cosh\vartheta d\phi\right)^2 \text{....(4.23a)}$$

where

$$y_1 = -1 - \frac{1}{3}\lambda\left(q'^2 + 5p_0^2\right) - 2\mathrm{Re}\left(\frac{M + in_0}{q' + ip_0}\right) + \left|\frac{e + ig}{q' + ip_0}\right|^2 \text{.........(4.23b)}$$

$$n_0 = -p_0 - \frac{4}{3}\lambda p_0^3 \text{.......................................(4.23c)}$$

The equations (4.19), (4.21) and (4.23) are the canonical representations of the generalized NUT solutions.

Now we would like to provide some comments concerning the interpretation of the generalized NUT solution described in canonical forms.

If $p_0 = 0$, then we have from (4.19)

$$ds^2 = q'^2\left(d\vartheta^2 + \sin^2\vartheta d\phi^2\right) + \frac{dq'^2}{y_1} - y_1 dt'^2 \text{.............(4.24a)}$$

$$y_1 = 1 - \frac{\lambda}{3}q'^2 - \frac{2M}{q'} + \frac{e^2 + g^2}{q'^2} \quad \text{..........................(4.24b)}$$

If g = 0 we recognize (4.24) as the Reissner-Nordstrom solution with the cosmological constant λ . The co-ordinate q' plays the role of the radial variable. The constant M and e are interpreted as mass and charge associated with this solution. If g is different from zero, then (4.24) represents a slight generalization of the cosmological Reisner-Nordstrom solution : g is interpreted as magnetic charge. For e = g = 0 the solution given by (4.24) becomes the Schwarzschild solution generalized by the cosmological constants. Further with e = g = λ =0, we obtained from (4.24), the basic Schwarzschild solution.

If $p_0 \neq 0$ but e = g = 0, then the solution given by (4.19) reduces to the form

$$ds^2 = \left(p_0^2 + q'^2\right)\left(d\vartheta^2 + \sin^2\vartheta d\phi^2\right) + \frac{dq'^2}{y_1} - y_1\left(dt' - 2p_0\cos\vartheta d\phi\right)^2 \quad \text{.......(4.25a)}$$

where

$$y_1 = 1 - \frac{\lambda}{3}\left(q'^2 + 5p_0^2\right) - 2\mathrm{Re}\left(\frac{M + in_0}{q' + ip_0}\right) \quad \text{.......................(4.25b)}$$

$$n_0 = p_0 - \frac{4}{3}\lambda p_0^3 \quad \text{...(4.25c)}$$

The solution Given by (4.25) is the NUT solution generalized by the presence of the cosmological constant. The parameter n_0 coincides with p_0 when $\lambda = 0$. This parameter is the NUT parameter or magnetic mass parameter.

It is clear that the generalized family of NUT solutions described in the canonical form by (4.19) and obtained by a contraction from Plebanski space-time (4.1) represents the combined NUT-Reissner-Nordstrom solution with the cosmological constant, additionally generalied by the possible presence of magnetic monopole. The parameters e and g have interpretation of the electric and magnetic charges; M and n_0 have interpretation of the mass and the NUT parameter.

If we put $p_0 = 0$ in (4.21) then we have

$$ds^2 = q'^2\left(d\vartheta^2 + \vartheta^2 d\phi^2\right) + \frac{dq'^2}{y_1} - y_1 dt'^2 \quad \text{.....................(4.26a)}$$

where

$$y_1 = -\frac{\lambda}{3}q'^2 - \frac{2M}{q'} + \frac{e^2 + g^2}{q'^2} \quad \text{.................................(4.26b)}$$

Equation (4.26) is the Kasner-type space-time, an anisotropic universe. Setting $\lambda = e = g = o$, equation (4.26) can be transformed to Kasner form.

Now if we put $p_0 = 0$ in (4.23) then we get

$$ds^2 = q'^2\left(d\vartheta^2 + \sinh^2\vartheta d\phi^2\right) + \frac{dq'^2}{y_1} - y_1 dt'^2 \quad \text{.......................(4.27a)}$$

where

$$y_1 = -1 - \frac{\lambda}{3}q'^2 - \frac{2M}{q'} + \frac{e^2 + g^2}{q'^2} \quad \text{....................................(4.27b)}$$

The solution (4.27) presents Levi-Civita's type of generalization of the cosmological Reissner-Noedstrom solution with charges of both types. The Levi-civita's metric is sometimes interpreted as the metric of a heavy tachyon.

(b) Now we consider "generalized anti-NUT solutions" given by (1.17) and restrict the parameter γ_1 to the discrete values

$\gamma_1 = 1,\ 0,\ -1$

Case I: $\gamma_1 = 1$

If we set $\alpha_1 = 1, \beta_1 = 0$ and consider the transformation $x'^2 = (p', \sigma', q', \tau')$ to $x''^\mu = (p', \phi, \vartheta, \tau')$ defined by

$$p' = p', \sigma' = \phi, q' = \sinh\vartheta, \tau' = \tau' \quad \text{...................................(4.28)}$$

then (4.17) reduce to the form

$$ds^2 = \frac{dp'^2}{x_2} + x_2\left(d\tau' + 2q_0 \sinh\vartheta d\phi\right)^2 + \left(q_0^2 + p'^2\right)\left(d\vartheta^2 - \cosh\vartheta d\phi^2\right) \quad \text{......(4.29a)}$$

where

$$x_2 = -1 - \frac{\lambda}{3}\left(p'^2 + 5q_0^2\right) + 2\mathrm{Re}\left(\frac{m_0 + in}{q_0 + ip'}\right) - \left|\frac{e + ig}{q_0 + ip'}\right|^2 \quad \text{................(4.29b)}$$

$$m_0 = q_0 + \frac{4}{3}\lambda q_0^3 \quad \text{...(4.29c)}$$

Case II: $\gamma_1 = 0$

If we now take $\alpha_1 = 1, \beta_1 = 0$ and introduce new coordinates $x''^{\mu} = (p', y, x, \tau')$ for $x'^{\mu} = (p', \sigma', q', \tau')$

defined by

$$p' = p', \sigma' = y, q' = x, \tau' = \tau' \quad \text{..(4.30)}$$

then (4.17) reduces to the form

$$ds^2 = \frac{dp'^2}{x_2} + x_2\,(d\tau' + 2q_0 x dy)^2 + (q_0^2 + p'^2)(dx^2 - dy^2) \quad \text{.............(4.31a)}$$

where

$$x_2 = -\frac{\lambda}{3}\left(p'^2 + 5q_0^2\right) + 2\,\mathrm{Re}\left(\frac{m_0 + in}{q_0 + ip'}\right) - \left|\frac{e + ig}{q_0 + ip'}\right|^2 \quad \text{............(4.31b)}$$

$$m_0 = \frac{4}{3}\lambda q_0^3 \quad \text{..(4.31c)}$$

Case III:

$\lambda_1 = -1$

In this case if we set $\lambda_1 = -1$, $\beta_1 = -1$ and consider the transformation defined by

$$p' = p', \sigma' = \phi, q' = \sinh\vartheta, \tau' = \tau'$$

then we have (4.17) as

$$ds^2 = \frac{dp'^2}{x_2} + x_2\left(d\tau' + 2q_0 \sinh\vartheta d\phi\right)^2$$
$$-\left(q_0^2 + p'^2\right)\left(d\vartheta^2 - \cosh^2\vartheta d\phi^2\right) \quad \text{................................(4.33a)}$$

where

$$x_2 = 1 - \frac{\lambda}{3}\left(p'^2 + 5q_0^2\right) + 2\,\mathrm{Re}\left(\frac{m_0 + in}{q_0 + ip'}\right) - \left|\frac{e + ig}{q_0 + ip'}\right|^2 \quad \text{........................(4.33b)}$$

$$m_0 = -q_o + \frac{4}{3}\lambda q_0^3 \qquad \text{(4.33c)}$$

Equations (4.29) , (4.31) and (4.33) are the canonical representations of the anti-NUT solution.

If we set $e = g = q_0 = \lambda = 0$, then (4.33) becomes to the form

$$ds^2 = \frac{dp'^2}{x_2} + x_2\, d\tau'^2 - p'^2(d\vartheta^2 - \cosh^2\vartheta d\phi^2) \qquad \text{(4.34a)}$$

where

$$x_2 = 1 + \frac{2n}{p'} \qquad \text{(4.34b)}$$

which is a solution of Einstein's equation in vacuum.

4.4 THE COMBIND NUT-KERR-NEWMAN-KASUYA SPACE-TIME

If we take

$$\epsilon = 1, \lambda = 0, b = a^2 - n^2 + g^2 \qquad \text{(4.35)}$$

then the metric (4.1) reduces to the form

$$ds^2 = \frac{p^2+q^2}{x}dp^2 + \frac{x}{p^2+q^2}(d\tau + q^2 d\sigma)^2 + \frac{p^2+q^2}{y}dq^2 - \frac{y}{p^2+q^2}(d\tau - p^2 d\sigma)^2 \,.. \qquad \text{......(4.36a)}$$

where

$$x = a^2 - (n-p)^2 \qquad \text{.....(4.36b)}$$

$$y = q^2 - 2Mq + a^2 - n^2 + e^2 + g^2 \qquad \text{.....(4.36c)}$$

and the parameter 'a' gives the interpretation of angular momentum per unit mass. The above equation represents the combined NUT-Kerr-Newman-Kasuya space-time in Boyer co-ordinates. If we set $n = e = g = 0$ then the equation (4.36) reduce to the kerr space-time [71].

The metric (4.36) takes the form

$$ds^2 = \Sigma d\vartheta^2 + \frac{\Sigma}{y}dr^2 + \frac{\sin^2\vartheta}{\Sigma}(a dt - \rho d\phi)^2 - \frac{y}{\Sigma}(dt - A d\phi)^2 \qquad \text{.............(4.37a)}$$

where

$$\Sigma = r^2 + (n + a\cos\vartheta)^2$$

$$y = r^2 - 2Mr + a^2 - n^2 + e^2 + g^2 \qquad \text{.......................(4.37b)}$$

$$\rho = r^2 + a^2 + n^2$$

$$A = a\sin^2\vartheta - 2n\cos\vartheta$$

by the co-ordinate transformation

$$p = n + a\cos\vartheta$$

$$q = r$$

$$\sigma = -\frac{\phi}{a} \qquad \text{.......................................(4.38)}$$

$$\tau = t - \frac{(n^2 + a^2)}{a}\phi$$

Equation (4.37) represents the NUT-Kerr-Newman-Kasuya (NUTKNK) space-time in Boyer-Lindquist co-ordinates. The NUTKNK space-time gives the following spacetimes.

(i) Kerr-Newman-Kasuya space-time [72] for $n = 0$

(ii) NUT-Ker-Newman space-time when $g = 0$

(iii) NUT-Kerr space-time [73] if $e = g = 0$

(iv) Kerr-Newman space-time [74] with $n = g = 0$

(v) Kerr space-time [75] for $n = g = e = 0$

(vi) Reissner-Nordstrom space-time [76,77] provided $n = g = a = 0$

(vii) Schwarzchild space-time [78] if $n = g = a = e = 0$

(viii) Charged NUT space-time [79] for $a = g = 0$

(ix) NUT space-time [80] when $a = e = g = 0$

So we observe that the NUTKNK space-time includes all the black hole space-time (iv) - (vii), which are asymptotically flat. In particular the NUTKNK space-time contains the NUT space-time which has peculiar properties.

4.5 THE COMBINED NUT-KERR-NEWMAN-KASUYA-DE SITTER SPACE-TIME

If we use

$$\epsilon = 1 - \frac{\lambda}{3}(a^2 + 6n^2)$$

$$b = a^2 - n^2 + g^2 - \frac{5}{3}\lambda n^2 (a^2 + n^2) \qquad (4.39)$$

and replace n by

$$n+\frac{\lambda}{3}\left(\frac{3a^2n^2}{p}+\frac{2n^4}{p}+2n^3-6n^2p-a^2n+2np^2\right) \quad \text{.....................(4.40)}$$

We find that the metric (4.1) reduces to the form

$$ds^2=\frac{p^2+q^2}{x}dp^2+\frac{x}{p^2+q^2}(d\tau+q^2d\sigma)^2+\frac{p^2+q^2}{y}dq^2-\frac{y}{p^2+q^2}(d\tau-p^2d\sigma)^2 \text{....}$$

$$\text{.........(4.41a)}$$

where

$$x=[a^2-(n-p)^2][1+\frac{\lambda}{3}(n-p)^2] \text{.........................(4.41b)}$$

$$y=(q^2+a^2+n^2)[1-\frac{\lambda}{3}(q^2+5n^2)]-2(Mq+n^2)+e^2+g^2 \text{........(4.41c)}$$

Equation (4.41) represents the combined NUT-Kerr-Newman-Kasuya-de sitter space-time in Boyer co-ordinates.

A co-ordinate transfomation

$$p=n+a\cos\vartheta$$

$$\sigma = -\exists^{-1}\left(\frac{\phi}{a}\right)$$

$$q = r$$

$$\tau = \exists^{-1}\left[t - \frac{\left(a^2 + n^2\right)}{a}\phi\right] \qquad (4.42)$$

where

$$\exists = 1 + \frac{\lambda}{3}a^2$$

brings the metric (4.41) to the form

$$ds^2 = \frac{\Sigma}{\Delta_\vartheta}d\vartheta^2 + \frac{\Sigma}{\Delta_r}dr^2 + \frac{\exists^{-2}\Delta_\vartheta \sin^2\vartheta}{\Sigma}(adt - \rho d\phi)^2 - \frac{\exists^{-2}\Delta_r}{\Sigma}(dt - Ad\phi)^2 \qquad (4.43a)$$

where

$$\Sigma = r^2 + (n + a\cos\vartheta)^2$$

$$\Delta_\vartheta = 1 + \frac{\lambda}{3}a^2\cos^2\vartheta$$

$$\Delta_r = (r^2 + a^2 + n^2)[1 - \frac{\lambda}{3}(r^2 + 5n^2)] - 2(Mr + n^2) + e^2 + g^2 \text{.........}(4.43b)$$

$$\rho = r^2 + a^2 + n^2$$

$$A = a \sin^2 \vartheta - 2n \cos\vartheta$$

The equation (4.43) represents the combined NUT-Kerr-Newman-Kasuya-de sitter space-time in Boyer-Lindquist co-ordinates. We call the metric (4.43) as hot NUT-Kerr-Newman-Kasuya (HNUTKNK) space-time since the de sitter space-time has been interpreted as being hot [81]. The HNUTKNK space-time includes:

(i) NUTKNK space-time when $\lambda = 0$

(ii) hot kerr-newman-Kasuya (HKNK) with $n = 0$

(iii) hot NUT-Kerr-Newman (HNUTKN) if $g = 0$

(iv) hot Kerr-Newman space-time [82, 83] for $n = g = 0$

(v) hot Kerr space-time [83] when $n = g = e = 0$

(vi) hot Reissner-Nordstrom space-time if $n = g = a = 0$

(vii) hot Schwarzschild space-time [83] with $n = g = a = e = 0$

(viii) hot NUT space-time [84] for $a = e = g = 0$

So we observe that the HNUTKNK space-time includes the NUTKNK, HKNK, HNUTKN, hot NUT space-times as well as all the black hole spacetimes (iv) - (vii) which are asymptotically de Sitter. Further if we put $\lambda = 0$ in the cases (ii) - (vii) we get the Kerr-Newman -Kasuya, NUT-Kerr-

Newman space-times and all the black hole space-times which are asymptotically flat. In the limit $\lambda = 0$ the case (viii) reduces to the NUT space-time which is considered as homogeneous anisotropic cosmological model [18].

Thus the Plebanski space-time (4.1) contains a large number of solutions of Einstein-Maxwell equations with or without cosmological constant which are important from the physical point of view. The metric (4.1) contains some space-times with cosmological parameter which may be found interesting from the point of view of its inflationary scenario of the early universe [85].

CHAPTER -V

COSMIC STRING IN NUT-KERR-NEWMAN SPACE-TIME

5.1 INTRODUCTION

Recently Frolov et al. [1] studied the possible equilibrium configurations of a cosmic string in the curved space-time such as Kerr-Newman black hole space-time. In this chapter we would like to study the equilibrium configurations of a cosmic string in the NUT-Kerr-Newman space-time which includes as special cases Kerr-Newman black hole space-time [1] as well as NUT space-time. The NUT space-time has very interesting properties.

5.2 MOTION OF A STRING

In the approximation that the gravitational field of the string is neglected, the motion of the string is described by the Nambu-action [21,37,86,87]

$$I=-\mu\int d^2 l\sqrt{-\det\left(g_{\alpha\beta}\frac{\partial x^\alpha}{\partial l^a}\frac{\partial x^\beta}{\partial l^b}\right)} \qquad (5.1)$$

where μ is the mass of the string per unit length and $g_{\alpha\beta}\,(\alpha,\beta=1,2,3,4)$ is an external gravitational field and l^a is for the world-sheet coodinates (a,b = 0,1; $l^0=\tau$, $l^1=\sigma$).

We consider the string to be open and infinite. In this case we also suppose that force is applied to the string at infinity so that the string will not fall to the source responsible for creating the space-time concerned.

In general a stationary space-time is given by

$$ds^2=-R(dt+L_i dx^i)^2+\frac{1}{R}l_{ij}dx^i dx^j \qquad (5.2)$$

where $\partial_t R=\partial_t L_i=\partial_t l_{ij}=0$ and i,j=2,3,4. For time-independent string configurations where $\tau=t$ and the space like coorinates x^i depend on σ, the Nambu-action can be written as

$$I=-\mu\int d\sigma\sqrt{l_{ij}\frac{dx^i}{d\sigma}\frac{dx^j}{d\sigma}}\Delta t \qquad (5.3)$$

since the equilibrium configurations corresponds to minimal energy, the problem is reduced to the investigation of the geodesics in a three-dimensional space with the metric

$$ds^2 = l_{ij} dx^i dx^j \quad \text{.........(5.4)}$$

5.3 EQUILIBRIUM CONFIGURATION

For the NUT-Kerr-Newman geometry we have

$$R = \frac{\Delta - a^2 \sin^2 \vartheta}{r^2 + (n - a\cos\vartheta)^2} \quad \text{.........(5.5)}$$

$$L_i = \delta_i^{\varphi} L_{\varphi} \quad \text{.........(5.6)}$$

where

$$L_{\varphi} = \frac{a \sin^2 \vartheta (2Mr - e^2 + n^2) + \Delta(\frac{n^2}{a} - 2n\cos\vartheta)}{\Delta - a^2 \sin^2 \vartheta} \quad \text{.........(5.7)}$$

and

$$\Delta = r^2 - 2Mr + a^2 + e^2 - n^2$$

Here M, a, e and n are the mass, angular momentum per unit mass, charge and NUT (magnetic mass) parameters respectively. The three-dimensional metric l_{ij} is given by

$$l_{ij} = 0 \text{ where } i \neq j \; .. \qquad \text{.................................(5.8a)}$$

$$l_{rr} = \frac{\Delta - a^2 \sin^2 \vartheta}{\Delta} \qquad \text{.......................................(5.8b)}$$

$$l_{\vartheta\vartheta} = \Delta - a^2 \sin^2 \vartheta \qquad \text{......................................(5.8c)}$$

$$l_{\phi\phi} = \Delta \sin^2 \vartheta \qquad \text{..(5..8d)}$$

For our study of the geodesics of the three-dimensional space metric l_{ij}, we will use the Hamilton-Jacobi method [88]. We can write the Hamilton-Jacobi equation of the metric l_{ij} as

$$\frac{\partial S}{\partial \sigma}+\frac{1}{2}l^{ij}\frac{\partial S}{\partial x^i}\frac{\partial S}{\partial x^j}=0 \qquad (5.9)$$

where σ is an affine parameter along the geodesic.

If we write

$$S=-\frac{1}{2}q^2\sigma+k\phi+P(r)+Q(\vartheta) \qquad (5.10)$$

then we can have from (5.8), (5.9) and (5.10)

$$\Delta\left(\frac{dP}{dr}\right)^2-\frac{a^2k^2}{\Delta}-q^2\Delta=-m^2 \qquad (5.11a)$$

$$\left(\frac{dQ}{d\vartheta}\right)^2+\frac{k^2}{\sin^2\vartheta}+q^2a^2\sin^2\vartheta=m^2 \qquad (5.11b)$$

where m^2 is the separation constant. The integral of motions k corresponds to the killing vector $\eta_\phi = \frac{\partial}{\partial \phi}$ and m is related to the existence of the killing tensor η_{ij}:

$$m^2 = \eta^{ij} p_i p_j \quad \text{...(5.12)}$$

where

$$p_i = \frac{\partial S}{\partial x^j} \quad \text{..(5.13)}$$

and

$$\eta_i^j = \text{diag}(a^2 \sin^2 \vartheta, \Delta, \Delta + a^2 \sin^2 \vartheta) \quad \text{.................(5.14)}$$

On integration from (5.11) we have

$$P(r) = \int^{r} dr \sqrt{H} \quad \text{..(5.15a)}$$

$$Q(\vartheta) = \int^{\vartheta} d\vartheta \sqrt{\theta} \quad \text{.......................................(5.15b)}$$

where

$$H = \frac{a^2k^2}{\Delta^2} - \frac{m^2}{\Delta} + q^2 \quad \text{.......................................(5.16)}$$

and

$$\theta = m^2 - \frac{k^2}{\sin^2 \vartheta} - q^2a^2 \sin^2 \vartheta \quad \text{...........................(5.17)}$$

Therefore the equation (5.10) can be written as

$$S = -\frac{1}{2}q^2\sigma + k\phi + \int^{r} \sqrt{H}dr + \int^{\vartheta} \sqrt{\theta}d\vartheta \quad \text{......... ....(5.18)}$$

By differentiating (5.18) with respect to q^2, m and k and setting each of the derivatives equal to zero, we obtain the equations

$$\sigma - \sigma_0 = \int_{r_0}^{r} \frac{dr}{\sqrt{H}} - a^2 \int_{\vartheta_0}^{\vartheta} \frac{\sin^2 \vartheta}{\sqrt{\theta}} d\vartheta \quad \text{............................(5.19)}$$

$$\int_{r_0}^{r} \frac{dr}{\Delta\sqrt{H}} = \int_{\vartheta_0}^{\vartheta} \frac{d\vartheta}{\sqrt{\theta}} \quad \text{...(5.20)}$$

$$\phi - \phi_0 = k\left(\int_{\vartheta_0}^{\vartheta} \frac{d\vartheta}{\sin^2\vartheta\sqrt{\theta}} - a^2\int_{r_0}^{r} \frac{dr}{\Delta^2\sqrt{H}}\right) \quad \text{.................(5.21)}$$

Equations (5.19) -- (5.21) describe the equilibrium configuration of a string passing through the point (r, ϑ_0, ϕ_0) where the value of the affine parameter σ is σ_0. The string lies on a rotational surface given by equation (5.20). The equation (5.21) provides a unique curve on this surface. Since q is an inessential parameter, it can be changed by redefinition of the affine parameter σ. From now on we set q = 1.

Equations (5.19) -- (5.21) can be put in the following form:

$$p_r^2 = \left(l_{rr}\frac{dr}{d\sigma}\right)^2 = H \quad \text{...(5.22)}$$

$$p_\vartheta^2 = \left(l_{\vartheta\vartheta}\frac{d\vartheta}{d\sigma}\right)^2 = \theta \quad \text{........... ....(5.23)}$$

$$p_\phi^2 = \left(l_{\phi\phi}\frac{d\phi}{d\sigma}\right)^2 = k^2 \quad \text{.. (5.24)}$$

To analyze the form of the rotational surface we rewrite (5.23) as

$$p_\vartheta^2 = m^2 - \frac{k^2}{\sin^2\vartheta} - a^2\sin^2\vartheta = m^2 - V(\vartheta)$$

where

$$V(\vartheta) = \frac{k^2}{\sin^2\vartheta} + a^2\sin^2\vartheta$$

If $m^2 = V(\vartheta)$ is the minimal value of the function $V(\vartheta)$ at $\vartheta = \vartheta_0$, the solution of (5.23) is $\vartheta = \vartheta_0$.In this particular case the surface on which the string lies is cone like.

When $k < a$, we get $\vartheta_0 = \arcsin\left(\left|\frac{k}{a}\right|^{\frac{1}{2}}\right)$ and $V(\vartheta_0) = 2a|k|$. In the case k > a we get $\vartheta_0 = \frac{\pi}{2}$ and $V(\vartheta_0) = k^2 + a^2$. For $k^2 > a^2$ and $m^2 = k^2 + a^2$, the string lies in the equatorial plane of the source responsible for the NUT- Kerr-Newman space-time.

5.4 DISCUSSION

The results obtained in this chapter go for the NUT space-time for $a = e = 0$ and for the Kerr-Newman for $n = 0$.

This study not only encompasses the known results of Frolov et al. [1] in the context of Kerr-Newman black hole but also provides a similar results for the NUT space-time which is considered as homogeneous anisotropic cosmological model [18].

CHAPTER - VI

COSMIC STRING IN KERR-NEWMAN-KASUYA SPACE-TIME

6.1 INTRODUCTION

In this chapter we would like to extend the results of Frolov et al. [1] in the Kerr-Newman-Kasuya space-time. The Kerr-Newman-Kasuya space-time is the Kerr-Newman space-time involved with extra magnetic monopole charge. This study will be interesting in that reasons to belive magnetic monopole exist have been on the grounds of the symmetry that they would introduce in the field equation of electromagnetism. This monopole hypothesis was propounded by Dirac relatively long ago.The ingenious suggestion by Dirac that the magnetic monopole exist was neglected due to the failure to detect such a particle. However in recent years the development of gauge theories have shed new light on this.

6.2 EQUILIBRIUM CONFIGURATION

In general a stationary space-time is given by (5.2)

For the Kerr-Newman-Kasuya space-time we have

$$R = \frac{\Delta - a^2 \sin^2 \vartheta}{r^2 + a^2 \cos^2 \vartheta} \qquad \text{.........(6.1)}$$

$$L_i = \delta_i^{\varphi} L_{\varphi} \qquad \text{.........(6.2)}$$

where

$$L_{\varphi} = \frac{a \sin^2 \vartheta (2Mr - e^2 - g^2)}{\Delta - a^2 \sin^2 \vartheta} \qquad \text{.........(6.3)}$$

and

$$\Delta = r^2 - 2Mr + a^2 + e^2 + g^2 \qquad \text{.........(6.4)}$$

here M, a, e and g are the mass, angular momentum per unit mass, electric and magnetic monopole charge parameters respectively. The three-dimensional metric l_{ij} become

$$l_{ij} = 0 \text{ where } i \neq j \;.. \qquad \text{.......(6.5a)}$$

$$l_{rr} = \frac{\Delta - a^2 \sin^2 \vartheta}{\Delta} \qquad \text{.......(6.5b)}$$

$$l_{\vartheta\vartheta} = \Delta - a^2 \sin^2 \vartheta \qquad \text{.......(6.5c)}$$

$$l_{\phi\phi} = \Delta \sin^2 \vartheta \qquad \text{.......(6.5d)}$$

For our study of the geodesics of the three-dimensional space l_{ij} we will use the Hamilton-Jacobi method [88]. We can write the Hamilton-Jacobi equation of the metric l_{ij} as

$$\frac{\partial S}{\partial \sigma} + \frac{1}{2} l^{ij} \frac{\partial S}{\partial x^i} \frac{\partial S}{\partial x^j} = 0 \qquad \text{.......(6.6)}$$

where σ is an affine parameter along the geodesic.

If we write

$$S = -\frac{1}{2}q^2\sigma + k\phi + P(r) + Q(\vartheta) \quad \text{.......................... (6.7)}$$

then we have from (6.5) - (6.7)

$$\Delta\left(\frac{dP}{dr}\right)^2 - \frac{a^2k^2}{\Delta} - q^2\Delta = -m^2 \quad \text{...............................(6.8a)}$$

$$\left(\frac{dQ}{d\vartheta}\right)^2 + \frac{k^2}{\sin^2\vartheta} + q^2a^2\sin^2\vartheta = m^2 \quad \text{.....................(6.8b)}$$

where m^2 is the separation constant. The integral of motions k corresponds to the killing vector $\eta_\phi = \frac{\partial}{\partial\phi}$ and m is related to the existence of the killing tensor η_{ij}:

$$m^2 = \eta^{ij}p_ip_j \quad \text{..(6.9)}$$

where

$$p_i = \frac{\partial S}{\partial x^j} \quad \text{...(6.10)}$$

and

$$\eta_i^j = \mathrm{diag}(a^2\sin^2\vartheta, \Delta, \Delta + a^2\sin^2\vartheta) \quad \text{................(6.11)}$$

On integration from (6.8) we have

$$P(r)=\int^{r} dr\sqrt{H} \quad \text{..............(6.12a)}$$

$$Q(\vartheta)=\int^{\vartheta} d\vartheta\sqrt{\theta} \quad \text{..............(6.12b)}$$

where

$$H=\frac{a^2k^2}{\Delta^2}-\frac{m^2}{\Delta}+q^2 \quad \text{..............(6.13)}$$

and

$$\theta=m^2-\frac{k^2}{\sin^2\vartheta}-q^2a^2\sin^2\vartheta \quad \text{..............(6.14)}$$

Therefore equation (6.7) can be written as

$$S=-\frac{1}{2}q^2\sigma+k\phi+\int^{r}\sqrt{H}dr+\int^{\vartheta}\sqrt{\theta}d\vartheta \quad \text{..............(6.15)}$$

By differentiating (6.15) with respect to q^2, m and k and setting each of the derivatives equal to zero, we obtain the equations

$$\sigma - \sigma_0 = \int_{r_0}^{r} \frac{dr}{\sqrt{H}} - a^2 \int_{\vartheta_0}^{\vartheta} \frac{\sin^2 \vartheta}{\sqrt{\Theta}} d\vartheta \quad \text{.................................(6.16)}$$

$$\int_{r_0}^{r} \frac{dr}{\Delta\sqrt{H}} = \int_{\vartheta_0}^{\vartheta} \frac{d\vartheta}{\sqrt{\Theta}} \quad \text{...(6.17)}$$

$$\phi - \phi_0 = k\left(\int_{\vartheta_0}^{\vartheta} \frac{d\vartheta}{\sin^2 \vartheta\sqrt{\Theta}} - a^2 \int_{r_0}^{r} \frac{dr}{\Delta^2\sqrt{H}} \right) \quad \text{.......................(6.18)}$$

Equation (6.16) - (6.18) describes the equilibrium configuration of a string passing through the point (r, ϑ_0, ϕ_0) where the value of the affine parameter σ is σ_0. The string lies on a rotational surface given by equation (6.17). Equation (6.18) provides a unique curve on this surface. Since q is an inessential parameter, it can be changed by redefination of the affine parameter σ. From now on we set $q = 1$.

Equations (6.16) - (6.18) can be put in the following form:

$$p_r^2 = \left(1_{rr}\frac{dr}{d\sigma}\right)^2 = H \quad \text{.......................................}(6.19)$$

$$p_\vartheta^2 = \left(1_{\vartheta\vartheta}\frac{d\vartheta}{d\sigma}\right)^2 = \theta \quad \text{.......................................}(6.20)$$

$$p_\phi^2 = \left(1_{\phi\phi}\frac{d\phi}{d\sigma}\right)^2 = k^2 \quad \text{.......................................}(6.21)$$

To analyze the form of the rotational surface we rewrite (6.20) as

$$p_\vartheta^2 = m^2 - \frac{k^2}{\sin^2\vartheta} - a^2\sin^2\vartheta = m^2 - V(\vartheta)$$

where

$$V(\vartheta) = \frac{k^2}{\sin^2\vartheta} - a^2\sin^2\vartheta$$

If $m^2 = V(\vartheta)$ is the minimal value of the function $V(\vartheta)$ at $\vartheta = \vartheta_0$, the solution of (6.20) is $\vartheta = \vartheta_0$. In this particular case the surface on which the string lies is conelike.

When $k < a$, we get $\vartheta_0 = \arcsin\left(\left| \frac{k}{a} \right|^{\frac{1}{2}} \right)$ and $V(\vartheta_0) = 2a|k|$. In the case, $k > a$ we get $\vartheta_0 = \frac{\pi}{2}$ and $V(\vartheta_0) = k^2 + a^2$. For $k^2 > a^2$ and $m^2 = k^2 + a^2$, the string lies in the equatorial plane of the source responsible for the Kerr-Newman-Kasuya space-time.

6.3 DISCUSSION

In the case g = 0, the result obtained in this chapter will be reduced to the result obtained by Frolov et al. [1].

This study not only encompasses the result obtained by Frolov et al. but also provides similar result if the Kerr-Newman space-time is involved with magnetic monopole. So it is interesting to note that the physical results remain the same whether or not the magnetic monopole does exist in nature.

DISCUSSION

In this thesis, we observe that the mathematical approach to study the equilibrium configuration of a cosmic string is the same in all cases such as Kerr-Newman black hole [1], NUT-Kerr-Newman space-time [chapter V] and Kerr-Newman-Kasuya space-time [chapter VI]. Under this observation we like to claim that the mathematical treatment for studying the equilibrium configuration of a cosmic string for the space-times having horizons are the same. In support of this claim, we would like to mention different works of Ahmed [4-12], Ahmed and Mondal [13,14] and Ahmed and Hossain [15,16]. Ahmed extensively studied the different problems such as superradiance phenomena, Hawking radiation in the space-times which are not black hole space-times but the space-times having horizons. Ahmed observed in his different works that the physical results such superradiance phenomena and Hawking radiance are not only true for the black hole space-times but also true for the space-times having horizons. The mathematical treatment followed by Ahmed in all of these cases are analogous to those used for the study of radiation for the black hole space-time.

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