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Analytical Invcstlgations in Tubulent and IVIlID Tttdmlent Flow

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ANALYTICAL INVESTIGATIONS IN TURBULENT AND MHD TURBULENT FLOW

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DEDICATED TO THE
MEMORY OF
MY BELOVED MOTHER

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Certified that the thesis entitled " **Analytical Investigations in Turbulent and MHD Turbulent Flow** " submitted by Mr. Md. Lutfor Rahman in fulfilment of the requirements for the degree of Doctor of Philosophy in Mathematics, University of Rajshahi , Rajshahi , has been completed under my supervision. I believe that this research work is an original one and it has not been submitted elsewhere for any degree.

(Dr. M.Shamsul Alam Sarker)
Supervisor.

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PREFACE

The thesis entitled "**Analytical Investigations in Turbulent and MHD Turbulent Flow**" is being presented for the award of the degree of Doctor of Philosophy in Mathematics. It is the outcome of my research conducted in the Department of Mathematics, Rajshahi University during the year 1994-1998 under the guidance of Dr. M. Shamsul Alam Sarker, Department of Mathematics, Rajshahi University, Rajshahi-6205, Bangladesh.

The whole thesis has been divided into six chapters. The first is an introductory chapter and gives the general idea of turbulence, magnetohydrodynamic turbulence and its principal concepts. Throughout the work we have considered the flow of fluids to be isotropic and homogeneous. The notions generally adopted are those used by Batchelor, Chandrasekhar and Deissler in their research papers. Number inside brackets [ ] refer to the references which are arranged alphabetical at the end of the thesis.

In the second chapter, we have derived the equation for the rate of change of magnetic field covariance in MHD turbulent flow. The result shows that the defining scalars of the magnetic field covariance depend on the defining scalar H of two point magnetic field correlation.

In the third chapter, the decay of turbulence at times before the final period in presence of dust particles is studied. Two and three point correlation equation is used to obtain a relation for the triple correlations and the equation is made determinate by neglecting the quadruple correlations. Finally, we obtained the energy decay law of dusty fluid turbulence before the final period.

In the fourth chapter, we have studied the decay of dusty fluid MHD turbulence before the final period. Three point correlation equation is used to obtain a relation for the triple correlations applicable at times before the final period. In this case the equation is made determinate by neglecting the quadruple correlations. Finally, we obtain the energy decay law of dusty fluid MHD turbulence at times before the final period.

In the fifth chapter, the decay of temperature fluctuation in a homogeneous MHD turbulence before the final period has been studied. We considered the two and three point correlation equations and solved them after neglecting the fourth order correlations in comparison with the second and third order correlations. Finally, the energy decay law of temperature fluctuation of MHD turbulence has been obtained.

In the sixth chapter, we have studied the thermal decay process of MHD turbulent flow in a rotating system in presence of dust particles. An early period decay equation for convective MHD turbulent flow in a rotating system at high Reynolds and Peclect number is used. The region where the variations of mean temperature, mean velocity and mean magnetic field is considered may be neglected because the transportation of the thermal energy from place to place is very rapid.

The following research papers which are extracted from this thesis have either been accepted for publication or communicated in different journals.

(1) Magnetic field covariance in magnetohydrodynamic turbulent flow.(Accepted in the Jour. Rajshahi Univ. Studies (1996))

(2) Decay of turbulence before the final period in presence dust particles. (Accepted in the Jour. Rajshahi Univ. Studies (1997))

(3) Decay of dusty fluid magnetohydrodynamic turbulence before the final period. (Communicated for publication)

(4) Decay of temperature fluctuations in magnetohydrodynamic turbulence before the final period.(Communicated for publication)

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CONTENTS

CHAPTER-1

INTRODUCTION

1.1 TURBULENCE AND ITS BACKGROUND

The notion of turbulence is generally accepted nowadays, and, broadly speaking, its meaning is understood, at least by technical people. Yet it is curious to note that the meaning of the word "turbulent" to characterize a certain type of flow, namely the counter part of stream line motion. According to webster's "New international Dictionary"turbulence means : agitation, commotion, disturbance....... In 1883 the first systematic experimental investigations of turbulent flow were made by Osborne Reynolds [51]. In his classical experiments, Reynolds used glass tube with flowing water from a reservoir and observed the flow pattern by injecting a thin stream of dye into the main stream. If the velocity of the water is sufficiently low, the coloured filament of dye remains straight and parallel to the walls of the tube which indicates that the flow is steady. Again if the velocity is increased beyond a certain value, the coloured filament begins to oscillate and finally loses its identity and diffuses through the tube. The first type of flow is clearly laminar and the second type of flow is called turbulent. The essential characteristic of turbulent flow is that the turbulent fluctuations are random in nature. In 1937, Taylor and Von Karman [62] gave the following definition : "Turbulence is an irregular motion which in general makes its appearance in fluids, gaseous or liquid, when they flow past solid surfaces or even when neighbouring streams of the same fluid flow past or over one another". According to this definition, the flow has satisfy the condition of irregularity. Indeed, this irregularity is a very important feature. Because of irregularity it is impossible to describe the motion in all details as a function of time and space co-ordinates. But, fortunately, turbulent motion is irregular in the sense that it is possible to describe it by laws of probability. It appears possible to indicate distinct average values of various quantities, such as velocity, pressure, temperature etc., and this is very important. Therefore it is not sufficient just to say that turbulence is an irregular motion. Yet we don't have a clearcut definition of turbulence. This is rather difficult. In his book "Turbulence" Hinze [24] suggests " Turbulent fluid motion is an irregular condition of flow in which variation with time and space coordinates, so that statistically distinct average values can be discerned". The addition "with time and space coordinates " is necessary ; it is not sufficient to define turbulent motion as irregular in time alone. According to the definition suggested by Taylor and Von Karman [62], turbulence can be generated by fluid flow past solid surfaces or by the flow of layers of fluids at different velocities past or over one another.

The definition above indicates that there are two distinct types of turbulence :

(i) Turbulence generated by the viscous effect due to the presence of a solid wall is designated by wall turbulence ; (2) Turbulence, in the absence of a wall,generated by the flow of layers of fluids at different velocities is called free turbulence. Turbulent flow through conduits and past bodies are examples of wall turbulence, and turbulent jet mixing regions and wakes fall into the category of free turbulence. In the previous discussion we have mentioned that Reynolds used a dye experiment to investigate the circumstances of the transition from laminar to turbulent flow. Based on his experimental results Reynolds concluded that transition from laminar to turbulent flow in pipes always occurred at nearly the same Reynolds number. The approximate value of the critical Reynolds number, R_{cr} at which the laminar regime breaks down was established to be the order of $2x10^3$. Later, with Reynolds, apparatus, Ekman was able to maintain laminar flow up to a critical Reynolds number of $4x10^4$ when the testing conditions were made extremely free from disturbances. This suggested that the upper limit of the critical Reynolds number depends very strongly on the initial disturbance as it increase with the decrease of the disturbance in the flow. In spite of the uncertainty of the upper limit of the critical Reynolds number, there exists a lower limit for a critical Reynolds number below which the flow always remains laminar. For flow through a circular pipe with smooth walls this lower critical Renolds number is established as being approximately

$2x10^3$. This led one to believe that laminar flow was stable for an infinitesimal disturbance, and transition occurred as a result of an external disturbance of finite magnitude. The critical Reynolds number which we have just discussed has considerable practical significance in connection with the origin of turbulence. The origin of the idea of statistical approach to the problem of turbulence may be traced back to Taylor's paper of 1921 [60] in which he has advanced the concept of the lagrangian correlation coefficient that provides a theoretical basis for turbulent diffusion. The most important work done by Taylor [61] is that he gave up the old theories of turbulence based on the Kinetic theory of gases and introduces the idea that the velocity of the fluid in turbulent motion is a random continuous function of position and time. He introduced the concept of correlation between the velocities at two points. To make the turbulent motion amenable to mathematical treatment, he assumed the turbulent fluid to be homogeneous and isotropic. In its support, he described the measurements showing that the turbulence generated down stream from a regular array of rods in a wind tunnel is approximately homogeneous and isotropic. Inspite of the fact that the turbulence in nature is not always exactly homogeneous and isotropic, it is essential to study homogeneous and isotropic turbulence as a first step to understand the more complicated phenomenon of non-homogeneous turbulence.

In the following, instead of giving a detailed account of the historical development of the subject, we shall confine to mere concepts and method of turbulence together with a few theories of turbulence which have been used in the subsequent chapters.

1.2 METHOD OF TAKING AVERAGES

To describe a turbulent motion quantitatively, it is necessary to introduce the motion of scale of turbulence; a certain scale in time and a certain scale in space. In the mathematical description of turbulent flow it is convenient to consider an instantaneous velocity component u_i is generally written as

$$u_i=\overline{u_i}+u_i' \tag{1.2.1}$$,

where u_i is the ith component of the total fluid velocity, $\overline{u_i}$ is the ith mean velocity component and u_i' is the ith component of fluctuating velocity. In taking the average of a turbulent quantity, the result depends not only on the scale used but also on the method of averaging. In practice, four different methods of averaging [47] have been used to obtain the mean value of a turbulent quantity (such as velocity, density etc).

If the turbulent flow field is quasi-steady or stationary random, averaging with respect to time can be used. In the case of homogeneous turbulence flow field, averaging with respect to space can be considered. If the flow field is steady and homogeneous, space-time average is used. Lastly, if the flow field is neither steady nor homogeneous, we assume that an average is taken over a large number of experiments that have the same initial and boundary conditions. We then speak of an ensemble average.

The methods of averaging are:

1.2(a): Time average in which we take the average at a fixed point in space over a long period of time, i.e.

$$[u(x,t)]_t = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} u(x,s)\,ds \qquad (1.2.2).$$

In practice, the value of the period 2T is determined by the scale used in the averaging process.

1.2(b): Space average in which we take the average over all the space at given time, i.e.,

$$[u(x,t)]_s = Lim_{V\to\infty} \frac{1}{V_B}\int_{V_B} u(s,t)\,ds \qquad (1.2.3).$$

In practice the volume of space V_B is determined by the scale used in averaging process.

1.2(c): Space-time average in which we take the average over a long period of time and over the space, i.e.,

$$[u(x,t)]_{s,t} = Lim_{T\to\infty V\to\infty} \frac{1}{2TV_B}\int_{-T}^{T}\int_{V_B} u(s,y)\,dsdy \qquad (1.2.4).$$

In practice both the values of T and of V_B are determined by the scale used.

1.2(d): Statistical average in which we take the average over the whole collection of sample turbulent functions for a fixed point space and at a fixed time, i.e.,

$$[u(x,t,\omega)]_{\omega}=\int_{\Omega}u(x,t,\omega)\,d\mu(\omega) \qquad (1.2.5).$$

Over the whole Ω -space of ω , the random parameter. The measure is

$$\int_{\Omega}d\mu(\omega)=1 \qquad (1.2.6).$$

Some explanations are neglected for the statistical average. The essential characteristic of the turbulent motion is that the turbulent fluctuations are random in nature. A turbulent velocity field can be regarded as a random vector field of a set of vectors in space and time. Any random vector field can be regarded as a field consisting of three random scalar fields as its components. A random scalar function $u(x,t,\omega)$ is a function of the spatial co-ordinates x and time t, which depends on a parameter ω . The parameter ω is chosen at random according to some probability law in a space. In the experimental investigation we use time averages almost exclusively, space averages seldom and never statistical averages. In the theory, we use almost exclusively the statistical averages.

1.3 REYNOLD'S RULES OF AVERAGE

Osborne Reynolds [51] was the first to introduce elementary statistical motions into the consideration of turbulent flow. In the theoretical investigations of turbulence, he assumed that the instantaneous fluid velocity satisfies the Navier-Stokes equations motion for a viscous incompressible fluid and that the instantaneous velocity may be separated into a mean velocity and a turbulent fluctuating velocity. Thus, the physical quantities characterizing the flow field are written as

$$u_i=\overline{u_i}+u_i', p=\overline{p}+p', \rho=\overline{\rho}+\rho', T=\overline{T}+T' etc. \qquad (1.3.1).$$

Here the quantities with bar denote the mean values and those with primes are fluctuations. Furthermore

$$\overline{u_i'}=\overline{p'}=\overline{\rho'}=\overline{T'}=0$$

In order to develop the rule of averaging, consider three arbitrary statistically dependent physical quantities, A, B and C, each consisting of a mean and a fluctuating part, i.e.,

$$A=\overline{A}+A', B=\overline{B}+B', C=\overline{C}+C' \qquad (1.3.2).$$

Then

$$\overline{A}=\overline{\overline{A}+A'}=\overline{\overline{A}}+\overline{A'}=\overline{A}.\ (since\ \overline{A'}=0) \qquad (1.3.3).$$

In the above relations we used the properties that the average of the sum is equal to the sum of the averages and the average of a constant times B is equal to the constant times the average of B.

Next

$$\overline{AB}=\overline{\overline{A}\,\overline{B}}+\overline{A'B'} \qquad (1.3.4).$$

Consequently

$$\overline{\overline{A}\,\overline{B}}=\overline{\overline{A}}.\overline{\overline{B}}=\overline{A}.\overline{B} \qquad (1.3.5).$$

Note that the average of a product is not equal to the product of the averages, terms such as $\overline{A'B'}$ are called "correlation".

For the product of three quantities, we have

$$\overline{ABC}=\overline{(\overline{A}+A')\,(\overline{B}+B')\,(\overline{C}+C')}=\overline{A}.\overline{B}.\overline{C}+\overline{A}\,\overline{B'C'}$$

$$+\overline{B}\,\overline{A'C'}+\overline{C}\,\overline{A'B'}+\overline{A'B'C'} \qquad (1.3.6).$$

Also, it can be shown that

$$\overline{\frac{\partial A}{\partial s}}=\frac{\partial \overline{A}}{\partial s} \qquad (1.3.7)$$

and

$$\overline{\int A ds}=\int \overline{A} ds \qquad (1.3.8).$$

1.4 REYNOLD'S EQUATIONS AND REYNOLD'S STRESS

It is assumed here that the fluids show Newtonian behaviour and that their flows are solutions of the equation of conservation of mass and of the Navier-Stokes equations of motion, satisfying prescribed boundary and initial conditions. The turbulent flows from a special class of such solutions, in which the dependent variables such as velocity, pressure and density are not unique functions of the space and time co-ordinates but must be described by probability laws (randomness of the motion). In turbulent flow, we usually assume that instantaneous velocity components satisfy the Navier-Stokes equations,

$$\frac{\partial U}{\partial t}+(U.\nabla)\,U=F-\frac{1}{\rho}\nabla p+\nu\nabla^2 U \qquad (1.4.1).$$

In tensor form the equation (1.4.1) can be written as

$$\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}=F-\frac{1}{\rho}\frac{\partial p}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_j \partial x_j} \qquad (1.4.2).$$

Substituting the expressions for the instantaneous velocity components $u_i=\overline{u_i}+u_i'$ into the Navier-Stokes equation (1.4.2) for an incompressible fluid after neglecting the body forces and taking the mean values of these equations according to Reynolds rule of averaging (1.3.1)-(1.3.5), we have the following Reynolds equation of motion for the turbulent flow of an incompressible fluid:

$$\rho\left(\frac{\partial \overline{u_i}}{\partial t}+\overline{u_j}\frac{\partial \overline{u_i}}{\partial x_j}\right)=-\frac{\partial \overline{p}}{\partial x_i}+\mu\frac{\partial^2 \overline{u_i}}{\partial x_j \partial x_j}+\frac{\partial}{\partial x_j}\left(-\rho\overline{u_i' u_j'}\right),$$

where i and j run from 1 to 3 and Einstein's summation convection is used. The bar represents the mean value and the prime represents the turbulent fluctuation. Additional terms over the Navier-Stokes equations are due to Reynolds stress are $-\rho\overline{u_i'^2}$ and the eddy stresses are $-\rho\overline{u_i' u_j'}(i\neq J)$, where ρ is the density of the fluid. These stresses represent the rate of transfer of momentum across the corresponding surfaces because of turbulent velocity fluctuations.

The solutions of Reynolds equation will be represents properly the turbulent flow. In general the Reynolds equations are not sufficient to determine the unknown variables u_i, u_j $(i,j=1,2,3)$, p and Reynold stresses. This is one of the main difficulties in the theoretical investigation of turbulent flow.

In similar way, Reynolds equation of motion for the turbulent flow of a compressible fluid may be obtained. But the expressions for the eddy stresses (Reynold stresses)of compressible fluid are much more complicated because the fluctuations of density should be considered.

As in the case of the Navier-Stokes equations, it is not at the present time possible to solve the Reynolds equations for any practical flow problem. Addition assumptions and hypothesis are necessary to simplify these equations, in order to obtain some approximate solutions for important practical cases.

1.5 CORRELATION FUNCTION

In 1935, in a most important series of papers, G.I.Taylor [61] introduced new notions into the study of the statistical theory of turbulence. Taylor successfully developed a statistical theory of turbulence which is applicable to continuous movements and which satisfies the equation of motion .

The first important new notion was that of studying the correlation, or coefficient of correlation between two fluctuating quantities in turbulent flow. In his theory, Taylor makes much use of the correlation between the components of the fluctuations at neighbouring points. Denote the components of the fluctuating velocity at one point P by u_1, u_2, u_3 and at another point P' by u'_1, u'_2, u'_3 . The correlation function between any of the u_i and u'_j where i,j=1,2 or 3, defined as

$$\rho_{ij}=\overline{u_i u_j} \qquad (1.5.1),$$

where the bar denotes the average by certain process.

Sometimes it is convenient to use the correlation coefficient such as

$$R_{ij}=\frac{\overline{u_i u_j}}{\sqrt{\overline{u_i^2}}\sqrt{\overline{u_j'^2}}} \qquad (1.5.2).$$

By Cauchy inequality, we have

$$\overline{u_i u_j}-\sqrt{\overline{u_i^2}}\,.\sqrt{\overline{u_j^2}}\leq 0 \qquad (1.5.3),$$

hence

$$1-\leq R_{ij}\leq 1 \qquad (1.5.4).$$

If we consider $u_i u_j$ as the velocity components in a flow field, the correlation of equation (1.5.1) is a tensor of rank two. By a different process of averaging we obtain different kinds of correlation functions. If we consider u_i and u_j as the velocity components at a given point in space, u_i and u_j are functions of time; hence, we should take the time average in equation (1.5.1) to get the correlation function ρ_{ij} .

If we consider u_i and u_j as the velocity components at a given time, u_i and u_j are functions of space co-ordinates $x(x_1,x_2,x_3)$; hence,

we should take the space average in equation to get the correlation function.

More generally if we consider u_i and u_j as functions of both time t and spatial co-ordinates $x(x_1,x_2,x_3)$, we obtain take a space-time average in equation (1.5.1) to get the correlation function.

The correlation function between the components of the fluctuating velocity at the same time at two different points of the fluid, first introduced by G. I. Taylor [61], has been investigated extensively in the isotropic turbulence.

The correlation function between two the fluctuating velocity components at the same point and at the same time gives the Reynolds stress. The correlation function between two fluctuating quantities may also be defined in a manner similar to above.

1.6 ISOTROPIC AND HOMOGENEOUS TURBULENCE

Isotropic turbulence is the simplest type of turbulence,since no preference for any specific direction and a minimum number of quantities and relation are required to describe its structure and behaviour. However, it is also a hypothetical type of turbulence, because no actual turbulent flow shows true isotropy, though conditions may be made such that isotropy is more or less closely approached.

From theoretical considerations and experimental evidence it is known that the fine structure of most actual non-isotropic turbulent flows is nearly isotropic (local isotropy). Hence many features of isotropic turbulence may apply to phenomena in actual turbulence that are determined mainly by the fine-scale structure, where local isotropy prevails.

In isotropic turbulence the mean value of any function of the velocity components and their space derivatives is unaltered by any rotation or reflection of the axes of references. Thus, in particular, $\overline{u^2}=\overline{v^2}=\overline{w^2}$ and $\overline{uv}=\overline{uw}=\overline{vw}=0$.

Isotropy introduces a great simplicity into the calculations. The study of isotropic turbulence may also be of practical importance, since far from solid boundaries it has been observed that $\overline{u_1^2},\overline{u_2^2},\overline{u_3^2}$ tend to become equal to one another, e.g. in the natural winds at a sufficient height above the ground and in a pipe flow near the axis.

Another simple type of turbulence is homogeneous turbulence. It is defined as the turbulence having quantitatively the same structure in all parts of the flow field. In a homogeneous turbulent flow field the statistical characteristics are invariant for any translation in the space occupied by the fluid.

The conception of homogeneous turbulence is idealized, in that there is no known method of realizing such a motion exactly. The various methods of producing turbulent motion in a laboratory or in nature all involve discrimination between different parts of the fluid, so that the average properties of the motion depend on position. However, in certain circumstances this departure from exact independence of position can be made very small, and it is possible to get a close approximation to homogeneous turbulence.

Most of the theoretical works in turbulence and MHD turbulence concern homogeneous and isotropic field in an incompressible fluid at rest.

1.7 SPECTRAL REPRESENTATION OF THE TURBULENCE

Theoretical treatment of the turbulence is merely related to the solutions of the Navier-Stokes equations. These equations, however, contain more unknowns than the number of equations and therefore additional assumptions must be made. This is known as the "closure problem". An alternative approach is based on the spectral form of the dynamic Navier-Stokes equation. The spectral form of the turbulence is still under-determined, but it has a simple physical interpretation and is more convenient. The spectral approach is, however, almost exclusively used for the description of homogeneous turbulence [45, 46]. The principal concepts of spectral representation in the study of turbulence are described below:

If we neglect the body forces from the Navier-stokes equation (1.4.1) and multiply the x_i-component of Navier-stokes equation written for the point p by $u_j^{/}$, adding and taking the ensemble average we get

$$\frac{\partial \overline{u_i u_j^{/}}}{\partial t} + (\overline{u_j^{/} u_l \frac{\partial u_i}{\partial x_l}} + \overline{u_i u_l^{/} \frac{\partial u_j^{/}}{\partial x_l^{/}}}) = -\frac{1}{\rho} (\overline{u_j^{/} \frac{\partial p}{\partial x_i}} + \overline{u_i \frac{\partial p^{/}}{\partial x_j^{/}}})$$

$$+\nu (\overline{u_j^{/} \frac{\partial^2 u_i}{\partial x_l^2}} + \overline{u_i \frac{\partial^2 u_j^{/}}{\partial x_l^{/2}}}) \qquad (1.7.1).$$

Since in the homogeneous turbulence, the statistical quantities are independent of position in space and considering the points p and p' separated by a distance vector r and applying the laws of spatial covariance, a simplified form of equation (1.7.1) is obtained as

$$\frac{\partial}{\partial t}\overline{u_i u_j'} = -\frac{\partial}{\partial r_l}(\overline{u_i u_j' u_l} - \overline{u_i u_j' u_l'})$$

$$+\frac{1}{\rho}\left(\frac{\partial}{\partial r_i}\overline{p u_j'} - \frac{\partial}{\partial r_j}\overline{p' u_i}\right) + 2\nu \frac{\partial^2}{\partial r_l^2}\overline{u_i u_j'} \qquad (1.7.2).$$

The covariance $\overline{u_i u_j'}$ is not suitable for direct analysis of quantitative estimate of the turbulent flows and it is better to use the three dimensional Fourier transforms of $\overline{u_i u_j'}$ with respect to r. The variable that corresponds to r in the three dimensional wave-number space is vector $k=(k_1,k_2,k_3)$. We define the wave-number spectral density as

$$\phi_{ij}(k) = \frac{1}{(2\pi)^3}\int \overline{u_i u_j'} \exp(-ik.r)\, dr$$

$$= \frac{1}{(2\pi)^3}\iiint \overline{u_i u_j} \exp\{-i(k_1 r_1 + k_2 r_2 + k_3 r_3)\}\, dr_1 dr_2 dr_3 \ .$$

It can be shown that if $\overline{u_i u_j'}$ has a continuous range of wave-length, $\phi_{ij}(k)$ has a continuous distribution in wave number space. We can rigorously regard $\phi_{ij}(k)\,dk_1 dk_2 dk_3$ as the contribution of the elementary volume $dk_1 dk_2 dk_3$ (centred at wave-number k and therefore representing a wave-number of length $\frac{2\pi}{|k|}$ in the direction of the vector k) to the value of $u_i u_j'$ hence the name "spectral density". This is consistent with the behaviour of the inverse transform

$$\overline{u_i u_j'(r)} = \int \phi_{ij}(k) \exp(ik.r)\, dk \qquad (1.7.4).$$

The one dimensional wave-number spectrum of $\overline{u_i u_j'}$ for a wave-number component in the x_1 direction is

$$\phi_{ij}(k_1) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{u_i u_j'(r_1)} \exp(-ik_1 r_1)\, dr_1 \qquad (1.7.5),$$

whose inverse is

$$\overline{u_i u_j'(r)} = \int_{-\infty}^{\infty} \phi_{ij}(k_1) \exp(ik_1 r_1)\, dk_1 \qquad (1.7.6).$$

The equation (1.7.2) for unstrained homogeneous turbulence becomes on Fourier transforming

$$\frac{\partial}{\partial t}=\Gamma_{ij}(k)+\Pi_{ij}(k)-2\nu k_i^2\phi_{ij}(k) \quad ,$$

where Γ and Π transforms of the triple product and pressure terms respectively.

1.8 FOURIER TRANSFORMATION OF THE NAVIER-STOKES EQUATIONS

The principal reason for using Fourier transforms is that they convert differential operators into multipliers. The equations are so complicated in configuration (or co-ordinate) space that very little can be done with them and the transformation to wave-number (or Fourier) space simplifies them very considerably.

Another and more mathematical argument shows that these transforms are right method of treating a homogeneous problem.

Associated with any correlation function $\phi(x,x')$ is a sequence of eigen functions $\phi(n,x)$ and their associated eigen-values $\lambda(n)$. These quantities satisfy the eigen-value equation

$$\int\phi(x,x')\,\psi(n,x)\,d^3x'=\lambda(n)\,\psi(n,x) \qquad (1.8.1)$$

and the ortho-normalization relation

$$\int \psi(n,x)\,\psi^*(n,x)\,d^3x$$

$=1$ if $m=n$

$=0$ otherwise

These equations imply that ϕ is a vector. Actually it is a tensor of order two, but this complicates the argument without introducing anything essentially new. The index n, is in general, a complex variable and ψ^* denotes the complex conjugate of ψ (strictly, ψ^* is the adjoint of ψ, but since ϕ is real and symmetric the adjoint is simply the complex conjugate). The integrations in equations (1.8.1) and (1.8.2) are over all space, which may be finite or infinite. If the space is finite, n is usually an infinite but countable sequence, while if space is infinite , n will be a continuous all have real eigen-values. It follows from (1.8.1) and (1.8.2) that

$$\phi(x,x') = \sum_n \lambda(n)\,\psi(n,x)\,\psi^*(n,x') \qquad (1.8.3)$$

and this is the diagonal representation of the correlation function in terms of its eigen functions.Evidently these functions are only defined "within a phase" that is , a factor exp (iY) can be added to $\psi(n,x)$ without altering $\phi(x,x')$ provided Y is real and independent of x. For a homogeneous field, ϕ is a function of $x-x'$ only, and the problem is to find eigen functions which are also homogeneous within a phase, in the sense that

$$\psi(n,x)=\exp(iY)\psi(n,x+a) \quad .$$

The equation is satisfied by the Fourier function,

$$(n,x)=\exp(in.x)=\exp(in_j x_j)$$

with Y=-n,a. In this instance,therefore, "the index" **n** is a wave-number equation (1.8.3) becomes,

$$\phi(x,x')=\sum\lambda(n)\exp\{in(x-x')\}$$

so that $\lambda(n)$ may be identified with $\phi(n)$, the Fourier transform of correlation function.

Since we are considering homogeneous isotropic turbulence, the turbulence field must be infinite in extent. This produces mathematical difficulties which can only be resolved by using functional calculus. This difficulty is avoided by supposing that the turbulence is confined to the inside of a large box with sides (a_1,a_2,a_3) and that it obeys cyclic boundary conditions on the sides of this box. The a_1 are allowed to tend to infinity at an appropriate point in the analysis. Thus the Fourier transform is defined by $U_i(x) = (2\pi)^3 (a_1 a_2 a_3)^{-1} \sum_k u_i(k) \exp(ik.x)$ (1.8.4).

Here, k is limited to wave vectors of the form

$$\frac{2n_1\pi}{a_1}, \frac{2n_2\pi}{a_2}, \frac{2n_3\pi}{a_3}$$

where the n_i are integers while the a_i are, as before, the sides of the elementary box. As these sides become infinitely large, equation (1.8.4) goes over into the standard form,

$$U_i(x) = \int u_i(k) \exp(ik.x)\, d^3k. \qquad (1.8.5).$$

The inverse of (1.8.5) is

$$u_i(k) = (2\pi)^{-3} \int_{box} U_i(x) \exp(-ik.x)\, d^3x. \qquad (1.8.6).$$

The Fourier transforms of the Navier-Stokes equation may be written as

$$\left(\frac{d}{dt} + \nu k^2\right) u_i(k) = M_{ijm}(k) \sum U_j(p)\, U_m(r) \qquad (1.8.7),$$

where Σ is a short notation for the integral operator in

$$\iint U_j(k)\, U_m \delta(k-p-r)\, d^3p . d^3r \ ,$$

where $\delta_{k,p+r}$ is the kronekar delta symbol which is zero unless

$k=p+r$

Here, M_{ijm} is a symple algebraic operator. We have

$$M_{ijm}(k) = -\frac{1}{2} i p_{ijm}(k)$$

where,

$$p_{ijm}(k) = k_m p_{ij}(k) + k_j p_{im}(k)$$

and

$$p_{ij} = \delta_{ij} - \frac{k_i k_j}{k^2}$$

$p_{ij}(k)$ is the Fourier transforms of p_{ij} (∇)but $p_{ijm}(k)$ is not

the transforms of p_{ijm}(∇).

As it stands, equation (1.8.7) can not describe stationary turbulence. Since it contains no input of energy to balance the dissipative effect of viscosity. In real life this input is provided by effects, such as the interaction of the Reynolds stress, which are incomparable with the ideas of homogeneity and isotropy. To avoid this difficulty, we introduce into the right - hand side of equation (1.8.7) a hypothetical homogeneous isotropic stirring force f_i. The equation then reads,

$$\left(\frac{d}{dt}+\nu k^2\right) u_i(k) = M_{ijm}(k) \sum u_j(p)\, u_m(r) + f_i(k) \,.$$

1.9 MAGNETOHYDRODYNAMIC TURBULENCE

The study of magnetohydrodynamic turbulence, i.e. the study of the interaction between a magnetic field and the turbulent motions of an electrically-conducting fluid, was first undertaken in connection with the implied existence of an interstellar magnetic field. The interaction between the velocity and magnetic fields results in a transfer of energy between the kinetic and magnetic spectra, and it is thought that the interstellar magnetic field is maintained by a "dynamo" action from turbulence in the interstellar gas.

Modern applications of magnetohydrodynamics in the field of propulsion, nuclear fission, and electrical power generation make the problem of magnetohydrodynamic turbulence one of considerable interest to engineers, since turbulence phenomena seem to be inherent in almost all types of flow problems.

It is generally supposed that in a medium of high electrical conductivity turbulence will give rise to a spontaneous generation of magnetic fields; that in the course of time these fields will be amplified; and that in an eventual equilibrium state the energy per unit volume in the magnetic field and in the velocity field will approach equality.

In drawing these conclusions one argues qualitatively interms of a picture, first extensively used by Alfven, that in a medium of high electrical conductivity the lines of magnetic force tend to be attached to the matter. They will, therefore, be dragged about in all directions by the random turbulent motions. In this manner an initial stray magnetic field will be amplified. This process of amplification will be checked when the prevailing magnetic field has increased to a certain strength; for, if the magnetic field is sufficiently strong it will prevent its further increase by suppressing the turbulent motions. In an equilibrium state, the amplification of the magnetic field by the turbulent motions and the suppression of the motions by the magnetic field will balance each other and one may expect that an equipartition between the two forms of energy will result. It is on such a picture that Fermi [18] postulated intestillar magnetic fields as a basis for his theory of the origin of the cosmic rays. But so far this picture has never been incorporated in a quantitative theory of hydromagnetic turbulence-even a heuristic theory of the type of Heisenberg's [22, 23] in ordinary hydrodynamics.

Here the theory of turbulence in an incompressible, viscous and electrically conducting fluid is formulated probabilistically through the use of the joint characteristics functional and the calculus. The use of the joint characteristics functional approach relies upon the fact that velocity and magnetic fields are both solenoidal, and, hence, in the probabilistic sense, are jointly distributed over the phase space consisting of the set of all solenoidal vector fields. The formulation of the problem in phase space is completely carried out. The full space time functional formulation of the problem as developed by Lewis and Kraichnan [40] for "ordinary" turbulence is extended to magnetohydrodynamic turbulence. This approach enables us to generate space time correlations between the velocity and magnetic field components rather than merely spatial correlations as were used in the original Hopf [25] Presentation. Dynamical equation for various order space-time correlation between velocity and magnetic field component are derived from the joint characteristic functional by its expansion in a Taylor series.

The concept of kolomogroff's [31] equilibrium hypothesis for ordinary turbulence are extended to magneto-hydrodynamic turbulence. The problem of predicting the form of the energy spectrum in the equilibrium range is taken up.

The fundamental equation of magnetohydrodynamics for an incompressible fluid are

$$\frac{\partial u}{\partial t}+(u.\nabla)u=-\frac{1}{\rho}\nabla p+\frac{\rho_e}{\rho}E+\frac{\mu}{\rho}j\times H+\nu\nabla^2 u+F \quad (1.9.1),$$

$$\nabla.u=0 \quad (1.9.2),$$

$$\frac{k}{c}\frac{\partial E}{\partial t}=\nabla\times H-4\pi j \quad (1.9.3),$$

$$\frac{\mu}{c}\frac{\partial H}{\partial t}=-\nabla\times E \quad (1.9.4),$$

$$\nabla . H=0 \quad (1.9.5),$$

$$J=\sigma\,(cE+\mu_e u\times H)+\rho_e\frac{u}{c} \quad (1.9.6),$$

where u is the velocity vector; F is the body force; p, the pressure; ρ, the fluid density; ρ_e , electric charge; E, the electric field strength; μ, the magnetic permeability; J, the electric current density; H, the magnetic field strength; ν, the kinematic viscosity; k, the di-electric constant; c, the velocity of light; σ, the electrical conductivity; ∇ , the gradient operator, and t is the time.

When conductivity σ of the fluid tends to infinity the electric field strength E, at each point must tend to the value $\mu\frac{u\times H}{c}$, otherwise the current j given by equation (1.9.6) will become very large even when the slightest mass motions are present.Hence when σ is large we may assume that,

$$E=-\mu\frac{u\times H}{c} \quad (1.9.7),$$

a rotation which will be increasingly valid as $\delta\to\infty$.

An important consequence of relation (1.9.7) is that under the circumstances in which this is a good approximation the energy in the electric field is of the order of $\frac{|\bar{u}|^2}{c^2}$ of the energy in the magnetic field and can therefore be neglected. Consequently in this approximation which is known as the approximation of magneto-hydrodynamics. We have to consider only the interaction between the two fields u and H. In the magneto-hydrodynamics approximation, Maxwell equation (1.9.3) becomes,

$$J=\frac{1}{4\pi}\nabla\times H \qquad (1.9.8).$$

In the frame work of the approximations (1.9.7) and (1.9.8), the Navier-Stokes equations are modified to take into account the electromagnetic body force (assuming that there is no body force F) and equation (1.9.1) becomes

$$\frac{\partial u}{\partial t}+(u.\nabla)\,u=\frac{\mu}{4\pi\rho}\,(\nabla\times H)\times H-\frac{1}{\rho}\nabla p+\nu\nabla^2 u \qquad (1.9.9).$$

Again, in the approximation (1.9.7), Maxwell equation (1.9.4) becomes

$$\frac{\partial H}{\partial t}=\nabla\times(u\times H) \qquad (1.9.10).$$

In a higher approximation in which the loss of energy by joule heat is allowed for equation (1.9.10) is modified to [3].

$$\frac{\partial H}{\partial t}=\nabla\times(u\times H)+\lambda\nabla^2 H \qquad (1.9.11),$$

where $\lambda=(4\pi\mu\sigma)^{-1}$.

The magnetic field intensity H is a solenoidal vector; and in an incompressible fluid the velocity u is also a solenoidal vector, when we use this property of u and H, equations (1.9.9) and (1.9.11) can be written in the forms [2] as

$$\frac{\partial u_i}{\partial t}+\frac{\partial u_i u_k}{\partial x_k}-\frac{\mu}{4\pi\rho}\frac{\partial H_i H_k}{\partial x_k}=-\frac{1}{\rho}\frac{\partial}{\partial x_i}\left(p+\mu\frac{|H|^2}{8\pi}\right)+\nu\nabla^2 u_i \qquad (1.9.12)$$

and

$$\frac{\partial H_i}{\partial t}+\frac{\partial}{\partial x_k}(H_i u_k-u_i H_k)=\lambda\nabla^2 H_i \qquad (1.9.13).$$

Equation (1.9.12) and (1.9.13) from the basis of Batchelor's discussion [3]. Chandrasekhar [7] extended the invariant theory of turbulence to the case of magneto-hydrodynamics. He introduced the new variable $h=\sqrt{\frac{\mu}{4\pi\rho}}H$ for H which has the dimensions of a

velocity known as Alfven velocity. Interms of h, equations (1.9.12) and (1.9.13) can be

written as

$$\frac{\partial u_i}{\partial t}+\frac{\partial}{\partial x_k}(u_i u_k-h_i h_k)=-\frac{\partial P_n}{\partial x_i}+\nu\nabla^2 u_i$$

or

$$\frac{\partial u_i}{\partial t}+u_k\frac{\partial u_i}{\partial x_k}-h_k\frac{\partial h_i}{\partial x_k}=-\frac{\partial P_n}{\partial x_i}+\nu\nabla^2 u_i \qquad (1.9.14)$$

and

$$\frac{\partial h_i}{\partial t}+\frac{\partial}{\partial x_k}(h_i u_k-u_i h_k)=\lambda\nabla^2 h_i$$

or

$$\frac{\partial h_i}{\partial t}+u_k\frac{\partial h_i}{\partial x_k}-h_k\frac{\partial u_i}{\partial x_k}=\lambda\nabla^2 h_i \qquad (1.9.15),$$

where, $P_n=\frac{p}{\rho}+\frac{1}{2}|h|^2$ is the total MHD pressure and $\lambda=(4\pi\mu\sigma)^{-1}$ is the magnetic diffusivity.

Chandrasekhar [10,11] in his theory, consider the correlations between u and h at two points p and p' in the field of isotropic turbulence in the same manner as in ordinary turbulence. Here, we have the double correlations, $\overline{u_i u_j'}, \overline{h_i h_j'}, \overline{u_i h_i'}$ and triple correlations

$$\overline{u_i u_j u_k'}, \overline{h_i h_j u_k'}, \overline{u_i u_j h_k'}, \overline{h_i h_j h_k'}, \overline{(h_i u_j-u_i h_j) h_k'} \text{ and } \overline{(h_j' u_k'-h_k' u_j') u_i},$$

where the subscripts refer to the components of the vectors i,j, k=1,2,3.

Each of these double and triple correlations depends on one scalar function in the case of isotropic turbulence because the divergence of both u and h is zero.

Equations (1.9.14) and (1.9.15) are derived by coupling Maxwell's equations for the electromagnetic field and the Navier-Stokes equations for the velocity field. The Maxwell equations are modified to include the induced electric field due to the fluid motion,and the Navier-Stokes equations are modified to include the

Lorentz force on fluid elements due to the magnetic field. The so-called "Magnetohydrodynamic approximation" is made, in which displacement currents are neglected in Maxwell' equations. This approximation is well-founded provided we are not dealing with very rapid oscillations of the electromagnetic field quantities, as is the case in the propagation of electromagnetic waves. Under this approximation, the energy in the electric field is of the order of $\frac{1}{c^2}$ times the energy in the magnetic field, where c is the speed of light and hence may be neglected. Therefore, we have only to consider the interaction between the velocity field **u** and the magnetic field **h**.

1.10 DECAY OF TURBULENCE BEFORE THE FINAL PERIOD

The energy spectrum at very small wave numbers suffers very little modulation during the whole of the decay process. On the other hand, the energy in higher wave-numbers of the spectrum is being rapidly dissipated by viscosity, and it follows that ultimately the big eddies will supply most of the remaining energy of the turbulence. If we choose the current time t as any instant after this ultimate state has been reached, we have the opportunity of formulating a decay problem in which the initial form of the spectrum (or, rather, the relevant part of it) can be prescribed form the relation $\Phi_{ij}(k) = k_l k_m C_{ijlm} + O(k^3)$.

This would not by itself make a tractable problem, but the assumption already made, that the decay is in an advanced stage, suggests that we might suppose with consistency that the turbulent velocities are so small as to make inertia forces negligible. On this basis the dynamical equation is linear, and we are able to get a complete solution of the decay of the turbulence at very large times after its formation. It happens that this final period of decay occurs at decay times which are within the reach of measurements in a wind-tunnel stream, and it has been possible to obtain valuable information about what, in the initial stages of decay, were the biggest eddies.

In the final period the inertia terms (triple correlations) in the two-point correlation equation obtained from the momentum and continuity equations can be neglected because the Renolds number of the eddies is small, and a solution can be obtained. However, at earlier times the inertia terms in the two-point correlation equation can not be neglected, so that in order to obtain a solution, an intuitive assumption is generally introduced to relate the triple correlations to the double correlations. The situation in homogeneous turbulence is therefore analogous to that in turbulent shear flow where intuitive assumptions have been introduced to relate the Reynolds stress or the eddy diffusivity to the mean flow; although one case of homogeneous turbulence, the turbulence in the final period, has been solved without introducing intuitive hypotheses where as those analysis aided greatly in unifying much of the information on turbulent flow and in clarifying some of the physical aspects of turbulence, they do not, of course, constitute deductive theories based on the momentum and continuity equations.

It should be possible to predict the turbulent decay at times before the final period from the momentum and continuity equations. If the initial distribution of velocities and pressures is known, the momentum and continuity equations could be used numerically to predict the distributions a short time later. However it appears that because of the small size in the calculations would have to be extremely small.

A better plan may be to construct, from the momentum and continuity equations, equations involving correlations between velocities and pressures at more than two points. Then, for instance, in the three-point correlation equation, one neglects the quadruple correlations and obtain's an equation for the triple correlations which should be applicable before the final period. In the final period the triple correlations are of course negligible.Using the expressions for the triple correlations so obtained, the two-point equation can be solved and the various quantities describing the turbulence at times before the final period can be obtained. Higher order approximations, valid at still earlier times, can be obtained in the same way by constructing four or five point correlations. Each time the set of equations is made determinate by neglecting the highest order correlation.

CHAPTER-2

MAGNETIC FIELD COVARIANCE IN MHD TURBULENT FLOW

2.1 INTRODUCTION

The main characteristic of the turbulent flow is that turbulent fluctuations are random in nature and the statistical property of a random variable may be described by the correlation function. It is generally supposed that in a medium of high electrical conductivity turbulence will give rise to a spontaneous generation of magnetic fields ; that in the course of time these fields will be amplified ; and that in an eventual equilibrium state the energy per unit volume in the magnetic field. Taylor [61] studied the correlation or coefficient of correlation between two fluctuating quantities in turbulent flow. Batchelor [4] determined an expression for acceleration covariance of the two particles at two different points x and x' provided that turbulence is isotropic and homogeneous. Jain [28] derived expression for pressure fluctuation and acceleration covariance by using Chandrasekhar's [11] theory of turbulence in turbulent medium which is isotropic homogeneous in space and stationary in time. Kishore and Sinha [33] studied the rate of change of vorticity covariance in ordinary turbulence. Kishore and Sarker [35] also studied the rate of change of vorticity covariance in MHD turbulence.

The main purpose of this chapter is to derive an equation for the rate of change of magnetic field covariance in MHD turbulent flow. Finally, we have shown the analogy between vorticity covariance in ordinary turbulent flow with the magnetic field covariance in MHD turbulent flow.

2.2 MATHEMATICAL MODEL OF THE PROBLEM

The induction equation of MHD turbulent flow is

$$\frac{\partial h_i}{\partial t}+u_k\frac{\partial h_i}{\partial x_k}-h_k\frac{\partial u_i}{\partial x_k}=\lambda\frac{\partial^2 h_i}{\partial x_k \partial x_k} \qquad (2.2.1),$$

where

$\lambda=(4\pi\mu\sigma)^{-1}$ =magnetic diffusivity

$u_i(x,t)$=component of turbulent velocity

$h_i(x,t)$=component of magnetic field

μ=magnetic permeability

σ=electric conductivity .

The equation (2.2.1) is the ith component of the induction equation for MHD turbulent flow. Now, a similar equation for the jth component is

$$\frac{\partial h_j'}{\partial t}+u_k'\frac{\partial h_j'}{\partial x_k'}=h_k'\frac{\partial u_j'}{\partial x_k'}+\lambda\frac{\partial^2 h_j'}{\partial x_k' \partial x_k'} \qquad (2.2.2).$$

Multiplying equation (2.2.1) by h_j' and equation (2.2.2) by h_i, we get

$$h_j' \frac{\partial h_i}{\partial t} + u_k h_j' \frac{\partial h_i}{\partial x_k} = h_j' h_k \frac{\partial u_i}{\partial x_k} + \lambda h_j' \frac{\partial^2 h_i}{\partial x_k \partial x_k} \qquad (2.2.3)$$

and

$$h_i \frac{\partial h_j'}{\partial t} + h_i u_k' \frac{\partial h_j'}{\partial x_k'} = h_i h_k' \frac{\partial u_j'}{\partial x_k'} + \lambda h_i \frac{\partial^2 h_j'}{\partial x_k' \partial x_k'} \qquad (2.2.4).$$

Adding equations (2.2.3) and (2.2.4), we obtain

$$h_j' \frac{\partial h_i}{\partial t} + h_i \frac{\partial h_j'}{\partial t} + u_k h_j' \frac{\partial h_i}{\partial x_k} + h_i u_k' \frac{\partial h_j'}{\partial x_k'} = h_j h_k \frac{\partial u_i}{\partial x_k} + h_i h_k' \frac{\partial u_j'}{\partial x_k'}$$

$$+ \lambda h_j' \frac{\partial^2 h_i}{\partial x_k \partial x_k} + \lambda h_i \frac{\partial^2 h_j'}{\partial x_k' \partial x_k'} \qquad (2.2.5).$$

For an incompressible fluid, we have

$$\frac{\partial h_k}{\partial x_k} = \frac{\partial u_k}{\partial x_k} = o \qquad (2.2.6).$$

Using equation (2.2.6) in equation (2.2.5), we get

$$\frac{\partial}{\partial t}(h_i h_j') + \frac{\partial}{\partial x_k}(h_i h_j' u_k) + \frac{\partial}{\partial x_k'}(h_i h_j' u_k') = \frac{\partial}{\partial x_k}(u_i h_k h_j') + \frac{\partial}{\partial x_k'}(u_j' h_i h_k')$$

$$+ \lambda \frac{\partial^2}{\partial x_k \partial x_k}(h_i h_j') + \lambda \frac{\partial^2}{\partial x_k' \partial x_k'}(h_i h_j') \qquad (2.2.7).$$

Taking ensemble average in equation (2.2.7), we have

$$\frac{\partial}{\partial t}(\overline{h_i h_j'}) + \frac{\partial}{\partial x_k}(\overline{h_i h_j' u_k}) + \frac{\partial}{\partial x_k'}(\overline{h_i h_j' u_k'}) = \frac{\partial}{\partial x_k}(\overline{u_i h_k h_j'}) + \frac{\partial}{\partial x_k'}(\overline{u_j' h_i h_k'})$$

$$+\lambda \frac{\partial^2}{\partial x_k \partial x_k}(\overline{h_i h_j'}) + \lambda \frac{\partial^2}{\partial x_k' \partial x_k'}(\overline{h_i h_j'}) \qquad (2.2.8).$$

For condition of homogeneity (J.O. Hinze [24]), we use

$$\xi_k = x_k' - x_k$$

and

$$\frac{\partial}{\partial \xi_k} = -\frac{\partial}{\partial x_k} = \frac{\partial}{\partial x_k'}$$

in equation (2.2.8), we have

$$\frac{\partial}{\partial t}(\overline{h_i h_j'}) - \frac{\partial}{\partial \xi_k}(\overline{h_i h_j' u_k}) + \frac{\partial}{\partial \xi_k}(\overline{h_i h_j' u_k'}) = -\frac{\partial}{\partial \xi_k}(\overline{u_i h_k h_j'})$$

$$+\frac{\partial}{\partial \xi_k}(\overline{h_i h_k' u_j'}) + 2\lambda \frac{\partial^2}{\partial \xi_k \partial \xi_k}(\overline{h_i h_j'}) \qquad (2.2.9).$$

Putting the following correlation tensors as (Chandrasekhar [11])

$$\overline{h_i h_j' u_k} = S_{ik,j},\ \overline{h_i h_j' u_k'} = S_{i,jk} = -S_{jk,i},\ \overline{u_i h_k h_j'} = S_{ik,j},$$

$$\overline{h_i h_k' u_j'} = S_{i,kj} = -S_{jk,i},\ \overline{h_i h_j'} = H_{ij} \qquad (2.2.10)$$

and substituting in equation (2.2.9), we get

$$\frac{\partial}{\partial t}\overline{h_i h_j'} - \frac{\partial}{\partial \xi_k} S_{ik,j} + \frac{\partial}{\partial \xi_k}(-S_{jk,i}) = -\frac{\partial}{\partial \xi_k} S_{ik,j} + \frac{\partial}{\partial \xi_k}(-S_{jk,i}) + 2\lambda \frac{\partial^2}{\partial \xi_k \partial \xi_k} H_{ij},$$

or

$$\frac{\partial}{\partial t}\overline{h_i h_j'}=\frac{\partial}{\partial \xi_k}S_{ik,j}+\frac{\partial}{\partial \xi_k}S_{jk,i}-\frac{\partial}{\partial \xi_k}S_{ik,j}-\frac{\partial}{\partial \xi_k}S_{jk,i}+2\lambda\frac{\partial^2}{\partial \xi_k \partial \xi_k}H_{ij},$$

or

$$\frac{\partial}{\partial t}\overline{h_i h_j'}=2\lambda\frac{\partial^2}{\partial \xi_k \partial \xi_k}H_{ij} \qquad (2.2.11).$$

The tensor H_{ij} are clearly symmetrical and solenoidal in their indices. Therefore, it can be expressed as [8]

$$H_{ij}=\frac{H'}{r}\xi_i\xi_j-(rH'+2H)\delta_{ij} \qquad (2.2.12),$$

where,

$r=|\xi_k|$ and H(r,t) is the defining scaler of the tensor H_{ij}. In equation (2.2.12) primes attached to scaler function such as H denote the differentiation with respect to r. Therefore,

$$\frac{\partial^2}{\partial \xi_k \partial \xi_k}H_{ij}=\left(\frac{H'''}{r}+4\frac{H''}{r^2}-4\frac{H'}{r^3}\right)\xi_i\xi_j-\left(rH'''+6H''+4\frac{H'}{r}\right)\delta_{ij} \qquad (2.2.13).$$

Substituting equation (2.2.13) in equation (2.2.11), we have

$$\frac{\partial}{\partial t}\overline{h_i h_j'}=2\lambda\left(\frac{H'''}{r}+4\frac{H''}{r^2}-4\frac{H'}{r^3}\right)\xi_i\xi_j-2\lambda\left(rH'''+6H''+4\frac{H'}{r}\right)\delta_{ij} \qquad (2.2.14).$$

Since $\overline{h_i h_j'}$ being an isotropic tensor of the second order depending on r and t, therefore, it can be expressed as

$$\overline{h_i h_j'}=\alpha(r,t)\xi_i\xi_k+\beta(r,t)\delta_{ij} \qquad (2.2.15),$$

which implies that

$$\frac{\partial}{\partial t}\overline{h_i h_j} = \frac{\partial}{\partial t}\alpha(r,t)\xi_i\xi_k + \frac{\partial}{\partial t}\beta(r,t)\delta_{ij} \qquad (2.2.16).$$

Now comparing equations (2.2.16) and (2.2.14) the expressions for $\frac{\partial}{\partial t}\alpha(r,t)$ and $\frac{\partial}{\partial t}\beta(r,t)$ are found as

$$\frac{\partial}{\partial t}\alpha(r,t) = 2\lambda\left(\frac{H'''}{r} + 4\frac{H''}{r^2} - 4\frac{H'}{r^3}\right) \qquad (2.2.17),$$

$$\frac{\partial}{\partial t}\beta(r,t) = -2\lambda\left(rH''' + 6H'' + 4\frac{H'}{r}\right) \qquad (2.2.18).$$

Thus with the help of the above two independent scaler equations (2.2.17) and (2.2.18), the rate of change of magnetic field covariance for MHD turbulent flow can be determined from the equation (2.2.16).

2.3 CONCLUSION

According to Ferraro and Plumpton [19] we know that the vorticity of a fluid is analogous to the magnetic field H. Kishore and Sinha [33] studied the vorticity covariance for ordinary turbulent flow and they obtained the equations

$$\frac{\partial}{\partial t}\alpha(r,t) = 2\nu\left(\frac{Q'''}{r} + 4\frac{Q''}{r^2} - 4\frac{Q'}{r^3}\right) \qquad (2.3.1)$$

and

$$\frac{\partial}{\partial t}\beta(r,t)=-2\nu\left(rQ'''+6Q''+4\frac{Q'}{r}\right) \qquad (2.3.2),$$

where, Q(r,t) is the defining scaler of the tensor $Q_{ij}=\overline{\omega_i\omega'_j}$. Thus we observed that the equation (2.2.17) and (2.2.18) for magnetic field covariance are analogous to the equations (2.3.1) and (2.3.2) of the vorticity covariance for turbulent flow obtained earlier by Kishore and Sinha [33].

CHAPTER-3

DECAY OF TURBULENCE BEFORE THE FINAL PERIOD IN PRESENCE OF DUST PARTICLES

3.1 INTRODUCTION

In the final period of decay, the inertia terms (triple correlations) in the two-point correlation equation obtained from the momentum and the continuity equations can be neglected because the Reynolds number of the turbulent motion is low enough. Batchelor and Townsend [1] studied the decay of turbulence in the final period and they neglected the inertia terms(three point correlation) from the equation of motion. Deissler [15] developed a theory "Decay of homogeneous turbulence for times before the final period". In his paper, he considered two and three point correlation terms and neglecting fourth and higher order correlation terms. Using Deissler's theory Kumar and Patel [36] studied the first order reactant in homogeneous turbulence before the final period for the case of multipoint and single time correlation. Saffman [52] derived an equation that describe the motion of a fluid containing small dust particles.

In this chapter, we have studied the decay of turbulence in presence of dust particles at times before the final period.

3.2 CORRELATION AND SPECTRAL EQUATIONS

The equations of motion of turbulent flow in presence of dust particles for the points P and P' separated by the vector r are

$$\frac{\partial u_i}{\partial t}+\frac{\partial(u_i u_k)}{\partial x_k}=-\frac{1}{\rho}\frac{\partial p}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_k \partial x_k}+\frac{KN}{\rho}(v_i-u_i) \qquad (3.2.1)$$

and

$$\frac{\partial u_j'}{\partial t}+\frac{\partial(u_j' u_k')}{\partial x_k'}=-\frac{1}{\rho}\frac{\partial p'}{\partial x_j'}+\nu\frac{\partial^2 u_j'}{\partial x_k' \partial x_k'}+\frac{KN}{\rho}(v_j'-u_j') \qquad (3.2.2),$$

where,

$u_i(x,t)$ =component of turbulent velocity

$p(x,t)$=hydrodynamic pressure

x_i=space co-ordinate

ν=kinematic viscosity

v_i=component of the fluctuating velocity of dust particles

N=number density of dust particles

ρ=density of the fluid

K=stock resistance.

Multiplying equation (3.2.1) by u_j' and equation (3.2.2) by u_i, we respectively have

$$u_j'\frac{\partial u_i}{\partial t}+u_j'\frac{\partial(u_i u_k)}{\partial x_k}=-\frac{u_j'}{\rho}\frac{\partial p}{\partial x_i}+\nu u_j'\frac{\partial^2 u_i}{\partial x_k \partial x_k}+f u_j'(v_i-u_i) \qquad (3.2.3)$$

and

$$u_i \frac{\partial u_j'}{\partial t} + u_i \frac{\partial (u_j' u_k')}{\partial x_k'} = -\frac{u_i}{\rho} \frac{\partial p'}{\partial x_j'} + \nu u_j' \frac{\partial^2 u_j'}{\partial x_k' \partial x_k'} + f u_i (v_j' - u_j') \qquad (3.2.4),$$

where,

$f = KN/\rho$ has the dimension of the frequency.

Adding (3.2.3) and (3.2.4) and taking the space or time averages, we get

$$\frac{\partial \overline{u_i u_j'}}{\partial t} + \frac{\partial \overline{u_i u_j' u_k}}{\partial x_k} + \frac{\partial \overline{u_i u_j' u_k'}}{\partial x_k'} = -\frac{1}{\rho} \left(\frac{\partial \overline{p u_j'}}{\partial x_i} + \frac{\partial \overline{p' u_i}}{\partial x_j'} \right) + \nu \left(\frac{\partial^2 \overline{u_i u_j'}}{\partial x_k \partial x_k} + \frac{\partial^2 \overline{u_i u_j'}}{\partial x_k' \partial x_k'} \right)$$

$$+ f \left(\left(\overline{v_i u_j'} - \overline{u_i u_j'} + \overline{u_i v_j'} - \overline{u_i u_j'} \right) \qquad (3.2.5).$$

By use of

$$\frac{\partial}{\partial r_i} = -\frac{\partial}{\partial x_i} = \frac{\partial}{\partial x_i'}$$

equation (3.2.5) can be written as

$$\frac{\partial \overline{u_i u_j'}}{\partial t} + \frac{\partial}{\partial r_k} \left(\overline{u_i u_j' u_k'} - \overline{u_i u_j' u_k'} \right) = -\frac{1}{\rho} \left(\frac{\partial \overline{p' u_i}}{\partial r_j} - \frac{\partial \overline{p u_j'}}{\partial r_i} \right) + 2\nu \frac{\partial^2 \overline{u_i u_j'}}{\partial r_k \partial r_K}$$

$$+ f \left(\overline{v_i u_j'} - 2\overline{u_i u_j'} + \overline{u_i v_j'} \right) \qquad (3.2.6).$$

Now we write equation (3.2.6) in spectral form in order to reduce it to an ordinary differential equation because of the physical significance of the spectral quantities. For this, we use three- dimensional Fourier transforms defined as follows:

$$\overline{u_i u_j^{/}(r)} = \int_{-\infty}^{\infty} \overline{\psi_i \psi_j^{/}(k)} \exp(ik.r)\, dk \qquad (3.2.7),$$

$$\overline{u_i u_k u_j^{/}(r)} = \int_{-\infty}^{\infty} \overline{\psi_i \psi_k \psi_j^{/}(k)} \exp(ik.r)\, dk \qquad (3.2.8),$$

$$\overline{p u_j^{/}} = \int_{-\infty}^{\infty} \overline{\lambda \psi_j^{/}(k)} \exp(ik.r)\, dk \qquad (3.2.9),$$

and

$$\overline{v_i u_j^{/}} = \int_{-\infty}^{\infty} \overline{\mu_i \psi_j^{/}(k)} \exp(ik.r)\, dk \qquad (3.2.10),$$

where **k** is known as a wave number vector and $dk = dk_1\, dk_2\, dk_3$.

From equation (3.2.8), we have

$$\overline{u_i u_k u_j^{/}(-r)} = \int_{-\infty}^{\infty} \overline{\psi_i \psi_k \psi_j^{/}(k)} \exp(-ik.r)\, dk = \int_{-\infty}^{\infty} \overline{\psi_i \psi_k \psi_j^{/}(-k)} \exp(ik.r)\, dk \ .$$

Interchanging the subscripts i and j and then interchanging the points P and $P^{/}$ give

$$\overline{u_i u_j^{/}(r)\, u_k^{/}(r)} = \overline{u_j u_k u_i^{/}(-r)} = \int_{-\infty}^{\infty} \overline{\psi_j \psi_k \psi_i^{/}(-k)} \exp(ik.r)\, dk \qquad (3.2.8a)$$

similarly,

$$\overline{u_i p^{/}(r)} = \overline{p u_i(-r)} = \int_{-\infty}^{\infty} \overline{\lambda \psi_i^{/}(-k)} \exp(ik.r)\, dk \qquad (3.2.9a)$$

and

$$\overline{u_i v_j^{/}} = \overline{v_j u_i^{/}(-r)} = \int_{-\infty}^{\infty} \overline{\mu_i \psi_i^{/}(-k)} \exp(ik.\ r)\, dk \qquad (3.2.10a).$$

Substituting equations (3.2.7), (3.2.8), (3.2.8a), (3.2.9), (3.2.9a), (3.2.10) and (3.2.10a) into (3.2.6) we get

$$\frac{d}{dt}\overline{\psi_i\psi'_j}+ik_k[\overline{\psi_j\psi_k\psi'_i(-k)}-\overline{\psi_i\psi_k\psi'_j}]=-\frac{1}{\rho}[ik_j\overline{\lambda\psi'_i(-k)}-ik_i\overline{\lambda\psi'_j}]-2\nu k^2\overline{\psi_i\psi'_j}$$

$$+f[\overline{\mu_i\psi'_j(k)}+\overline{\mu_i\psi'_i(-k)}-2\overline{\psi_i\psi'_j(k)}] \qquad (3.2.11).$$

The tensor equation (3.2.11) becomes a scaler equation by contraction of the indices i and j

$$\frac{d}{dt}\overline{\psi_i\psi'_i}+2\nu k^2\overline{\psi_i\psi'_i}=ik_k[\overline{\psi_i\psi_k\psi'_i}-\overline{\psi_i\psi_k\psi'_i(-k)}]$$

$$+f[\overline{\mu_i\psi'_i(k)}+\overline{\mu_i\psi_i(-k)}-2\overline{\psi_i\psi'_i(k)}] \qquad (3.2.12).$$

The pressure terms drop out of equation (3.2.12) because of the continuity relation

$$\frac{\partial u_i}{\partial x_i}=\frac{\partial u'_i}{\partial x'_i}=0 \ .$$

The first term on the right hand side of equation (3.2.12) is called energy transfer term and the second term comes out for dusty fluid.In the present investigation it is proposed to obtain an expression for the transfer term applicable at times before the final period in presence of dust particles from the three point correlation. To obtain the three- point equation, we consider the equation of motion of turbulent flow in presence of dust particles at points P, P', and P'' as

$$\frac{\partial u_i}{\partial t}+\frac{\partial(u_i u_l)}{\partial x_l}=-\frac{1}{\rho}\frac{\partial p}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_l \partial x_l}+f(v_i-u_i) \qquad (3.2.12a),$$

$$\frac{\partial u_j'}{\partial t}+\frac{\partial(u_j' u_l')}{\partial x_l'}=-\frac{1}{\rho}\frac{\partial p'}{\partial x_j'}+\nu\frac{\partial^2 u_j'}{\partial x_l' \partial x_l'}+f(v_j'-u_j') \qquad (3.2.12b)$$

and

$$\frac{\partial u_k''}{\partial t}+\frac{\partial(u_k'' u_l'')}{\partial x_l''}=-\frac{1}{\rho}\frac{\partial p''}{\partial x_k''}+\nu\frac{\partial^2 u_k''}{\partial x_l'' \partial x_l''}+f(v_k''-u_k'') \qquad (3.2.13).$$

Multiplying (3.2.12a) by $u_j' u_k''$, (3.2.12b) by $u_i u_k''$ and (3.2.13) by $u_i u_j'$, adding the three equations and taking space or time averages, we obtain

$$\frac{\partial}{\partial t}\overline{u_i u_j' u_k''}+\frac{\partial}{\partial x_l}\overline{u_i u_j' u_k'' u_l}+\frac{\partial}{\partial x_l'}\overline{u_i u_j' u_k'' u_l'}+\frac{\partial}{\partial x_l''}\overline{u_i u_j' u_k'' u_l''}=-\frac{1}{\rho}\left(\frac{\partial}{\partial x_i}\overline{p u_j' u_k''}\right.$$

$$\left.+\frac{\partial}{\partial x_j'}\overline{p' u_i u_k''}+\frac{\partial}{\partial x_k''}\overline{p'' u_i u_j'}\right)+\nu\left(\frac{\partial^2 \overline{u_i u_j' u_k''}}{\partial x_l \partial x_l}+\frac{\partial^2 \overline{u_i u_j' u_k''}}{\partial x_l' x \partial x_l'}+\frac{\partial^2 \overline{u_i u_j' u_k''}}{\partial x_l'' \partial x_l''}\right)$$

$$+f(\overline{v_i u_j' u_k''}+\overline{v_j' u_i u_k''}+\overline{v_k'' u_i u_j'}-\overline{u_i u_j' u_k''}-\overline{u_j' u_i u_k''}-\overline{u_i u_j' u_k''}) \qquad (3.2.14).$$

Using the transformations

$$\frac{\partial}{\partial x_l'}=\frac{\partial}{\partial r_l},\ \frac{\partial}{\partial x_l''}=\frac{\partial}{\partial r_l'}$$

and

$$\frac{\partial}{\partial x_1} = -\frac{\partial}{\partial r_1} - \frac{\partial}{\partial r_1'}$$

into equation (3.2.14), we get

$$\frac{\partial}{\partial t}\overline{u_1 u_j' u_k''} - \frac{\partial}{\partial r_1}\overline{u_1 u_j' u_k'' u_1} - \frac{\partial}{\partial r_1'}\overline{u_1 u_j' u_k'' u_1} + \frac{\partial}{\partial r_1}\overline{u_1 u_j' u_k'' u_1'} + \frac{\partial}{\partial r_1'}\overline{u_1 u_j' u_k'' u_1''}$$

$$= -\frac{1}{\rho}\left(-\frac{\partial}{\partial r_1}\overline{p u_j' u_k''} - \frac{\partial}{\partial r_j'}\overline{p u_j' u_k''} + \frac{\partial}{\partial r_j}\overline{p' u_1 u_k''} + \frac{\partial}{\partial r_k'}\overline{p'' u_1 u_j'}\right) + 2\nu\left(\frac{\partial^2 \overline{u_1 u_j' u_k''}}{\partial r_1 \partial r_1}\right.$$

$$\left. + \frac{\partial^2 \overline{u_1 u_j' u_k''}}{\partial r_1 \partial r_1'} + \frac{\partial^2 \overline{u_1 u_j' u_k''}}{\partial r_1' \partial r_1'}\right) + f\left(\overline{v_1 u_j' u_k''} + \overline{v_j' u_1 u_k''} + \overline{v_k'' u_1 u_j'} - 3\overline{u_1 u_j' u_k''}\right) \quad (3.2.15).$$

In order to convert equation (3.2.15) to spectral form, we can define the following six dimensional Fourier transforms:

$$\overline{u_1 u_j'(r)\, u_k''(r')} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\beta_1 \beta_j'(k)\, \beta_k''(k')}\,.\exp[i(k.r + k'.r')]\, dk dk' \quad (3.2.16),$$

$$\overline{u_1 u_1 u_j'(r)\, u_k''(k')} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\beta_1 \beta_1 \beta_j'(k)\, \beta_k''(k')}\,.\exp[i(k.r + k'.r')]\, dk dk'$$

.....(3.2.17),

$$\overline{p u_j'(r)\, u_k''(r')} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\alpha \beta_j'(k)\, \beta_k''(k')}\,.\exp[i(k.r + k'.r')]\, dk.dk' \quad (3.2.18)$$

and

$$\overline{v_1 u_j^{/}(r)\, u_k^{//}(r^{/})} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\gamma_1 \beta_j^{/}(k)\, \beta_k^{//}(k^{/})}\,.\exp[i(k.r+k^{/}.r^{/})]\,dkdk^{/} \quad (3.2.19).$$

By using the method used in obtaining equation (3.2.8a), the following relations result from equations (3.2.17), (3.2.18) and (3.2.19):

$$\overline{u_1 u_1^{/}(r)\, u_j^{/}(r)\, u_k^{//}(r^{/})} = \overline{u_j u_1 u_1^{/}(-r)\, u_k^{//}(r^{/}-r)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\beta_j \beta_1 \beta_1^{/}(-k-k^{/})\, \beta_k^{//}(k^{/})}$$

$$.\exp[i(k.r+k^{/}r^{/})]\,dkdk^{/} \quad (3.2.17a),$$

$$\overline{u_1 u_j^{/}(r)\, u_k^{//}(r^{/})\, u_1^{//}(r^{/})} = \overline{u_k u_1 u_1^{/}(-r^{/})\, u_j^{//}(r-r^{/})}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\beta_k \beta 1 \beta_1^{/}(-k-k^{/})\, \beta_j^{//}(k)}\,.\exp[i(k.r+k^{/}.r^{/})]\,dkdk^{/} \quad (3.2.17b),$$

where the points P and $P^{/}$ are interchanged to obtain equation (3.2.17a). For obtaining (3.2.17b) P is replaced by $P^{/}$, $P^{/}$ is replaced by $p^{//}$, and $P^{//}$ is replaced by P.

Similarly,

$$\overline{u_1 p^{/}(r)\, u_k^{//}(r^{/})} = \overline{p u_1^{/}(-r)\, u_k^{//}(r^{/}-r)}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\alpha \beta_1^{/}(-k-k^{/})\, \beta_k^{//}(k^{/})}\,.\exp[i(k.r+k^{/}.r^{/})]\,dkdk^{/} \quad (3.2.18a),$$

$$\overline{u_1 u_j^{/}(r)\, p^{//}(r^{/})} = \overline{p u_1^{/}(-r^{/})\, u_j^{//}(r-r^{/})}$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\overline{\alpha\beta'_i(-k-k')\beta''_j(k')}.\exp[i(k.r+k'.r')]\,dkdk' \qquad (3.2.19a),$$

$$\overline{v''_k(r')u_iu'_j(r)}=\overline{v''_k(r''-r)u'_i(-r')u''_j(r-r')}$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\overline{\gamma''_k(k')\beta'_i(-k-k')\beta''_j}.\exp[i(k.r+k'.r')]\,dkdk' \qquad (3.2.19b),$$

$$\overline{v'_ju_iu''_k(r')}=\overline{v''_j(r-r')u'_i(-r')u''_k(r'-r)}$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\overline{\gamma''_j(k)\beta'_i(-k-k')\beta''_k(k)}.\exp[i(k.r+k'.r')]\,dkdk' \qquad (3.2.19c).$$

Substituting the preceding relations into equation (3.2.15), we get

$$\frac{d}{dt}\overline{\beta_i\beta'_j\beta''_k}+2\nu(k^2+k_lk'_l+k'^2)\overline{\beta_i\beta'_j\beta''_k}=[i(k_l+k'_l)\overline{\beta_l\beta_i\beta'_j\beta''_k}$$

$$-ik_l\overline{\beta_j\beta_l\beta'_i(-k-k')\beta''_k(k')}-ik'_l\overline{\beta_k\beta_l\beta'_i(-k-k')\beta''_j(k)}]$$

$$-\frac{1}{\rho}[-i(k_i+k'_i)\overline{\alpha\beta'_j\beta''_k}+ik_j\overline{\alpha\beta'_i(-k-k')\beta''_k(k')}+ik_k\overline{\alpha\beta'_i(-k-k')\beta''_j(k)}]$$

$$+f[\overline{\gamma_i\beta'_j(k)\beta''_k(k')}+\overline{\gamma''_j\beta'_i(-k-k')\beta''_k(k)}+\overline{\gamma''_k(k')\beta'_i(-k-k')\beta''_j(k)}$$

$$-3\overline{\beta_i\beta'_j(k)\beta''_k(k')}] \qquad (3.2.20).$$

The tensor equation (3.2.20) can be converted to a scaler equation by contraction of the indexes i and j and inner multiplication by k_k:

$$\frac{d}{dt}(k_k\overline{\beta_i\beta_i'\beta_k''})+2\nu(k^2+k_ik_i'+k'^2)k_k\overline{\beta_i\beta_i'\beta_k''}$$

$$=ik_k(k_i+k_i')\overline{\beta_i\beta_i\beta_i'\beta_k''}-ik_kk_i\overline{\beta_i\beta_i\beta_i'(-k-k')\beta_k''(k)}-ik_kk_i'\overline{\beta_k\beta_i\beta_i'(-k-k')\beta_i'}$$

$$-\frac{1}{\rho}[-ik_k(k_i+k_i')\overline{\alpha\beta_i'\beta_k''}+ik_kk_i\overline{\alpha\beta_i'(-k-k')\beta_k''(k')}+ik_kk_k'\overline{\alpha\beta_i'(-k-k')\beta_i''(k)}]$$

$$+fk_k[\overline{\gamma_i\beta_i'(k)\beta_k''(k')}+\overline{\gamma_i''(k)\beta_i'(-k-k')\beta_k''(k)}+\overline{\gamma_k''(k')\beta_i'(-k-k')\beta_i''(k)}$$

$$-3\overline{\beta_i\beta_i'\beta_k''(k')}] \qquad (3.2.21).$$

To obtain a relation between the terms on the right hand side of equation (3.2.21) derived from the quadruple correlation terms, pressure terms and the dust particle term in equation (3.2.15), take the divergence of the equation of motion and combine with the continuity equation to give

$$\frac{1}{\rho}\frac{\partial^2 p}{\partial x_i \partial x_i}=-\frac{\partial^2(u_iu_i)}{\partial x_i\partial x_i}+f\frac{\partial}{\partial x_i}(v_i-u_i) \qquad (3.2.22).$$

Multiplying the equation (3.2.22) by $u_i'u_k''$, taking space or time averages and writing the resulting equation in terms of the independent variables r and r', give

$$\frac{1}{\rho}\left(\frac{\partial^2 \overline{pu_1'u_k''}}{\partial r_1 \partial r_1}+2\frac{\partial^2 \overline{pu_1'u_k''}}{\partial r_1 \partial r_1'}+\frac{\partial^2 \overline{pu_1'u_k''}}{\partial r_1' \partial r_1'}\right)$$

$$=-\frac{\partial^2 \overline{u_1u_1u_1'u_k''}}{\partial r_1 \partial r_1}-\frac{\partial^2 \overline{u_1u_1u_1'u_k''}}{\partial r_1' \partial r_1}-\frac{\partial^2 \overline{u_1u_1u_1'u_k''}}{\partial r_1 \partial r_1'}-\frac{\partial^2 \overline{u_1u_1u_1'u_k''}}{\partial r_1' \partial r_1'}$$

$$-f\left(\frac{\partial}{\partial r_1}+\frac{\partial}{\partial r_1'}\right)\left(\overline{v_1u_1u_k''}-\overline{u_1u_1'u_k''}\right) \qquad (3.2.23).$$

The Fourier transform of equation (3.2.23) is

$$-\frac{1}{\rho}(k^2+2k_1k_1'+k'^2)\overline{\alpha\beta_1\beta_k''}=(k_1k_1+k_1'k_1+k_1k_1'+k_1'k_1')\overline{\beta_1\beta_1\beta_1'\beta_k''}$$

$$-if(k_1+k_1')\left(\overline{\gamma_1\beta_1'\beta_k''}-\overline{\beta_1\beta_1'\beta_k''}\right)$$

$$-\frac{1}{\rho}\overline{\alpha\beta_1\beta_k''}=\frac{(k_1k_1+k_1'k_1+k_1k_1'+k_1'k_1')\overline{\beta_1\beta_1\beta_1'\beta_k''}-if(k_1+k_1')\left(\overline{\gamma_1\beta_1'\beta_k''}-\overline{\beta_1\beta_1'\beta_k''}\right)}{k^2+2k_1k_1'+k'^2}$$

..... (3.2.24).

Equation (3.2.24) can be used to eliminate the quantities $\overline{\alpha\beta_1'\beta_k''}$, $\overline{\alpha\beta_1'(-k-k')\beta_k''}$, etc., from equation (3.2.21).

3.3 SOLUTION FOR TIMES BEFORE THE FINAL PERIOD.

To obtain the equation for final period of decay the third order correlation terms are neglected compared to the second order correlation terms. Analogously, it would be possible to obtain a solution for times before the final period of decay by neglecting the fourth order correlation terms in comparison with third order correlation terms. If this assumption is made, all the fourth order correlation terms in the right side of equation (3.2.24) should be neglected. Thus from (3.2.21) and (3.2.24) we obtain

$$\frac{d}{dt}(\overline{k_k\beta_i\beta_i'\beta_k''}) + [2\nu(k^2+k_ik_i'+k'^2) - Mf]\,\overline{k_k\beta_i\beta_i'\beta_k''}=0 \qquad (3.3.1),$$

where,

$$M=[\frac{(-k_i+k_i')^2+k_i(k_i+k_i')+k_k'(k_i+k_i')}{k^2+2k_ik_i'+k'^2}(R-1)+S-3] \ ,$$

$$\overline{\gamma_i\beta_i'\beta_k''}=R\overline{\beta_i\beta_i'\beta_k''}$$

and

$$\overline{\gamma_i\beta_i'(k)\beta_k''(k')}+\overline{\gamma_i''(k)\beta_i'(-k-k')\beta_k''(k)}+\overline{\gamma_k''(k')\beta_i'(-k-k')\beta_i''(k)}=S\overline{\beta_i\beta_i'\beta_i'} \ ,$$

also M, R and S are arbitrary constants.

Integrating the equation (3.3.1) between t_0 and t to give

$$\overline{k_k\beta_i\beta_i'\beta_k''}=k_k(\overline{\beta_i\beta_i'\beta_k''})_0\exp[-(2\nu(k^2+kk'\cos\theta+k'^2)-Mf)(t-t_0)] \qquad (3.3.2),$$

where θ is the angle between **k** and **k**$'$.

Now, by letting $r'=0$ in equation (3.2.16) and comparing with equations (3.2.8) and (3.2.8a) we obtains

$$\overline{\psi_i\psi_k\psi'_i(k)}=\int_{-\infty}^{\infty}\overline{\beta_i\beta'_i(k)\,\beta''_k(k')}\,dk' \qquad (3.3.3),$$

$$\overline{\psi_i\psi_k\psi_i(-k)}=\int_{-\infty}^{\infty}\overline{\beta_i\beta'_i(-k)\,\beta''_k(-k')}\,dk' \qquad (3.3.4).$$

Substituting the equations (3.3.2), (3.3.3) and (3.3.4) in equation (3.2.12) we have

$$\frac{d}{dt}\overline{\psi_i\psi'_i(k)}+2\nu k^2\overline{\psi_i\psi'_i(k)}=\int_0^{\infty}2\pi k'^2\,[ik_k(\beta_i\beta'_i\beta''_k-\beta_i\beta'_i(-k)\,\beta''_k(-k'))_0$$

$$\cdot\int_{-1}^{1}[\exp(-\{2\nu(k^2+kk'\cos\theta+k'^2)-Mf\})\,(t-t_0)]\,d(\cos\theta)]\,dk \qquad (3.3.5),$$

where $dk=dk_1'dk_2'dk_3'$ is written interms of k' and θ (cf. Deissler [15])as $dk'=-2\pi k'^2 d(\cos\theta)dk'$ .....(3.3.6).

In order to find the solution completely and following Loeffler and Deissler [3], we assume that

$$ik_k[\overline{\beta_i\beta'_i\beta''_k}-\overline{\beta_i\beta'_i(-k)\,\beta''_k(-k')}]_0=-\beta_o(k^4k'^6-k^6k'^4) \qquad (3.3.7),$$

where β_0 is a constant determined by the initial conditions.

Substituting the equation (3.3.7) in equation (3.3.5) and completing the integration with respect to $\cos\theta$, we have

$$\frac{d}{dt}(2\pi k^2\overline{\psi_i\psi'_i})+2\nu k^2(2\pi k^2\overline{\psi_i\psi'_i})=-\frac{\beta_0}{2\nu(t-t_o)}\int_o^{\infty}(k^5k'^7-k^7k'^5)$$

$$.\,[\exp[-(t-t_0)\,\{2\nu(k^2-kk'+k'^2)-Mf\}]$$

$$-\exp[-(t-t_0)\,\{2\nu(k^2+kk'+k'^2)-Mf]]dk' \qquad (3.3.8)$$

or

$$\frac{dE}{dt}+2\nu k^2E=W \qquad (3.3.9),$$

where, $E=2\pi k^2\overline{\psi_i\psi'_i}$ is the energy spectrum function and W is the energy transfer term given by

$$W=-\frac{\beta_0}{2\nu(t-t_0)}\int_0^\infty (k^5k'^7-k^7k'^5).[\exp[-(t-t_0)\{2\nu(k^2-kk'+k'^2)-Mf\}]$$

$$-\exp[-(t-t_0)\{2\nu(k^2+kk'+k'^2)-Mf\}]]dk' \qquad (3.3.10).$$

Integrating equation (3.3.10) with respect to k', we get

$$W=-\sqrt{\frac{\pi}{2}}\frac{\beta_0}{256}\exp[-\frac{3}{2\nu k^2(t-t_0)}+Mf(t-t_0)].[105\frac{k^6}{(t-t_0)^{\frac{9}{2}}}+45\frac{k^5}{(t-t_0)^{\frac{7}{2}}}$$

$$-19\frac{k^{10}}{(t-t_0)^{\frac{5}{2}}}-3\frac{k^{12}}{(t-t_0)^{\frac{3}{2}}}] \qquad (3.3.11).$$

The series of equation (3.3.11) contains only even power of k. It is interesting to note that

$$\int_0^\infty W dk=0 \qquad (3.3.12).$$

This indicates that the conditions of continuity and homogeneity are maintained.

The linear equation (3.3.9) can be solved to give

$$E=\exp[-2\nu k^2(t-t_0)]\int \exp[2\nu k^2(t-t_0)]\,Wdt+c(k)\exp[-2\nu k^2(t-t_0)]$$

......(3.3.13),

where $C(k)=\frac{(j_0k^4)}{3\pi}$ is a constant of integration and can be obtained following corrsin [12].

Substituting the values of W from equation (3.3.11) in equation (3.3.13) and integrating with respect to t, we get

$$E=\frac{j_0k^4}{3\pi}\exp[-2\nu k^2(t-t_0)]-\frac{\sqrt{\pi}\,\beta_0}{256\nu}\exp[-\frac{3}{2\nu k^2\nu(t-t_0)}+Mf(t-t_0)]$$

$$.[-\frac{15\sqrt{2}k^6}{\nu^{\frac{7}{2}}(t-t_0)^{\frac{7}{2}}}-\frac{12\sqrt{2}k^8}{\nu^{\frac{5}{2}}(t-t_0)^{\frac{5}{2}}}+\frac{7\sqrt{2}k^{10}}{3\nu^{\frac{3}{2}}(t-t_0)^{\frac{3}{2}}}+\frac{16\sqrt{2}k^{12}}{3\nu^{\frac{1}{2}}(t-t_0)^{\frac{1}{2}}}-\frac{32k^{13}}{3}F(\omega)]$$

.....(3.3.14),

where ,

$$F(\omega)=\exp(-\omega^2)\int_0^{\omega}\exp(x^2)\,dx,\ \omega=k[\nu\frac{(t-t_0)}{2}]^{\frac{1}{2}}\ .$$

By setting r=0, j=i, $dk=-2\pi k^2 d(\cos\theta)dk$ and

$$E=2\pi k^2\overline{\psi_i\psi_i'}$$

in equation (3.2.7). We get energy decay as

$$\frac{\overline{u_iu_i'}}{2}=\int_0^{\infty}Edk \qquad (3.3.15).$$

Substituting equation (3.3.14) into (3.3.15) and after integration, we have the energy decay law

$$\frac{\overline{u_i u_i'}}{2}=\frac{j_0 \nu^{-\frac{5}{2}}(t-t_0)^{-\frac{5}{2}}}{32\sqrt{2\pi}}+\exp\{Mf(t-t_o)\}\times 0.2296\beta_o \nu^8 (t-t_o)^{-7}.$$

Thus, the energy decay law of velocity fluctuation before the final period in presence of dust particle may be written as

$$\overline{u^2}=A(t-t_o)^{-\frac{5}{2}}+B(t-t_o)^{-7}\exp\{Mf(t-t_o)\} \qquad (3.3.16),$$

where $\overline{u^2}$ is the mean square of the velocity fluctuation, t is the time, $A=\frac{J_o \nu^{-\frac{5}{2}}}{32\sqrt{2\pi}}$, $B=0.2296\beta_o \nu^8 (t-t_o)^{-7}$ and t_0 are constants determined by the initial conditions.

3.4 CONCLUSION

By neglecting the fourth order correlation terms in the three point correlation equations, results applicable to the turbulence in presence of dust particles before the final period of decay were obtained.For clean fluid, i.e. in absence of dust particles we put f=0, the equation (3.3.16) becomes

$$\overline{u^2}=A(t-t_o)^{-\frac{5}{2}}+B(t-t_o)^{-7},$$

which was obtained earlier by Deissler [15]. At large time the results reduced to those for the final period.

CHAPTER-4

DECAY OF DUSTY FLUID MHD TURBULENCE BEFORE THE FINAL PERIOD

4.1 INTRODUCTION

Saffman [52] observed the effect of dust particles on the stability of the laminar flow of an incompressible fluid with constant mass concentration of dust particles and gave an equation which described the motion of a fluid containing small dust particles. It is a great interest of the behavior of dust particles in turbulent flow to many branches of science and technology, particularly if there is a substantial difference in density between the particles and the fluid. The behavior of dust particles in turbulent flow depends on the concentration of the particles and the size of the particles with respect to the scale of turbulent flow. Deissler [15] developed a theory "Decay of homogeneous turbulence for times before the final period". In his paper, he considered two and three point correlation equations and neglecting fourth and higher order correlation terms. Using Deissler's theory Kumar and Patel [36] studied the " first order reactants in homogeneous turbulent flow before the final period" for the case of multipoint and single time correlation. Loeffler and Deissler [39] studied the decay of temperature fluctuation in homogeneous turbulence before the final period. In their approach they considered the two and three point correlation equations and solved these equations after neglecting the fourth and higher order correlation terms. Following Deissler's approach Sarker and Kishore [54] also studied the decay of MHD turbulence before the final period.

In this chapter, we studied the decay of dusty fluid magneto-hydrodynamic turbulence before the final period. This is the extension work of Sarker and Kishore [54]. The energy decay law for magnetic field fluctuation of dusty fluid MHD turbulence before the final period is in the form $\overline{h^2}=A(t-t_0)^{-\frac{3}{2}}+B(t-t_0)^{-5}.\exp\{Rf(t-t_0)\}$,

where $|\overline{h^2}|$ denotes the total energy, t is the time, A, B, t_0 and R are constants and $f=KN/\rho$ has the dimension of frequency.

4.2 TWO POINT CORRELATION AND SPECTRAL EQUATIONS.

The induction equation of a magnetic field at the point P and $P^{/}$separated by the vector r are

$$\frac{\partial h_i}{\partial t}+u_k\frac{\partial h_i}{\partial x_k}-h_k\frac{\partial u_i}{\partial x_k}=(\frac{\nu}{P_M})\frac{\partial^2 h_i}{\partial x_k \partial x_K} \qquad (4.2.1)$$

and

$$\frac{\partial h_j^{/}}{\partial t}+u_k^{/}\frac{\partial h_j^{/}}{\partial x_k^{/}}-h_k^{/}\frac{\partial u_j^{/}}{\partial x_k^{/}}=(\frac{\nu}{P_M})\frac{\partial^2 h_j^{/}}{\partial x_k^{/}\partial x_k^{/}} \qquad (4.2.2),$$

where,

$u_k(x,t)$=turbulent velocity,

$h_i(x,t)$=magnetic field fluctuation,

$P_M = \frac{\nu}{\lambda}$ =magnetic prandtl number,

ν =kinematic viscosity,

λ =magnetic diffusivity.

Multiplying equation (4.2.1) by h_j' and (4.2.2) by h_i, adding and taking ensemble average, we get

$$\frac{\partial \overline{h_i h_j'}}{\partial t} + \frac{\partial (\overline{u_k h_i h_j'})}{\partial x_k} + \frac{\partial (\overline{u_k' h_i h_j'})}{\partial x_k'} - \frac{\partial (\overline{u_i h_k h_j'})}{\partial x_k} - \frac{\partial (\overline{u_j' h_i h_k'})}{\partial x_k'}$$

$$= \frac{\nu}{P_M} \left(\frac{\partial^2 \overline{h_i h_j'}}{\partial x_k \partial x_k} + \frac{\partial^2 \overline{h_i h_j'}}{\partial x_k' \partial x_k'} \right) \qquad (4.2.3).$$

Using the transformations,

$$\frac{\partial}{\partial r_k} = -\frac{\partial}{\partial x_k} = \frac{\partial}{\partial x_k'}$$

and the relations (cf.Chandrasekhar[8])

$$\overline{u_k h_i h_j'} = -\overline{h_i u_k' h_j'}$$

and

$$\overline{h_i u_j' h_k'} = -\overline{u_i h_k h_j'}$$

in equation (4.2.3), we get

$$\frac{\partial(\overline{h_i h_j'})}{\partial t}+2\left[\frac{\partial}{\partial r_k}(\overline{u_k' h_i h_j'})-\frac{\partial}{\partial r_k}(\overline{u_i h_k h_j'})\right]=2\frac{\nu}{p_M}\frac{\partial^2(\overline{h_i h_j'})}{\partial r_k \partial r_k} \qquad (4.2.4).$$

Now, we write equation (4.2.4) in spectral form by use of the three dimensional Fourier transforms

$$\overline{h_i h_j'}(r)=\int_{-\infty}^{\infty}\overline{\psi_i \psi_j'(k)}\exp(ik.r)\,dk \qquad (4.2.5),$$

$$\overline{u_i h_k h_j'}=\int_{-\infty}^{\infty}\overline{\alpha_i \psi_k \psi_j'(k)}\exp(ik.r)\,dk \qquad (4.2.6).$$

Interchanging the subscripts i and j and then interchanging the points P and P$'$, we have

$$\overline{u_k' h_i h_j'}(r)=\overline{u_k h_i h_j'(-r)}=\int_{-\infty}^{\infty}\overline{\alpha_k \psi_i \psi_j'(-k)}\,.\exp(ik.r)\,dk \qquad (4.2.7).$$

Putting (4.2.5), (4.2.6) and (4.2.7) into equation (4.2.4), we get

$$\frac{\partial}{\partial t}\overline{\psi_i \psi_j'(k)}+2\frac{\nu}{P_M}k^2\overline{\psi_i \psi_j'(k)}=2ik_k[\overline{\alpha_i \psi_k \psi_j'(k)}-\overline{\alpha_k \psi_i \psi_j'(-k)}] \qquad (4.2.8).$$

The tensor equation (4.2.8) becomes a scaler equation by contraction of the indices i and j

$$\frac{\partial}{\partial t}\overline{\psi_i \psi_i'(k)}+2\frac{\nu}{P_M}k^2\overline{\psi_i \psi_j'(k)}=2ik_k[\overline{\alpha_i \psi_k \psi_i'(k)}-\overline{\alpha_k \psi_i \psi_i'(-k)}] \qquad (4.2.9).$$

The term on the right side of equation (4.2.9) is called energy transfer term while the second term on the left hand side is the dissipation term.

4.3 THREE POINT CORRELATION AND EQUATIONS

The momentum equation of MHD turbulence in presence of dust particles at the point P and the induction equations of magnetic field fluctuation at $P^{/}$ and $P^{//}$ as

$$\frac{\partial u_i}{\partial t}+u_k\frac{\partial u_i}{\partial x_k}-h_k\frac{\partial h_i}{\partial x_k}=-\frac{\partial W}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_k\partial x_k}+\frac{KN}{\rho}(v_i-u_i) \qquad (4.3.1),$$

$$\frac{\partial h_i^{/}}{\partial t}+u_k^{/}\frac{\partial h_i^{/}}{\partial x_k^{/}}-h_k^{/}\frac{\partial u_i^{/}}{\partial x_k^{/}}=\frac{\nu}{P_M}\frac{\partial^2 h_i^{/}}{\partial x_k^{/}\partial x_k^{/}} \qquad (4.3.2)$$

and

$$\frac{\partial h_j^{//}}{\partial t}+u_k^{//}\frac{\partial h_j^{//}}{\partial x^{//}}-h_k^{//}\frac{\partial u_j^{//}}{\partial x_k^{//}}=\frac{\nu}{P_M}\frac{\partial^2 h_j^{//}}{\partial x_k^{//}\partial x_k^{//}} \qquad (4.3.3),$$

where,

$W=\dfrac{p}{\rho}+\dfrac{1}{2}|\bar{h}|^2$ =total MHD pressure,

$p(x,t)$=hydrodynamic pressure,

ρ=fluid density,

K =stock resistance,

N =number density of dust particles.

v=component of the fluctuating velocity of dust particles.

Multiplying the equation (4.3.1) by $h_i h_j^{//}$, (4.3.2) by $u_i h_j^{//}$,and (4.3.3) by $u_i h_i^{/}$, adding three equations and taking space or time averages, we obtain

$$\frac{\partial}{\partial t}\overline{(u_i h_j^{/} h_j^{//})}+\frac{\partial}{\partial x_k}\overline{(u_i u_k h_i^{/} h_j^{//})}-\frac{\partial}{\partial x_k}\overline{(h_i h_k h_i^{/} h_j^{//})}+\frac{\partial}{\partial x_k^{/}}\overline{(u_i u_k^{/} h_i^{/} h_j^{//})}$$

$$-\frac{\partial}{\partial x_k^{/}}\overline{(u_i u_i^{/} h_k^{/} h_j^{//})}+\frac{\partial}{\partial x_k^{//}}\overline{(u_i u_k^{//} h_i^{/} h_j^{//})}-\frac{\partial}{\partial x_k^{//}}\overline{(u_i u_j^{//} h_i^{/} h_k^{//})}=-\frac{\partial}{\partial x_i}\overline{(W h_i^{/} h_j^{//})}$$

$$+\nu\frac{\partial^2}{\partial x_k \partial x_k}\overline{(u_i h_i^{/} h_j^{//})}+\frac{\nu}{P_M}\left[\frac{\partial^2}{\partial x_k^{/}\partial x_k^{/}}\overline{(u_i h_i^{/} h_j^{//})}+\frac{\partial^2}{\partial x_k^{//}\partial x_k^{//}}\overline{(u_i h_i^{/} h_j^{//})}\right]$$

$$+f(\overline{v_i h_i h_j^{//}}-\overline{u_i h_i^{/} h_j^{//}}) \qquad (4.3.4).$$

where,

$f=KN/\rho$ has the dimension of the frequency.

Substituting the relations

$$\frac{\partial}{\partial x_k^{/}}=\frac{\partial}{\partial r_k},\ \frac{\partial}{\partial x_k^{//}}=\frac{\partial}{\partial r_k^{/}}$$

and

$$\frac{\partial}{\partial x_k}=-\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k^{/}}\right)$$

into equation (4.3.4), we get

$$\frac{\partial}{\partial t}\overline{(u_i h_i^{/} h_j^{//})}-\frac{\nu}{P_M}[(1+P_M)\frac{\partial^2}{\partial r_k \partial r_k}\overline{(u_i h_i^{/} h_j^{//})}+(1+P_M)\frac{\partial^2}{\partial r_k^{/}\partial r_k^{/}}\overline{(u_i h_i^{/} h_j^{//})}$$

$$+2P_M\frac{\partial^2}{\partial r_k \partial r_k^{/}}\overline{(u_i h_i^{/} h_j^{//})}]=\frac{\partial}{\partial r_k}\overline{(u_i u_k h_i^{/} h_j^{//})}+\frac{\partial}{\partial r_k^{/}}\overline{(u_i u_k h_i^{/} h_j^{//})}$$

$$-\frac{\partial}{\partial r_k^{/}}\overline{(h_i h_k h_i^{/} h_j^{//})}-\frac{\partial}{\partial r_k}\overline{(u_i u_k^{/} h_i^{/} h_j^{//})}+\frac{\partial}{\partial r_k}\overline{(u_i u_i^{/} h_k^{/} h_j^{//})}-\frac{\partial}{\partial r_k^{/}}\overline{(u_i u_k^{//} h_i h_j^{//})}$$

$$+\frac{\partial}{\partial r_k^{/}}\overline{(u_i u_j^{//} h_i^{/} h_k^{//})}+\frac{\partial}{\partial r_i}\overline{(w h_i^{/} h_j^{//})}+\frac{\partial}{\partial r_i^{/}}\overline{(w h_i^{/} h_j^{//})}$$

$$+f(\overline{v_i h_i^{/} h_j^{//}}-\overline{u_i h_i^{/} h_j^{//}}) \qquad (4.3.5).$$

Now, we write equation (4.3.5) in spectral form in order to reduce it to an ordinary differential equation and because of the physical significance of spectral quantities. For this, we use six dimensional Fourier transforms:

$$\overline{u_i h_i^{/}(r)\, h_j^{//}(r^{/})}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\overline{\phi_i \beta_i^{/}(k)\,\beta_j^{//}(k^{/})}.\exp[i(k.r+k^{/}.r^{/})]\,dk dk^{/}$$

$$\ldots\ldots(4.3.6),$$

$$\overline{u_i u_k^{/}(r)\, h_i^{/}(r)\, h_j^{//}(r^{/})}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\overline{\phi_i \phi_k^{/}(k)\,\beta_i^{/}(k)\,\beta_j^{//}(k^{/})}.\exp[i(k.r+k^{/}.r^{/})]\,dk dk$$

$$\ldots\ldots(4.3.7),$$

$$\overline{u_i u_i^{\prime}(r) h_k^{\prime}(r) h_j^{\prime\prime}(r^{\prime})} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\phi_i \phi_i^{\prime}(k) \beta_k^{\prime}(k) \beta_j^{\prime\prime}(k^{\prime})} . \exp[i(k.r+k^{\prime}.r^{\prime})]\, dkdk$$

$$\ldots\ldots(4.3.8),$$

$$\overline{Wh_i^{\prime}(r) h_j^{\prime\prime}(r^{\prime})} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\gamma \beta_i^{\prime}(k) \beta_j^{\prime\prime}(k^{\prime})} . \exp[i(k.r+k^{\prime}.r^{\prime})]\, dkdk^{\prime} \qquad (4.3.9),$$

$$\overline{h_i h_k h_i^{\prime}(r) h_j^{\prime\prime}(r^{\prime})} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\beta_i \beta_k \beta_i^{\prime} \beta_j^{\prime\prime}(k^{\prime})} . \exp[i(k.r+k^{\prime}.r^{\prime})]\, dkdk^{\prime} \; (4.3.10),$$

$$\overline{wh_i^{\prime}(r) h_j^{\prime\prime}(r^{\prime})} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\gamma \beta_i^{\prime}(k) \beta_j^{\prime\prime}(k^{\prime})} . \exp[i(k.r+k^{\prime}.r^{\prime})]\, dkdk^{\prime} \qquad (4.3.11)$$

and

$$\overline{v_i h_i^{\prime}(r) h_j^{\prime\prime}(r^{\prime})} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\mu_i \beta_i^{\prime}(k) \beta_j^{\prime\prime}(k^{\prime})} . \exp[i(k.r+k^{\prime}.r^{\prime})]\, dkdk^{\prime} \; (4.3.12).$$

Interchanging of points $P^{\prime}$ and $P^{\prime\prime}$ along with the indices i and j, result in the relations

$$\overline{u_i u_k^{\prime\prime} h_j^{\prime\prime} h_i^{\prime}} = \overline{u_i u_k^{\prime} h_i^{\prime} h_j^{\prime\prime}}$$

and

$$\overline{u_i u_j^{\prime\prime} h_i^{\prime} h_k^{\prime\prime}} = \overline{u_i u_i^{\prime} h_k^{\prime} h_j^{\prime\prime}}.$$

By use of these facts and relations (4.3.6)-(4.3.12), we can write equation (4.3.5) in the form

$$\frac{d}{dt}\overline{\phi_i \beta_i^{\prime} \beta_i^{\prime\prime}} + \frac{\nu}{P_M}[(1+P_M)k^2+(1+P_M)k^{\prime 2}+2P_M k_k k_k^{\prime}]\overline{\phi_i \beta_i^{\prime} \beta_j^{\prime\prime}}$$

$$= i(k_k+k_k^{\prime})\overline{\phi_i \phi_k \beta_i^{\prime} \beta_j^{\prime\prime}} - i(k_k+k_k^{\prime})\overline{\beta_i \beta_k \beta_i^{\prime} \beta_j^{\prime\prime}} - i(k_k+k_k^{\prime})\overline{\phi_i \phi_k^{\prime} \beta_i^{\prime} \beta_j^{\prime\prime}}$$

$$+ i(k_k+k_k^{\prime})\overline{\phi_i \phi_i^{\prime} \beta_k^{\prime} \beta_j^{\prime\prime}} + i(k_i+k_i^{\prime})\overline{\gamma \beta_i^{\prime} \beta_j^{\prime\prime}} + f(\overline{\mu_i \beta_i^{\prime} \beta_i^{\prime\prime}} - \overline{\phi_i \beta_i^{\prime} \beta_i^{\prime\prime}}) \qquad (4.3.13).$$

The tensor equation (4.3.13) can be converted to the scaler equation by contraction of the subscripts i and j

$$\frac{d}{dt}\overline{\phi_i\beta'_i\beta''_i}+\frac{\nu}{P_M}[(1+P_M)(k^2+k'^2)+2P_M k_k k'_k]\overline{\phi_i\beta'_i\beta''_i}=i(k_k+k'_k)\overline{\phi_i\phi_k\beta'_i\beta''_i}$$

$$-i(k_k+k'_k)\overline{\beta_i\beta_k\beta'_i\beta''_i}-i(k_k+k'_k)\overline{\phi_i\phi'_k\beta'_i\beta''_i}+i(k_k+k'_k)\overline{\phi_i\phi'_i\beta'_k\beta''_i}$$

$$+i(k_i+k'_i)\overline{\gamma\beta'_i\beta''_i}+f(\overline{\mu_i\beta'_i\beta''_i}-\overline{\phi_i\beta'_i\beta''_i}) \qquad (4.3.14).$$

If we take the derivative with respect to x_i of the momentum equation (4.3.1) at P, we obtain

$$-\frac{\partial^2 w}{\partial x_i\partial x_i}=\frac{\partial^2}{\partial x_i\partial x_k}(u_i u_k-h_i h_k)-\frac{\partial}{\partial x_i}f(v_i-u_i) \qquad (4.3.15).$$

Multiplying equation (4.3.15) by $h'_i h''_j$, taking time averages and writing the equation in terms of the independent variables r and r′ we get,

$$-[\frac{\partial^2}{\partial r_i\partial r_i}+\frac{\partial^2}{\partial r'_i\partial r'_i}+2\frac{\partial^2}{\partial r_i\partial r'_i}]\overline{wh'_i h''_j}$$

$$=[\frac{\partial^2}{\partial r_i\partial r_k}+\frac{\partial^2}{\partial r'_i\partial r_k}+\frac{\partial^2}{\partial r_i\partial r'_k}+\frac{\partial^2}{\partial r'_i\partial r'_k}](\overline{u_i u_k h'_i h''_j}-\overline{h_i h_k h'_i h''_j})$$

$$+f(\frac{\partial}{\partial r_i}+\frac{\partial}{\partial r'_i})(\overline{v_i h'_i h''_j}-\overline{u_i h'_i h''_j}) \qquad (4.3.16).$$

Taking the Fourier transforms of equation (4.3.16), we get

$$-\overline{\gamma\beta'_i\beta''_j}=[(k_ik_k+k'_ik_k+k_ik'_k+k'_ik'_k)\,(\overline{\phi_i\phi_k\beta'_i\beta''_j}-\overline{\beta_i\beta_k\beta'_i\beta''_j})$$

$$-if(k_i+k'_i)\,(\overline{\mu\beta'_i\beta''_j}-\overline{\phi_i\beta'_i\beta''_j}]/(k^2+k'^2+2k_ik'_i) \qquad (4.3.17).$$

Equation (4.3.17) can be used to eliminate $\overline{\gamma\beta'_i\beta''_j}$ from equation (4.3.13).

4.4 SOLUTION FOR TIMES BEFORE THE FINAL PERIOD

To study the decay of MHD dusty fluid turbulence for times before the final period, the three point correlation are considered and the quadruple correlation are neglected. If this is happened then equation (4.3.17) shows that the term $\overline{\gamma\beta'_i\beta''_j}$ associated with the pressure correlations, should also be neglected. Thus we have from the equation (4.3.14)

$$\frac{d}{dt}\overline{\phi_i\beta'_i\beta''_i}+[\frac{\nu}{p_M}\{(1+p_M)\,(k^2+k'^2)+2p_Mkk'\}-Rf]\,\overline{\phi_i\beta'_i\beta''_i}=0 \qquad (4.4.1),$$

where,

$$R=\{\frac{(k_i+k'_i)^2}{k^2+k'^2+2k_ik'_i}-1\}\,(S-1)$$

and

$$\overline{\mu\beta'_i\beta''_i}=S\overline{\phi_i\beta'_i\beta''_i}\ ,$$

also R and S are arbitrary constants.

Integrating the equation (4.4.1) between t_0 and t with inner multiplication by k_k and gives

$$\overline{k_k\phi_i\beta'_i\beta''_i}=k_k[\overline{\phi_i\beta_i\beta''_i}]_0\exp[\{-\frac{\nu}{p_M}[(1+p_M)(k^2+k'^2)+2p_M kk'\cos\theta]$$

$$+Rf\}(t-t_0)] \qquad (4.4.2),$$

where θ is the angle between k and k'. Now letting $r'=0$ in equation (4.3.6) and comparing (4.2.6) and (4.2.7), we have

$$\overline{\alpha_i\psi_k\psi'_i(k)}=\int_{-\infty}^{\infty}\overline{\phi_i\beta'_i\beta''_i}\,dk' \qquad (4.4.3)$$

and

$$\overline{\alpha_k\psi_i\psi'_i(-k)}=\int_{-\infty}^{\infty}\overline{\phi_i\beta'_i(-k)\beta''_i(-k')}\,dk' \qquad (4.4.4).$$

Substituting equations (4.4.2), (4.4.3) and (4.4.4) in equation (4.2.9), we get

$$\frac{d}{dt}\overline{\psi_i\psi'_i(k)}+2\frac{\nu}{p_M}k^2\overline{\psi_i\psi'_i(k)}=\int_{-\infty}^{\infty}2ik_k[\overline{\phi_i\beta'_i\beta''_i}-\overline{\phi_i\beta'_i(-k)\beta''_i(-k')}]_0$$

$$.\exp[\{-\frac{\nu}{p_M}[(1+p_M)(k^2+k'^2)+2p_M kk'\cos\theta]+Rf\}(t-t_0)]dk' \qquad (4.4.5).$$

Now, $dk'=dk_1'dk_2'dk_3'$ can be expressed interms of $k'$and θ (cf.Deissler [15]) as

$$dk'=-2\pi k'^2 d(\cos\theta)\,dk' \qquad (4.4.6).$$

Substituting (4.4.6) to equation (4.4.5) to give

$$\frac{d}{dt}\overline{\psi_i\psi_i'(k)}+2\frac{\nu}{p_M}k^2\overline{\psi_i\psi_i'(k)}=2\int_0^\infty 2\pi i k_k[\overline{\phi_i\beta_i'\beta_i''}-\overline{\phi_i\beta_i(-k)\beta_i''(-k')}]_0 k'^2$$

$$.\,[\int_{-1}^{1}\exp[\{-\frac{\nu}{p_M}[(1+p_M)(k^2+k'^2)+2p_M kk'\cos\theta]+Rf\}(t-t_0)]d(\cos\theta)]\,dk'$$

$$\qquad .....(4.4.7).$$

In order to find the solution completely and following Loeffler and deissler [3], we assume that

$$ik_k[\overline{\phi_i\beta_i'\beta_i''}-\overline{\phi_i\beta_i'(-k)\beta_i''}]_0=-\frac{\xi_0}{(2\pi)^2}(k^2k'^4-k^4k'^2) \qquad (4.4.8),$$

where ξ_0 is a constant depending on the initial conditions.Putting (4.4.8) in equation (4.4.7) and completing the integration with respect to $\cos\theta$, we get

$$\frac{d}{dt}(2\pi\overline{\psi_1\psi'_1(k)})+2\frac{\nu}{p_M}k^2(2\pi\overline{\psi_1\psi'_1(k)})=-\frac{\xi_0}{\nu(t-t_0)}\int_0^\infty(kk'^5-k^3k'^3)$$

$$.[\exp[\{-\frac{\nu}{p_M}[(1+p_M)(k^2+k'^2)-2p_Mkk']+Rf\}(t-t_0)]$$

$$-\exp[\{-\frac{\nu}{p_M}((1+p_M)(k^2+k'^2)+2p_Mkk')+Rf\}(t-t_0)]]dk' \qquad (4.4.9).$$

Multiplying both sides by k^2, we have

$$\frac{dH}{dt}+2\frac{\nu}{p_M}k^2H=G \qquad (4.4.10),$$

where $H=2\pi k^2\overline{\psi_1\psi'_1(k)}$ is the magnetic energy spectrum function and G is the energy transfer term given by

$$G=-\frac{\xi_0}{\nu(t-t_0)}\int_0^\infty(k^3k'^5-k^5k'^3)\cdot\exp[\{-\frac{\nu}{p_M}[(1+p_M)(k^2+k'^2)$$

$$-2p_Mkk']Rf\}(t-t_0)]dk' \qquad (4.4.11).$$

Integrating equation (4.4.11) with respect to k', we obtain

$$G=-\frac{\sqrt{\pi}\,\xi_0 p_M^{\frac{5}{2}}}{\nu^{\frac{3}{2}}(t-t_0)^{\frac{3}{2}}(1+p_M)^{\frac{5}{2}}}\exp[\{-\frac{\nu k^2}{p_M}(\frac{1+2p_M}{1+p_M})+Rf\}(t-t_0)]$$

$$.[\frac{15p_M k^4}{4\nu^2(t-t_0)^2(1+p_M)}+\{\frac{5p_M^2}{(1+p_M)^2\nu(t-t_0)}-\frac{3}{2\nu(t-t_0)}\}k^6$$

$$+\frac{p_M}{1+p_M}\{\frac{p_M^2}{(1+p_M)^2}-1\}k^8] \qquad (4.4.12).$$

The series of equation (4.4.12) contains only even power of k. It is interesting to note that

$$\int_0^\infty G.\,dk=0 \qquad (4.4.13).$$

The linear equation (4.4.10) can be solved to give

$$H=\exp[-\frac{2\nu k^2(t-t_0)}{p_M}]\int G.\exp[\frac{2\nu k^2(t-t_0)}{p_M}]\,dt$$

$$+J(k)\exp[-\frac{2\nu k^2(t-t_0)}{p_M}] \qquad (4.4.14),$$

where $J(k)=\frac{N_0 k^2}{\pi}$ is a constant of integration and can be obtained as by Corrsin [12]. Substituting the values of G from equation (4.4.12) in equation (4.4.14) and integrating with respect to t,we get

$$H=\frac{N_0k^2}{\pi}\exp[-\frac{2\nu}{p_M}k^2(t-t_0)]+\frac{\sqrt{\pi}\,\xi_0 p_M^{\frac{5}{2}}}{\nu^{\frac{3}{2}}(1+p_M)^{\frac{7}{2}}}\exp[\{-\frac{\nu k^2}{p_M}(\frac{1+2p_M}{1+p_M})+Rf\}(t-t_0)]$$

$$.[\frac{3p_M k^4}{2\nu^2(t-t_0)^{\frac{5}{2}}}+\frac{(7p_M^2-6p_M)k^6}{3\nu(1+p_M)(t-t_0)^{\frac{3}{2}}}-\frac{4}{3}\frac{(3p_M^2-2p_M+3)k^8}{(1-p_M)^2(t-t_0)^{\frac{1}{2}}}$$

$$+\frac{8\sqrt{\nu}}{3}\frac{(3p_M^2-2p_M+3)k^9}{(1+p_M)^{\frac{5}{2}}p_M^{\frac{1}{2}}}F(\omega)] \qquad (4.4.15),$$

where,

$$F(\omega)=\exp(-\omega^2)\int_0^\infty \exp(X^2)\,dX$$

and

$$\omega=k[\frac{\nu(t-t_0)}{p_M(1+p_M)}]^{\frac{1}{2}}\ .$$

By setting $r=0, dk=2\pi k^2 d(\cos\theta)\,dk$ and $H=2\pi k^2\overline{\psi_i\psi_i'}$ in equation (4.2.5), we get energy decay as

$$\frac{\overline{h_i h_i'}}{2}=\int_0^\infty H\,dk \qquad (4.4.16).$$

substituting equation (4.4.15) into the equation (4.4.16) and after integration, we can obtain

$$\frac{\overline{h_i h_i'}}{2}=\frac{N_0 p_M^{\frac{3}{2}} \nu^{-\frac{3}{2}} (t-t_0)^{-\frac{3}{2}}}{8\sqrt{2\pi}}+\exp\{Rf(t-t_0)\}\xi_0 Q\nu^{-6}(t-t_0)^{-5} \qquad (4.4.17),$$

where,

$$Q=\frac{\pi p_M^6}{(1+p_M)(1+2p_M)^{\frac{5}{2}}}\left[\frac{9}{16}+\frac{5p_M(7p_M-6)}{16(1+2p_M)}-\frac{35}{8}\frac{p_M(3p_M^2-2p_M+3)}{(1+2p_M)^2}+.....\right].$$

Thus, the decay law for magnetic energy fluctuation before the final period in presence of dust particle may be written as

$$\overline{h^2}=A(t-t_0)^{-\frac{3}{2}}+B(t-t_0)^{-5}\exp\{Rf(t-t_0)\} \qquad (4.4.18),$$

where, $\overline{h^2}$ is the mean square of the magnetic field fluctuation, t is the time, $A=\frac{N_0 p_M^{\frac{3}{2}} \nu^{-\frac{3}{2}}}{8\sqrt{2}\pi}$, $B=\xi_0 Q\nu^{-6}$ and t_0 are constants determined by the initial conditions.

4.5 CONCLUSION

The results of the present study, obtained by neglecting the quadruple correlations in the three point correlation equation, appear to represent the MHD dusty fluid turbulence for times before the final period.For clean fluid, i.e. in absence of dust particles, we put f=0, the equation (4.4.18) becomes

$$\overline{h^2}=A(t-t_0)^{-\frac{3}{2}}+B(t-t_0)^{-5},$$

which was obtained earlier by Sarker and Kishore [54]. For large times, the last term in the equation becomes negligible,giving the -3/2 power decay law for the final period.

CHAPTER-5

DECAY OF TEMPERATURE FLUCTUATIONS IN MHD TURBULENCE BEFORE THE FINAL PERIOD

5.1 INTRODUCTION

The problem of the decay of temperature fluctuations in homogeneous turbulence would appear to be one of the initial steps required for understanding the important process of heat transfer in shear turbulence.As pointed out in [12], such a study would also be applicable to concentration fluctuations during the mixing of equidense fluids, for the case of constant mutual diffusion coefficient and no interfacial tension. Deissler [15], the decay of homogeneous turbulence before the final period was analyzed by utilizing correlation equations for fluctuating quantities at two and three points in the fluid. The set of equations was made determinate by neglecting the quadruple correlations in the three point equation. Corrsin [12,13] has already made an analytical attempt on the problem of turbulent temperature fluctuations using the approaches employed in the statistical theory of turbulence. Loeffler and Deissler [39] presented a theory "Decay of temperature fluctuation in homogeneous turbulence before the final period".In their approach they considered fourth-order correlation terms are negligible compared to the third order correlation terms.

In this chapter, we also studied the decay of temperature fluctuations in magneto hydrodynamics turbulence before the final period. Finally energy decay law for temperature field fluctuation of MHD turbulence before the final period is obtained.

5.2 CORRELATION AND SPECTRAL EQUATIONS

The induction equation of a magnetic field at the point P is

$$\frac{\partial h_i}{\partial t}+u_k\frac{\partial h_i}{\partial x_k}-h_k\frac{\partial u_i}{\partial x_k}=\left(\frac{\nu}{P_M}\right)\frac{\partial^2 h_i}{\partial x_k \partial x_k} \tag{5.2.1}$$

and the energy equation at the point P' is

$$\frac{\partial T'_j}{\partial t}+u'_k\frac{\partial T'_j}{\partial x'_k}=\left(\frac{\nu}{P_r}\right)\frac{\partial^2 T'_j}{\partial x'_k \partial x'_k} \tag{5.2.2},$$

where,

u_i =component of turbulent velocity,

h_i =component of magnetic field,

$P_M = \nu/\lambda$=magnetic prandtl number,

$P_r = \nu/\gamma$ =prandtl number,

ν =kinematic viscosity,

$\lambda = (4\pi\mu\sigma)^{-1}$= magnetic diffusivity,

$\gamma = \frac{k}{\rho c_p}$ =thermal diffusivity ,

C_p=heat capacity at constant pressure,

x_k =space co-ordinate.

Multiplying equation (5.2.1) by T_j' and (5.2.2)by h_i , adding and taking ensemble average, we get

$$\frac{\partial \overline{h_i T_j'}}{\partial t}+u_k\frac{\partial \overline{h_i T_j'}}{\partial x_k}+u_k'\frac{\partial \overline{h_i T_j'}}{\partial x_k'}-h_k\frac{\partial \overline{u_i T_j'}}{\partial x_k'}$$

$$=\nu\left[\frac{1}{p_M}\frac{\partial^2 \overline{h_i T_j'}}{\partial x_k \partial x_k}+\frac{1}{p_r}\frac{\partial^2 \overline{h_i T_j'}}{\partial x_k' \partial x_k'}\right] \qquad (5.2.3).$$

The continuity equation is

$$\frac{\partial u_k}{\partial x_k}=\frac{\partial u_k'}{\partial x_k'}=0 \qquad (5.2.4).$$

Substituting equation (5.2.4) in to equation (5.2.3) yields

$$\frac{\partial}{\partial t}\overline{(h_i T_j')}+\frac{\partial}{\partial x_k}\overline{(u_k h_i T_j')}+\frac{\partial}{\partial x_k'}\overline{(u_k' h_i T_j')}-\frac{\partial}{\partial x_k}\overline{(u_i h_k T_j')}$$

$$=\nu\left[\frac{1}{p_M}\frac{\partial^2 \overline{(h_i T_j')}}{\partial x_k \partial x_k}+\frac{1}{p_r}\frac{\partial^2 \overline{(h_i T_j')}}{\partial x_k' \partial x_k'}\right] \qquad (5.2.5).$$

Using the transformation

$$\frac{\partial}{\partial r_k} = -\frac{\partial}{\partial x_k} = \frac{\partial}{\partial x'_k}$$

and relations (cf. Chandrasekhar [8])

$$\overline{u_k h_i T'_j} = -\overline{h_i u'_k T'_j}$$

equation (5.2.5) becomes

$$\frac{\partial}{\partial t}\overline{(h_i T'_j)} + 2\frac{\partial}{\partial r_k}\overline{(u'_k h_i T'_j)} + \frac{\partial}{\partial r_k}\overline{(u_i h_k T'_j)}$$

$$= \nu\left[\frac{\partial^2 \overline{(h_i T'_j)}}{\partial r_k \partial r_k}\left(\frac{1}{p_M} + \frac{1}{p_r}\right)\right] \qquad (5.2.6).$$

It is convenient to write this equation in spectral form by use of the following three dimensional Fourier transforms

$$\overline{h_i T'_j(r)} = \int_{-\infty}^{\infty} \overline{l_i \tau'_j(k)} \exp(ik.r)\, dk \qquad (5.2.7),$$

$$\overline{u_i h_k T'_j} = \int_{-\infty}^{\infty} \overline{\phi_i l_k \tau'_j(k)} \exp(ik.r)\, dk \qquad (5.2.8)$$

and since it is obvious by interchanging p and p$'$ that

$$\overline{u'_k h_i T'_j} = \overline{u_k h_i T_j(-r)} = \int_{-\infty}^{\infty} \overline{\phi_k l_i \tau'_j(-k)} \exp(ik.r)\, dk \qquad (5.2.9).$$

Substituting of equation (5.2.7) to (5.2.9) in to equations (5.2.6) leads to the spectral equation

$$\frac{\partial}{\partial t}\overline{(l_i\tau'_j(k))}+ik_k[2\overline{\phi_k l_i\tau_j(-k)}+\overline{\phi_i l_k\tau'_j(k)}]$$

$$=-\nu[(\frac{1}{p_M}+\frac{1}{p_r})k^2\overline{l_i\tau'_j(k)}] \qquad (5.2.10).$$

The tensor equation (5.2.10) becomes a scalar equation by contraction of the indices i and j

$$\frac{\partial}{\partial t}\overline{l_i\tau_i(k)}+ik_k[2\overline{\phi_k l_i\tau_i(-k)}+\overline{\phi_i l_k\tau'_i(k)}]$$

$$=-\nu[(\frac{1}{p_M}+\frac{1}{p_r})k^2\overline{l_i\tau'_i(k)}] \qquad (5.2.11).$$

5.3 THREE POINT CORRELATION AND EQUATION

The momentum equation of MHD turbulence at the point P, the induction equation at the point P' and the energy equation at P'' as

$$\frac{\partial u}{\partial t}+u_k\frac{\partial u_i}{\partial x_k}-h_k\frac{\partial h_i}{\partial x_k}=-\frac{\partial W}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_k\partial x_k} \qquad (5.3.1),$$

$$\frac{\partial h'_i}{\partial t}+u'_k\frac{\partial h'_i}{\partial x'_k}-h'_k\frac{\partial u'_i}{\partial x'_k}=(\frac{\nu}{p_M})\frac{\partial^2 h'_i}{\partial x'_k\partial x'_k} \qquad (5.3.2)$$

and

$$\frac{\partial T_j^{\prime\prime}}{\partial t}+u_k^{\prime\prime}\frac{\partial T_j^{\prime\prime}}{\partial x_k^{\prime\prime}}=\left(\frac{\nu}{P_r}\right)\frac{\partial^2 T_j^{\prime\prime}}{\partial x_k^{\prime\prime}\partial x_k^{\prime\prime}} \quad (5.3.3),$$

where,

$W=\dfrac{p}{\rho}+\dfrac{1}{2}|\bar{h}|^2$ =total MHD pressure and

$p(x,t)$=hydrodynamic pressure.

Multiplying equation (5.3.1) by $h_i^{\prime}T_j^{\prime\prime}$, (5.3.2)by $u_iT_j^{\prime\prime}$, and (5.3.3) by $u_ih_i^{\prime}$, adding the equation and taking space or time averages, we obtain

$$\frac{\partial}{\partial t}(\overline{u_ih_i^{\prime}T_j^{\prime\prime}})+\frac{\partial}{\partial x_k}(\overline{u_iu_kh_i^{\prime}T_j^{\prime\prime}})-\frac{\partial}{\partial x_k}(\overline{h_ih_kh_i^{\prime}T_j^{\prime\prime}})+\frac{\partial}{\partial x_k^{\prime}}(\overline{u_iu_k^{\prime}h_i^{\prime}T_j^{\prime\prime}})$$

$$-\frac{\partial}{\partial x_k^{\prime}}(\overline{u_iu_i^{\prime}h_k^{\prime}T_j^{\prime\prime}})+\frac{\partial}{\partial x_k^{\prime\prime}}(\overline{u_ih_i^{\prime}u_k^{\prime\prime}T_j^{\prime\prime}})=-\frac{\partial}{\partial x_i}(\overline{Wh_i^{\prime}T_j^{\prime\prime}})+\nu\frac{\partial^2}{\partial x_k\partial x_k}(\overline{u_ih_i^{\prime}T_j^{\prime\prime}})$$

$$+\nu\left[\frac{1}{P_M}\frac{\partial^2}{\partial x_k^{\prime}\partial x_k^{\prime}}(\overline{u_ih_i^{\prime}T_j^{\prime\prime}})+\frac{1}{P_r}\frac{\partial^2}{\partial x_k^{\prime\prime}\partial x_k^{\prime\prime}}(\overline{u_ih_i^{\prime}T_j^{\prime\prime}})\right] \quad (5.3.4).$$

Substituting the relations

$$\frac{\partial}{\partial x_k^{\prime}}=\frac{\partial}{\partial r_k},\ \frac{\partial}{\partial x_k^{\prime\prime}}=\frac{\partial}{\partial r_k^{\prime}}$$

and

$$\frac{\partial}{\partial x_k} = -\left(\frac{\partial}{\partial r_k} + \frac{\partial}{\partial r'_k}\right)$$

into equation (5.3.4), we get

$$\frac{\partial}{\partial t}(\overline{u_i h'_i T''_j}) - \nu\left[\left(1+\frac{1}{P_M}\right)\frac{\partial^2}{\partial r_k \partial r_k}(\overline{u_i h'_i T''_j}) + \left(1+\frac{1}{P_r}\right)\frac{\partial^2}{\partial r'_k \partial r'_k}(\overline{u_i h'_i T''_j})\right.$$

$$\left.+2\frac{\partial^2}{\partial r'_k \partial r'_k}(\overline{u_i h'_i T''_j})\right] = \frac{\partial}{\partial r_k}(\overline{u_i u_k h'_i T''_j}) + \frac{\partial}{\partial r'_k}(\overline{u_i u_k h'_i T''_j}) - \frac{\partial}{\partial r_k}(\overline{h_i h_k h'_i T''_j})$$

$$-\frac{\partial}{\partial r_k}(\overline{h_i h_k h'_i T''_j}) - \frac{\partial}{\partial r'_k}(\overline{h_i h_k h'_i T''_j}) - \frac{\partial}{\partial r_k}(\overline{u_i u'_k h'_i T''_j}) + \frac{\partial}{\partial r_k}(\overline{u_i u'_i h'_k T''_j})$$

$$-\frac{\partial}{\partial r'_k}(\overline{u_i u''_k h'_i T''_j}) + \frac{\partial}{\partial r_i}(\overline{W h'_i T''_j}) + \frac{\partial}{\partial r'_i}(\overline{W h'_i T''_j}) \qquad (5.3.5).$$

Six-dimensional Fourier transforms for quantities in this equation may be defined as

$$\overline{u_i h'_i(r)\, T''_j(r')} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\phi_i \beta'_i(k)\, \theta''_j(k')}\,.\exp[i(k.r+k'.r')]\,dk\,dk' \qquad (5.3.6),$$

$$\overline{u_i u_k h_i' T_j''} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\phi_i \phi_k \beta_i'(k) \theta_j''(k')} . \exp[i(k.r + k'.r')]\, dk dk' \quad (5.3.7),$$

$$\overline{h_i h_k h_i' T_j''} = \int_{-\infty}^{\infty} \overline{\beta_i \beta_k \beta_i'(k) \theta_j''(k')} . \exp[i(k.r + k'.r')]\, dk dk' \quad (5.3.8),$$

$$\overline{u_i u_k' h_i' T_j''} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\phi_i \phi_k'(k) \beta_i'(k) \theta_j''(k')} . \exp[i(k.r + k'.r')]\, dk dk' \quad (5.3.9),$$

$$\overline{u_i u_i' h_k' T_j''} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\phi_i \phi_i'(k) \beta_k'(k) \theta_j''}(k') . \exp[i(k.r + k'.r')]\, dk dk' \quad (5.3.10),$$

$$\overline{W h_i'(r) T_j''(r')} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\gamma \beta_i'(k) \theta_j''(k')} . \exp[i(k.r + k'.r')]\, dk dk' \quad (5.3.11).$$

Interchange of points P' and P'' along with the indices i and j, result in the relations

$$\overline{u_i u_k'' h_i' T_j''} = \overline{u_i u_k' h_i' T_j''} \; .$$

By use of these facts and equations (5.3.6) to (5.3.11), equation (5.3.5) may be transformed to

$$\frac{\partial}{\partial t}(\overline{\phi_i \beta_i' \theta_j''}) + \nu[(1+\frac{1}{p_M})k^2 + (1+\frac{1}{p_r})k'^2 + 2k_k k_k']\overline{\phi_i \beta_i' \theta_j''} = i(k_k + k_k')\overline{\phi_i \phi_k \beta_i' \theta_j''}$$

$$-i(k_k + k_k')\overline{\beta_i \beta_k \beta_i' \theta_j''} - i(k_k + k_k')\overline{\phi_i \phi_k' \beta_i' \theta_j''} + i k_k \overline{\phi_i \phi_i' \beta_k' \theta_j''} + i(k_k + k_k')\overline{\gamma \beta_i' \theta_j''}$$

.....(5.3.12).

The tensor equation (5.3.12) can be converted to scalar equation by contraction of the indices i and j

$$\frac{\partial}{\partial t}(\overline{\phi_i\beta'_i\theta''_i})+\nu[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2+2k_kk'_k]\overline{\phi_i\beta_i\theta''_i}$$

$$=i(k_k+k'_k)\overline{\phi_i\phi_k\beta'_i\theta''_i}-i(k_k+k'_k)\overline{\beta_i\beta_k\beta'_i\theta''_i}-i(k_k+k'_k)\overline{\phi_i\phi_k\beta'_i\theta''_i}$$

$$+ik_k\overline{\phi_i\phi'_i\beta'_k\theta''_i}+i(k_k+k'_k)\overline{\gamma\beta'_i\theta''_i} \qquad (5.3.13).$$

If the derivative with respect to x_i is taken of the momentum equation (5.3.1) for point P, the equation multiplied through by $h_iT''_j$ and time average taken, the resulting equation

$$-\frac{\partial^2(\overline{wh'_iT''_j})}{\partial x_i\partial x_i}=\frac{\partial^2}{\partial x_i\partial x_k}(\overline{u_iu_kh'_iT''_j}-\overline{h_ih_kh'_iT''_j}) \qquad (5.3.14)$$

or, interms of the displacement vector r and r' this becomes

$$-[\frac{\partial^2}{\partial r_i\partial r_i}+2\frac{\partial^2}{\partial r'_i\partial r_i}+\frac{\partial^2}{\partial r'_i\partial r'_i}](\overline{wh'T''_j})$$

$$=[\frac{\partial^2}{\partial r_i\partial r_k}+\frac{\partial^2}{\partial r'_i\partial r_k}+\frac{\partial^2}{\partial r_i\partial r'_k}+\frac{\partial^2}{\partial r'_i\partial r'_k}](\overline{u_iu_kh'_iT''_j}-\overline{h_ih_kh'_iT''_j}) \qquad (5.3.15).$$

Taking the Fourier transforms of equation (5.3.15),

$$-\overline{\gamma\beta'_i\theta''_j}=\frac{(k_ik_k+k'_ik_k+k_ik'_k+k'_ik'_k)(\overline{\phi_i\phi_k\beta'_i\theta''_j}-\overline{\beta_i\beta_k\beta'_i\theta''_j})}{k_ik_i+2k'_ik_i+k'_ik'_i} \qquad (5.3.16).$$

Equation (5.3.16) can be used to eliminate $\overline{\gamma\beta'_i\theta''_j}$ from equation (5.3.12).

5.4 SOLUTION FOR TIMES BEFORE THE FINAL PERIOD

It is known that the equation for final period decay is obtained by considering the two point correlation terms after neglecting the third order correlations. Analogously, it would be anticipated that for times before the final period the fourth-order correlation terms should be negligible in comparison with the third order terms. If this assumption is made then equation (5.3.16) shows that term $\overline{\gamma\beta'_i\theta''_j}$ associated with the pressure fluctuations, should also be neglected. Thus, neglecting all the terms on the right hand side of equation (5.3.13), we get

$$\frac{\partial}{\partial t}(\overline{\phi_i\beta'_i\theta''_i})+\nu\left[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2+2k_kk'_k\right]\overline{\phi_i\beta'_i\theta''_i}=0 \qquad (5.4.1).$$

Integrating the equation (5.4.1) between t_0 and t with inner multiplication by k_k and gives

$$\overline{k_i\phi_i\beta'_i\theta''_i}=k_k[\overline{\phi_i\beta'_i\theta''_i}]_0.\exp[-\nu\{(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2+2kk'\cos\theta\}(t-t_o)]$$

.....(5.4.2),

where θ is the angle between k and k'.

Letting r'=o in equation (5.3.6) and comparing with equations (5.2.8) and (5.2.9) we get

$$\overline{\phi_i l_k\tau'_i(k)}=\int_{-\infty}^{\infty}\overline{\phi_i\beta'_i\theta''_i}\,dk' \qquad (5.4.3)$$

$$\overline{\phi_i l_i\tau'_i(-k)}=\int_{-\infty}^{\infty}\overline{\phi_k\beta'_i(-k)\theta''_i(-k')}\,dk' \qquad (5.4.4).$$

Substituting equation (5.4.2), (5.4.3) and (5.4.4) in equation (5.2.11) we get

$$\frac{\partial}{\partial t}\overline{l_i\tau'_i(k)}+\nu\left(\frac{1}{p_M}+\frac{1}{p_r}\right)k^2\overline{l_i\tau'_i(k)}=-\int_{-\infty}^{\infty}[\overline{\phi_i\beta'_i\theta''_i}+2\overline{\phi_k\beta'_i(-k)\theta''_i(-k')}]_c$$

$$.\exp[-\nu(t-t_o)\{(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2+2kk'\cos\theta\}]\,dk' \qquad (5.4.5).$$

Now $dk'(=dk'_1dk'_2dk'_3)$ can be expressed interms of k' and θ(cf.Deissler [15]) as

$$dk'=-2\pi k'^2 d(\cos\theta)dk' \; . \qquad (5.4.6).$$

Putting equation (5.4.6) in equation (5.4.5) yields

$$\frac{\partial}{\partial t}\overline{l_i\tau'_i(k)}+\nu\left(\frac{1}{p_M}+\frac{1}{p_r}\right)k^2\overline{l_i\tau'_i(k)}=-\int_0^{\infty}2\pi ik_k[\overline{\phi_i\beta'_i\theta''_i}$$

$$+2\overline{\phi_k\beta'_i(-k)\theta''_i(-k')}]_0k'^2.\,[\int_{-1}^{1}\exp\{-\nu(t-t_0)\,[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'$$

$$+2kk'\cos\theta]\}\,d(\cos\theta)]\,dk' \qquad (5.4.7).$$

In order to find the solution completely and following Loeffler and Deissler [3], we assume that

$$ik_k[\overline{\phi_i\beta'_i\theta''_i}+2\overline{\phi_k\beta'_i(-k)\theta''_i(-k')}]_0=\frac{\beta_0}{(2\pi)^2}(k^2k'^4-k^4k'^2) \qquad (5.4.8),$$

where β_0 is a constant depending on the initial condition.

Substituting equation (5.4.8) in equation (5.4.7) and completing the integration with respect to cosθ, we get

$$\frac{\partial}{\partial t}(\overline{2\pi l_i\tau'_i})+\nu\left(\frac{1}{p_M}+\frac{1}{p_r}\right)k^2(2\pi\overline{l_i\tau'_i(k)})=-\frac{\beta_0}{2\nu(t-t_0)}\int_0^\infty(kk'^5-k^3k'^3)$$

$$.[\exp\{-\nu(t-t_0)[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k_/^2-2kk']\}-\exp\{-\nu(t-t_0)[(1+\frac{1}{p_M})k^2$$

$$+(1+\frac{1}{p_r})k'^2+2kk']\}]dk' \qquad (5.4.9).$$

Multiplying both sides by k^2, we have

$$\frac{\partial Q}{\partial t}+\nu\left(\frac{1}{p_M}+\frac{1}{p_r}\right)k^2Q=F \qquad (5.4.10),$$

where

$$Q=2\pi k^2\overline{l_i\tau'_i(k)} \qquad (5.4.11)$$

and

$$F=-\frac{\beta_0}{2\nu(t-t_0)}\int_0^\infty(k^3k'^5-k^5k'^3)[\exp\{-\nu(t-t_0)[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2$$

$$-2kk']\}-\exp\{-\nu(t-t_0)[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2+2kk']\}]dk' \qquad (5.4.12).$$

Integrating equation (5.4.12) with respect to k', we have

$$F=-\frac{\sqrt{\pi}\,\beta_0 p_r^{\frac{5}{2}}}{2\nu^{\frac{3}{2}}(t-t_o)^{\frac{3}{2}}(1+p_r)^{\frac{5}{2}}}\exp\{-\nu(t-t_o)(1+\frac{1}{p_M}-\frac{p_r}{1+p_r})k^2\}$$

$$\cdot\left[\frac{15p_r k_4}{4\nu^2(t-t_o)^2(1+p_r)}+\{\frac{5p_r^2}{(1+p_r)^2}-\frac{3}{2}\}\frac{k^6}{\nu(t-t_o)}+\{\frac{p_r^3}{(1+p_r)^3}-\frac{p_r}{(1+p_r)}\}k^8\right]$$

.....(5.4.13).

The series of equation (5.4.13) contains only even powers of k and start with k^4 and the equation represents the transfer function arising owing to consideration of magnetic field at three points at a time.

It is interesting to note that

$$\int_o^{\infty} F.\,dk=0 \qquad (5.4.14),$$

this indicates that the conditions of continuity and homogeneity are maintained. Physically, it was to be expected, since F is a measure of transfer of energy and the total energy transferred to all wave number must be zero.

The linear equation (5.4.10) can be solved to give

$$Q=\exp[-\nu k^2(t-t_o)(\frac{1}{p_M}+\frac{1}{p_r})]\int F\exp[\nu k^2(\frac{1}{p_M}+\frac{1}{p_r})(t-t_o)]\,dt$$

$$+C(k)\exp[-\nu k^2(\frac{1}{p_M}+\frac{1}{p_r})(t-t_0)] \qquad (5.4.15),$$

where $C(k)=(N_0k^2)/\pi$ is a constant of integration and can be obtained as by Corrsin [19]. Substituting the values of F from equation (5.4.13) in equation (5.4.15) and integrating with respect to t, we get

$$Q(k,t)=\frac{N_o k^2}{\pi}\exp\{-\nu k^2(\frac{1}{p_M}+\frac{1}{p_r})(t-t_o)\}+\frac{\sqrt{\pi}\,\beta_o p_r^{\frac{5}{2}}}{2\nu^{\frac{3}{2}}(1+p_r)^{\frac{7}{2}}}$$

$$.\exp[-\nu k^2(t-t_o)\{\frac{1+p_r+p_M}{p_M(1+p_r)}\}].[\frac{3p_r k^4}{2\nu^2(t-t_o)^{\frac{5}{2}}}+\frac{p_r(7p_r-6)k_6}{3\nu(1+p_r)(t-t_o)^{\frac{3}{2}}}$$

$$-\frac{4(3p_r^2-2p_r+3)k^8}{3(1+p_r)^2(t-t_o)^{\frac{1}{2}}}+\frac{8\sqrt{\nu}(3p_r^2-2p_r+3)k^9}{3(1+p_r)^{\frac{5}{2}}\sqrt{p_r}}N(\omega)] \qquad (5.4.16),$$

where,

$$N(\omega)=\exp(-\omega^2)\int_o^\infty\exp(x^2)\,dx,\ \omega=k\frac{\sqrt{\nu(t-t_o)}}{p_r(1+p_r)}.$$

The function $F(\omega)$ has been calculated numerically and tabulated in [5]. If in equation r is set equal to zero and use is made of the definition of Q as given by equation (5.4.11).

The result is

$$\frac{\overline{T^2}}{2}=\frac{\overline{T_i T_j'}}{2}=\int_o^\infty Q(k)\,dk \qquad (5.4.17).$$

Substituting equations (5.4.16) in to (5.4.17) and after integration, we get

$$\frac{\overline{T^2}}{2}=\frac{N_o p_r^{\frac{3}{2}} p_M^{\frac{3}{2}}(t-t_o)^{-\frac{3}{2}}}{4\sqrt{\pi}\,v^{\frac{3}{2}}(p_r+p_M)^{\frac{3}{2}}}+\frac{\beta_o \pi p_r^{\frac{7}{2}} p_M^{\frac{5}{2}}(t-t_o)^{-5}}{2v^6(1+p_r)(1+p_r+p_M)^{\frac{5}{2}}}\{\frac{9}{6}+\frac{5p_M(7p_r-6)}{16(1+p_r+p_M)}$$

$$-\frac{35p_M^2(3p_r^2-2p_r+3)}{8p_r(1+p_r+p_M)^2}+\frac{8p_M^3(3p_r^2-2p_r+3)}{3.2^6 p_r^2(1+p_r+p_M)^3}\cdot\sum_{n=o}^{\infty}\frac{1.3.5.\ldots..(2n+9)}{\lfloor n(2n+1)2^2 n(1+p_r)^n}\}$$

or

$$\frac{\overline{T^2}}{2}=\frac{N_o p_r^{\frac{3}{2}} p_M^{\frac{3}{2}}(t-t_o)^{-\frac{3}{2}}}{4\sqrt{\pi}\,v^{\frac{3}{2}}(p_r+p_M)^{\frac{3}{2}}}+\beta_o S v^{-6}(t-t_o)^{-5} \qquad (5.4.18),$$

where,

$$S=\frac{\pi p_r^{\frac{7}{2}} p_M^{\frac{5}{2}}}{2(1+p_r)(1+p_r+p_M)^{\frac{5}{2}}}[\frac{9}{16}+\frac{5p_M(7p_r-6)}{16(1+p_r+p_M)}-\frac{35p_M^2(3p_r^2-2p_r+3)}{8p_r(1+p_r+p_M)^2}+\ldots\ldots..].$$

Thus the energy decay law for temperature field fluctuation of turbulent flow in presence of magnetic field before the final period may be written as

$$\overline{T^2}=X(t-t_o)^{-\frac{3}{2}}+Y(t-t_o)^{-5}. \qquad (5.4.19),$$

where,

$$X=\frac{N_0 P_r^{\frac{3}{2}} P_M^{\frac{3}{2}}}{2\sqrt{\pi}\,\nu^{\frac{3}{2}}\,(P_r+P_M)^{\frac{3}{2}}} \text{ and } Y=2\beta_0 S\nu^{-6} .$$

For large times, the last terms in the equation becomes negligible, leaving the -3/2 power decay law for the final period.

5.5 CONCLUSION

By neglecting the quadruple correlations in the three point correlation equation, the results (5.4.19) applicable to the temperature fluctuation in MHD turbulence before the final period were obtained. If the equation (5.4.10) is integrated with respect to k from zero to infinity and use is made of equations (5.4.14) and (5.4.17), the resulting equation is

$$-\frac{\partial}{\partial t}\frac{\overline{T^2}}{2}=\nu\left(\frac{1}{p_M}+\frac{1}{p_r}\right)\int_0^\infty k^2 Q dK .$$

This equation points out the interesting fact that for a given viscosity and temperature fluctuation spectrum the decay rate is inversely proportional to the Prandtl number. The results of this analysis however are not comparable on this basis since the manner in which the initial conditions were imposed (equation(5.4.8)) precludes comparing two different Prandtl number fluids with the same spectral curve. However, the results of this analysis do show that the decay rate decreases relative to the final period rate with increasing Prandtl numbers. Corrsin [12] has previously pointed out that for the final period, as well as for self-preserving and initial spectrums at very large Reynold number, temperature fluctuations die out more slowly than velocity fluctuations. This analysis indicates that the same is true for times before the final period, as can be seen by comparison of equation (5.4.19) for $\overline{T^2}$ is analogous to the equation for $\overline{u^2}$ (equation (38) of [15]).

In absence of a magnetic field, magnetic prandtl number coincides with the prandtl number (i.e. $p_r = p_M$), then the equation becomes

$$\frac{\overline{T^2}}{2} = \frac{N_o p_r^{\frac{3}{2}}}{8\sqrt{2\pi}\,\nu^{\frac{3}{2}}(t-t_o)^{\frac{3}{2}}} + \frac{\beta_o S}{\nu^6 (t-t_o)^5} ,$$

which was obtained earlier by Loeffler and Deissler [15].

CHAPTER-6

THERMAL DECAY PROCESS OF MHD TURBULENT FLOW IN A ROTATING SYSTEM IN PRESENCE OF DUST PARTICLES

6.1 INTRODUCTION

Turbulent flows are always discipative in nature. Deformation occurs as a result of viscous shear stress. This deformation increases the thermal energy of the fluid at the cost of kinetic energy of turbulence. To compensate for these viscous losses turbulence requires a continuous supply of energy. If there is no supply of energy turbulence decays rapidly. Saffman [52] observed the effect of dust particles on the stability of the laminar flow of an incompressible fluid with constant mass concentration of dust particles and gave an equation which described the motion of a fluid containing small dust particles. Corrsin [13] considered the problem of temperature fluctuation is isotropic turbulence. Jain [28] studied the temperature fluctuation in turbulence and the results so obtained have been compared with those obtained by corrsin [12]. By using Millionschikov's hypothesis [43] of quasinormality in the fluctuating components of velocities. Ghosh [21] obtained a dynamical equation for the early period decay of turbulence. Mazumder [42] derived the early-period decay equations for general type of turbulence by superimposing a scalar field (i.e. temperature) on an incompressible velocity field. The approach is phenomenological in the sense that he has considered the region where the variations of the mean temperature and mean velocity may be neglected because the transportation of the thermal energy from place to place is very rapid. Sinha [57] obtained an early-period decay equation for MHD turbulent flow. Sarker [55] also derived an equation for MHD turbulent flow in a rotating system.

In this chapter we have considered the convective MHD turbulent flow in a rotating system in presence of dust particles. The coriolis force due to rotation plays an important role in a rotating system of turbulent flow. The main object of this chapter is to derive an early-period decay equation for MHD turbulent flow in presence of dust particles in a rotating system at high Reynolds and Peclect numbers. We have considered the region where the variations of mean temperature, mean velocity and mean magnetic field may be neglected because the transportation of the thermal energy from place to place is very rapid.

6.2 FUNDAMENTAMENTAL EQUATION

The temperature diffusion equation and the equations of motion and continuity for viscous, incompressible and conducting fluids for MHD turbulent flow in a rotating system are

$$\frac{\partial\theta}{\partial t}+u_n\frac{\partial\theta}{\partial x_n}=\alpha\nabla_x^2\theta \qquad (6.2a),$$

$$\frac{\partial u_i}{\partial t}+\frac{\partial}{\partial x_k}(u_i u_k-h_i h_k)=-\frac{\partial p^*}{\partial x_i}+\nu\nabla_x^2 u_i-2\varepsilon_{mki}\omega_m u_K \qquad (6.2b),$$

$$\frac{\partial h_i}{\partial t}+\frac{\partial}{\partial x_k}(h_i u_k-u_i h_k)=\lambda\nabla_x^2 h_i \qquad (6.2c),$$

and

$$\frac{\partial u_i}{\partial x_i}=\frac{\partial h_i}{\partial x_i}=0 \qquad (6.2d),$$

where

θ=temperature field fluctuation,

$\alpha=\frac{K_T}{\rho c_p}$ =thermal diffusivity,

$p^*=\frac{p}{\rho}+\frac{1}{2}|h|^2+|\omega\times x|^2$ =generalised pressure inclusive of potential of centrifugal force,

$u_i(x,t)$=fluctuating velocity component,

$h_i(x,t)$=fluctuation of magnetic field.

ω_m =the component of constant angular velocity of uniform rotation,

ϵ_{mki}=alternating tensor,

ν=kinematic viscosity,

λ=magnetic diffusivity,

k_T=thermal conductivity,

c_p=specific heat at constant pressure,

ρ =fluid density.

For MHD turbulent flow of a dusty incompressible fluid, equations (2) can be written as

$$\frac{\partial\theta}{\partial t}+u_n\frac{\partial\theta}{\partial x_n}=\alpha\nabla_x^2\theta \qquad (6.2.1),$$

$$\frac{\partial u_i}{\partial t}+\frac{\partial}{\partial x_k}(u_iu_k-h_ih_k)=-\frac{\partial P^*}{\partial x_i}+\nu\nabla_x^2u_i-2\epsilon_{mki}\omega_mu_k+\frac{KN}{\rho}(v_i-u_i) \qquad (6.2.2),$$

$$\frac{\partial h_i}{\partial t}+\frac{\partial}{\partial x_k}(h_iu_k-u_ih_k)=\lambda\nabla_x^2h_i \qquad (6.2.3),$$

with

$$\frac{\partial u_i}{\partial x_i}=\frac{\partial h_i}{\partial x_i}=0 \qquad (6.2.4),$$

where v_i is the ith component of the fluctuating velocity of dust particles, K is the stock resistance coefficient, N is the number density of the dust particles, $x=(x_1,x_2,x_3)$ and

$$\nabla_x^2=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{\partial^2}{\partial x_3^2}.$$

The third term on the right side of equation (6.2.2) represents the coriolis force and fourth term is due to the presence of dust particles.

6.3 DYNAMICAL EQUATION

To derive the dynamical equation, we can write an equation, for temperature fluctuation θ' at the point $p'(x', t)$ similar to (6.2.1) as

$$\frac{\partial\theta'}{\partial t}+u_l'\frac{\partial\theta'}{\partial x_l'}=\alpha\nabla_{x'}^2\theta' \qquad (6.3.1),$$

where,

$x'=(x_1, x_2, x_3)$

and

$$\nabla_{x'}^2=\frac{\partial^2}{\partial x_1'^2}+\frac{\partial^2}{\partial x_2'^2}+\frac{\partial^2}{\partial x_3'^2}.$$

Multiplying the equation (4.2.1) by θ' and (6.3.1) by θ, then adding and taking average we obtain

$$\frac{\partial\overline{\theta\theta'}}{\partial t}+\frac{\partial\overline{\theta u_n\theta'}}{\partial x_n}+\frac{\partial\overline{\theta u_l'\theta'}}{\partial x_l'}=\alpha\,(\nabla_x^2+\nabla_{x'}^2)\,\overline{\theta\theta'} \qquad (6.3.2).$$

If we put the following correlation tensors,

$\overline{\theta\theta'}=F_{\theta,\theta}(x, x', t)$, $\overline{\theta u_n\theta'}=F_{\theta n,\theta}(x, x', t)$

and

$\overline{\theta u_l'\theta}=F_{\theta,\theta l}(x, x, t)$

in the equation (6.3.2), we have

$$\frac{\partial F_{\theta,\theta}}{\partial t}+\frac{\partial F_{\theta n,\theta}}{\partial x_n}+\frac{\partial F_{\theta,\theta 1}}{\partial x_1'}=\alpha\,(\nabla_x^2+\nabla_{x'}^2)\,F_{\theta,\theta} \qquad (6.3.3).$$

The equation for the velocity fluctuation u_k'' at the point $p''(x'',t)$ can be written as

$$\frac{\partial u_k''}{\partial t}+\frac{\partial}{\partial x_j''}(u_k''u_j''-h_k''h_j'')=-\frac{\partial P^{*''}}{\partial x_k''}+\nu\nabla_{x''}^2 u_k''$$

$$-2\epsilon_{mjk}\omega_m u_j''+f(v_k''-u_k'') \qquad (6.3.4),$$

where $\frac{KN}{\rho}=f, X''=(X_1'',X_2'',X_3'')$ and

$$\nabla_{x''}^2=\frac{\partial^2}{\partial x_1''^2}+\frac{\partial^2}{\partial x_2''^2}+\frac{\partial^2}{\partial x_3''^2}\ .$$

Multiplying the equation (6.2.1) by $\theta' u_k''$, (6.3.1) by $\theta u_k''$ and (6.3.4) by $\theta\theta'$ and then adding and taking averages, we have

$$\frac{\partial}{\partial t}(\overline{\theta\theta' u_k''})+\frac{\partial}{\partial x_n}(\overline{\theta u_n\theta' u_k''})+\frac{\partial}{\partial x_1'}(\overline{\theta' u_1'\theta u_k''})+\frac{\partial}{\partial x_j''}(\overline{u_k''u_j''\theta\theta'}-\overline{h_k''h_j''\theta\theta'})$$

$$=-\frac{\partial}{\partial x_k''}(\overline{P^{*''}\theta\theta'})+\{\alpha(\nabla_x^2+\nabla_{x''}^2)+\nu\nabla_{x''}^2\}(\overline{\theta\theta' u_k''})-2\epsilon_{mjk}\omega_m(\overline{\theta\theta' u_j''})$$

$$+f(\overline{v_k''\theta\theta'}-\overline{u_k''\theta\theta'}) \qquad (6.3.5).$$

After using tensor notations, equation (6.3.5) can be written as

$$\frac{\partial}{\partial t}F_{\theta,\theta,k}+\frac{\partial}{\partial x_n}F_{\theta n,\theta,k}+\frac{\partial}{\partial x_1^{/}}F_{\theta,\theta 1,k}+\frac{\partial}{\partial x_j^{//}}(F_{\theta,\theta,kj}-H_{\theta,\theta,kj})$$

$$=-\frac{\partial}{\partial x_k^{//}}P_{\theta,\theta,\omega}+\{\alpha(\nabla_x^2+\nabla_{x^/}^2)+\nu\nabla_{x^{//}}^2\}F_{\theta,\theta,k}-2\varepsilon_{mjk}\omega_m F_{\theta,\theta,j}$$

$$+f(Q_{\theta,\theta,k}-F_{\theta,\theta,k}) \qquad (6.3.6),$$

where,

$\overline{\theta\theta^{/}u_k^{//}}=F_{\theta,\theta,k}(x,x^{/},x^{//},t)$, $\overline{\theta u_n\theta^{/}u_k^{//}}=F_{\theta n,\theta,k}(x,x^{/},x^{//},t)$,

$\overline{\theta^{/}u_1^{/}\theta u_k^{//}}=F_{\theta,\theta 1,k}(x,x^{/},x^{//},t)$, $\overline{u_k^{//}u_j^{//}\theta\theta^{/}}=F_{\theta,\theta,kj}(x,x^{/},x^{//},t)$,

$\overline{h_k^{//}h_j^{//}\theta\theta^{/}}=H_{\theta,\theta,kj}(x,x^{/},x^{//},t)$, $\overline{P^{*}\theta\theta^{/}}=P_{\theta,\theta,\omega}(x,x^{/},x^{//},t)$

and

$\overline{\theta\theta^{/}v_k^{//}}=Q_{\theta,\theta,k}(x,x^{/},x^{//},t)$.

Now taking $\frac{\partial}{\partial x_k^{//}}$ of equation (6.3.4) and using the continuity

condition at $p^{//}(x^{//},t)$, we have

$$\frac{\partial}{\partial x_k^{//}}\frac{\partial}{\partial x_j^{//}}(\overline{u_k^{//}u_j^{//}}-\overline{h_k^{//}h_j^{//}})=-\frac{\partial^2 P^{*//}}{\partial x_k^{//}\partial x_k^{//}}-2\varepsilon_{mjk}\omega_m\frac{\partial u_j^{//}}{\partial x_k^{//}}+f\frac{\partial v_k^{//}}{\partial x_k^{//}} \qquad (6.3.7).$$

Multiplying (6.3.7) by $\theta\theta'$ and taking averages, we obtain

$$\frac{\partial}{\partial x_k^{//}}\frac{\partial}{\partial x_j^{//}}(\overline{u_k^{//}u_j^{//}}-\overline{h_k^{//}h_j^{//}})\overline{\theta\theta'}=-\frac{\partial^2 \overline{P^{*//}\theta\theta'}}{\partial x_k^{//}\partial x_k^{//}}-2\varepsilon_{mjk}\omega_m\frac{\partial \overline{u_j^{//}\theta\theta'}}{\partial x_k^{//}}+f\frac{\partial \overline{v_k^{//}\theta\theta'}}{\partial x_k^{//}}$$

.....(6.3.8).

In tensor notation this equation becomes

$$\frac{\partial}{\partial x_k^{//}}\frac{\partial}{\partial x_j^{//}}(F_{\theta,\theta,kj}-H_{\theta,\theta,kj})=-\frac{\partial^2 P_{\theta,\theta,\omega}}{\partial x_k^{//}\partial x_k^{//}}-2\varepsilon_{mjk}\omega_m\frac{\partial F_{\theta,\theta,j}}{\partial x_k^{//}}+f\frac{\partial Q_{\theta,\theta,k}}{\partial x_k^{//}} \qquad (6.3.9).$$

Now we use the Fourier transforms of two point correlations that appear in equation (6.3.3) as

$$\left.\begin{aligned}
\psi_{\theta,\theta}(k,k',t)&=\frac{1}{(2\pi)^6}\iint F_{\theta,\theta}(x,x',t).\exp[i(k.x+k'.x')]\,dxdx' \\
\psi_{\theta n,\theta}(k,k',t)&=\frac{i}{(2\pi)^6}\iint F_{\theta n,\theta}(x,x',t).\exp[i(k.x+k.x')]\,dxdx' \\
\psi_{\theta,\theta 1}(k,k',t)&=\frac{i}{(2\pi)^6}\iint F_{\theta,\theta 1}(x,x',t).\exp[i(k.x+k'.x')]\,dxdx'
\end{aligned}\right\} (6.3.10).$$

Accordingly, the Fourier inverse transforms of the above relations are:

$$\left.\begin{aligned}
F_{\theta,\theta}(X,X',t) &= \iint \psi_{\theta,\theta}(k,k',t)\,.\exp[-i(k.X+k'.X'+k''.X'')]\,dkdk' \\
F_{\theta n,\theta}(x,x',t) &= -i\iint \psi_{\theta n,\theta}(k,k',t)\,.\exp[-i(k.x+k'.x')]\,dkdk' \\
F_{\theta,\theta l}(x,x',t) &= -i\iint \psi_{\theta,\theta l}(k,k',t)\,.\exp[-i(k.x+k'.x')]\,dkdk'
\end{aligned}\right\}(6.3.11).$$

It is to be noted that the investigations in (6.3.10) are performed over the whole of x,x' -spaces and their respective volume elements are $dx=dx_1dx_2dx_3$
and $dx'=dx_1'dx_2'dx_3'$.
Obviously the integrations in (6.3.11) are performed over the whole of k,k'-spaces and their respective volume elements are
$dk=dk_1dk_2dk_3$
and
$dk'=dk_1'dk_2'dk_3'$.
Similarly, the Fourier transforms of three-point correlations that appear in (6.3.6) can be written as

$$\psi_{\theta,\theta,k}(k,k',k'',t)=\frac{i}{(2\pi)^9}\iiint F_{\theta,\theta,k}(x,x',x'',t)\,.\exp[i(k.x+k'.x'$$

$$+k''.x'')]\,dxdx'dx'' \qquad (6.3.12),$$

$$\psi_{\theta n,\theta,k}(k,k',k'',t)=\frac{1}{(2\pi)^9}\iiint F_{\theta n,\theta,k}(x,x',x'',t).\exp[i(k.x+k'.x'$$

$$+k''.x'')]\,dxdx'dx'' \qquad (6.3.13),$$

$$\psi_{\theta,\theta l,k}(k,k',k'',t)=\frac{1}{(2\pi)^9}\iiint F_{\theta,\theta l,k}(x,x',x'',t).\exp[i(k.x+k'.x'$$

$$+k''.x'')]\,dxdx'dx'' \qquad (6.3.14),$$

$$\psi_{\theta,\theta,kj}(k,k',k''t)=\frac{1}{(2\pi)^9}\iiint F_{\theta,\theta,kj}(x,x',x'',t).\exp[i(k.x+k'.x'$$

$$+k''.x'')]\,dxdx'dx'' \qquad (6.3.15),$$

$$\phi_{\theta,\theta,kj}(k,k',k'',t)=\frac{1}{(2\pi)^9}\iiint H_{\theta,\theta,kj}(x,x',x'',t).\exp[i(k.x+k'.x'$$

$$+k''.x'')]\,dxdx'dx'' \qquad (6.3.16),$$

$$\pi_{\theta,\theta,\omega}(k,k',k'',t)=\frac{1}{(2\pi)^9}\iiint P_{\theta,\theta,\omega}(x,x',x'',t).\exp[i(k.x+k'.x'$$

$$+k''.x'')]\,dxdx'dx'' \qquad (6.3.17),$$

$$\beta_{\theta,\theta,k}(k,k^{/},k^{//},t)=\frac{i}{(2\pi)^9}\iiint Q_{\theta,\theta,k}(x,x^{/},x^{//},t).\exp[i(k.x+k^{/}.x^{/}$$

$$+k^{//}.x^{//})]\,dxdx^{/}dx^{//} \qquad (6.3.18),$$

where the integrations appearing in (6.3.12) to (6.3.17) are performed over the whole of $x, x^{/}, x^{//}$,-spaces, and their respective volume elements are $dx_1dx_2dx_3$, $dx_1^{/}dx_2^{/}dx_3^{/}$, $dx_1^{//}dx_2^{//}dx_3^{//}$.

The Fourier inverse transform of the above relations are

$$F_{\theta,\theta,k}(x,x^{/},x^{//})=-i\iiint\psi_{\theta,\theta,k}(k,k^{/},k^{//},t).\exp[-i(k.x+k^{/}.x^{/}$$

$$+k^{//}.x^{//})]\,dkdk^{/}dk^{//} \qquad (6.3.19),$$

$$F_{\theta n,\theta,k}(x,x^{/},x^{//},t)=\iiint\psi_{\theta n,\theta,k}(k,k^{/},k^{//}).\exp[-i(k.x+k^{/}.x^{/}$$

$$+k^{//}.x^{//})]\,dkdk^{/}dk^{//} \qquad (6.3.20),$$

$$F_{\theta,\theta l,k}(x,x^{/},x^{//},t)=\iiint\psi_{\theta,\theta l,k}(k,k^{/},k^{//},t).\exp[-i(k.x+k^{/}.x^{/}$$

$$+k^{//}.x^{//})]\,dkdk^{/}dk^{//} \qquad (6.3.21),$$

$$F_{\theta,\theta,kj}(x,x^{/},x^{//},t)=\iiint\psi_{\theta,\theta,kj}(k,k^{/},k^{//},t).\exp[-i(k,x+k^{/}.x^{/}$$

$$+k^{//}.x^{//})]dkdk^{/}dk^{//} \quad (6.3.22),$$

$$H_{\theta,\theta,kj}(x,x^{/},x^{//},t)=\iiint\phi_{\theta,\theta,kj}(k,k^{/},k^{//},t).\exp[-i(k.x+k^{/}.x^{/}$$

$$+k^{//}.x^{//})]dkdk^{/}dk^{//} \quad (6.3.23),$$

$$P_{\theta,\theta,\omega}(x,x^{/},x^{//},t)=\iiint\pi_{\theta,\theta,\omega}(k,k^{/},k^{//},t).\exp[-i(k.x+k^{/}.x^{/}$$

$$+k^{//}.x^{//})]dkdk^{/}dk^{//} \quad (6.3.24),$$

and

$$Q_{\theta,\theta,k}(x,x^{/},x^{//},t)=\iiint\beta_{\theta,\theta,k}(k,k^{/},k^{//},t)\exp[-i(k.x+k^{/}.x^{/}$$

$$+k^{//}.x^{//})]dkdk^{/}dk^{//} \quad (6.3.25).$$

The integrations appearing in (6.3.19) to (6.3.25) are taken over the whole of $k,k^{/},k^{//}$-spaces.

Using the relation (6.3.11) in the equation (6.3.3) we have

$$\frac{\partial}{\partial t}\psi_{\theta,\theta}(k,k^{/},t)=k_n\psi_{\theta n,\theta}(k,k^{/},t)+k_l^{/}\psi_{\theta,\theta l}(k,k^{/},t)$$

$$-\alpha(k^2+k^{/2})\psi_{\theta,\theta}(k,k^{/},t) \quad (6.3.26).$$

Similarly, with the help of equations (6.3.19) to (6.3.25), equations (6.3.6) and (6.3.9) can be written as

$$\frac{\partial}{\partial t}\psi_{\theta,\theta,k}(k,k^{/},k^{//},t)+k_n\psi_{\theta n,\theta,k}(k,k^{/},k^{//},t)+k_l^{/}\psi_{\theta,\theta l,k}(k,k^{/},k^{//},t)$$

$$+k_j^{//}\psi_{\theta,\theta,kj}(k,k^{/},k^{//},t)-k_j^{//}\phi_{\theta,\theta,kj}(k,k^{/},k^{//},t)+k_k^{//}\pi_{\theta,\theta,\omega}(k,k^{/},k^{//},t)$$

$$=-\{\alpha(k^2+k^{/2})+\nu k^{//2}\}\psi_{\theta,\theta,k}(k,k^{/},k^{//},t)+2\epsilon_{mjk}\omega_m\psi_{\theta,\theta j}(k,k^{/},k^{//},t)$$

$$-f\beta_{\theta,\theta,k}(k,k^{/},k^{//},t)+f\psi_{\theta,\theta,k}(k,k^{/},k^{//},t) \qquad (6.3.27)$$

and

$$\frac{k_r^{//}k_k^{//}k_j^{//}}{k^{//2}}(\psi_{\theta,\theta,kj}-\phi_{\theta,\theta,kj})(k,k^{/},k^{//},t)=-k_r^{//}\pi_{\theta,\theta,\omega}(k,k^{/},k^{//},t)$$

$$-2\epsilon_{mjk}\omega_m\frac{k_r^{//}k_k^{//}}{k^{//2}}\psi_{\theta,\theta,j}(k,k^{/},k^{//},t)+f\frac{k_r^{//}k_k^{//}}{k^{//2}}\beta_{\theta,\theta,k}(k,k^{/},k^{//},t) \qquad (6.3.28).$$

Substituting (6.3.28) in (6.3.27), we get

$$\frac{\partial}{\partial t}\psi_{\theta,\theta,k}(k,k^{/},k^{//},t)+k_n\psi_{\theta n,\theta,k}(k,k^{/},k^{//},t)+k_l^{/}\psi_{\theta,\theta l,k}(k,k^{/},k^{//},t)$$

$$+k_j^{//}\psi_{\theta,\theta,kj}(k,k^{/},k^{//},t)-k_j^{//}\phi_{\theta,\theta,kj}(k,k^{/},k^{//},t)-\frac{k_k^{//}k_r^{//}k_j^{//}}{k^{//2}}(\psi_{\theta,\theta,rj}$$

$$-\phi_{\theta,\theta,rj})(k,k',k'',t)=-\{\alpha(k^2+k'^2)+\nu k''^2\}\psi_{\theta,\theta,k}(k,k',k'',t)$$

$$+2e_{mjk}\omega_m\psi_{\theta,\theta,j}(k,k',k'',t)-2e_{mjk}\omega_m\frac{k_r''k_k''}{k''^2}\psi_{\theta,\theta,j}(k,k',k'',t)$$

$$-f\beta_{\theta,\theta,k}+f\frac{k_r''k_k''}{k''^2}\beta_{\theta,\theta,k}(k,k',k'',t) \qquad (6.3.29).$$

Now, consider the case when both Reynolds and Peclect numbers are very high, so that the molecular effects are very negligible. From this consideration, we can put $\alpha=\nu=o$ in the equation (6.3.26) and (6.3.27) and they reduced respectively to the forms:

$$\frac{\partial}{\partial t}\psi_{\theta,\theta}(k,k',t)=k_n\psi_{\theta n,\theta}(k,k',t)+k_l'\psi_{\theta,\theta l}(k,k',t) \qquad (6.3.30)$$

and

$$\frac{\partial}{\partial t}\psi_{\theta,\theta,k}(k,k',k'',t)=-k_n\psi_{\theta n,\theta,k}(k,k',k'',t)-k_l'\psi_{\theta,\theta l,k}(k,k',k'',t)$$

$$-k_j''\psi_{\theta,\theta,kj}(k,k',k'',t)+k_j''\phi_{\theta,\theta,kj}(k,k',k'',t)+\frac{k_k''k_r''k_j''}{k''^2}(\psi_{\theta,\theta,rj}$$

$$-\phi_{\theta,\theta,rj})(k,k^{/},k^{//},t)+2\varepsilon_{mjk}\omega_{m}\psi_{\theta,\theta,j}(k,k^{/},k^{//},t)-2\varepsilon_{mjk}\omega_{m}\frac{k_{r}^{//}k_{k}^{//}}{k^{//2}}$$

$$\cdot\psi_{\theta,\theta,j}(k,k^{/},k^{//},t)+f(\frac{k_{r}^{//}k_{k}^{//}}{k^{//2}}-1)\beta_{\theta,\theta,k}(k,k^{/},k^{//},t)$$

$$+f\psi_{\theta,\theta,k}(k,k^{/},k^{//},t) \qquad (6.3.31).$$

The triple correlation $\overline{\theta,\theta^{/}u_{k}^{//}}=F_{\theta,\theta,k}(x,x^{/},x^{//},t)$ formed of the two

temperature fluctuations relating to the points p and $p^{/}$and one velocity component relating to the point $p^{//}$, and its Fourier transform $\psi_{\theta,\theta,k}$ $(k,k^{/},k^{//},t)$is being considered as follows:

when the point $p^{//}$coincides with the point p or in the alternative way p coincides with the point $p^{//}$, we respectively obtain the relations

$$\int\psi_{\theta,\theta,k}(k,k^{/},k^{//},t)=\psi_{\theta k,\theta}(k,k^{/},t) \qquad (6.3.32a)$$

and

$$\int\psi_{\theta,\theta,k}(k,k^{/},k^{//},t)=\psi_{\theta,\theta k}(k,k^{/},t) \qquad (6.3.32b).$$

Let us consider the velocity component $u_l^{/}$ at a fourth point $p^{///}(x^{///},t)$ and form the quadruple correlation

$$\overline{\theta\theta^{/}u_k^{//}u_l^{//}}=F_{\theta,\theta,k,l}(x,x^{/},x^{//},x^{///},t).$$

The Fourier transform of $F_{\theta,\theta,k,l}(x,x^{/},x^{//},x^{///},t)$ in wave-number is

usually denoted by $\theta,\theta,k,l(k,k^{/},k^{//},k^{///},t)$. The quasi-normality

hypothesis as required may be expressed in wave-number space by the relation

$$\psi_{\theta,\theta,k,l}(k,k^{/},k^{//},k^{///},t)=\psi_{\theta,\theta}(k,k^{/},t)\,\psi_{k,l}(k^{//},k^{///},t)+\psi_{\theta,k}(k,k^{//}t)$$

$$.\psi_{\theta,l}(k,k^{//},t)+\psi_{\theta,l}(k,k^{///},t)\,\psi_{\theta,k}(k^{/},k^{///},t) \qquad (6.3.33).$$

When $p^{///}$coincides with p, the equation (6.3.33) can be written as

$$\psi_{\theta l,\theta,k}(k,k^{/},k^{//},t)=\int\psi_{\theta,\theta}(k-k^{///},k^{/},t)\,\psi_{k,l}(k^{//},k^{///},t)\,dk$$

$$+\int\psi_{\theta,k}(k-k^{///},k^{//},t)\,\psi_{\theta,l}(k^{/},k^{///},t)\,dk^{///}+\psi_{\theta,k}(k^{/},k^{//},t)\,\psi_{\theta l}(k,t)$$

$$(6.3.34).$$

Similarly, we obtain

$$\psi_{\theta,\theta l,k}(k,k^{/},k^{//},t)=\int\psi_{\theta,\theta}(k,k^{/}-k^{//},t)\,\psi_{k,l}(k^{//},k^{///},t)\,dk^{///}$$

$$+\int\psi_{\theta,l}(k,k^{///},t)\,.\,\psi_{\theta,k}(k^{/}-k^{///},k^{//},t)\,dk^{///}+\psi_{\theta,k}(k,k^{//},t)\,\psi_{\theta,l}(k^{/},t)$$

$$\ldots\ldots(6.3.35)$$

and

$$\psi_{\theta,\theta,kl}(k,k^{/},k^{//},t)=\int\psi_{\theta,k}(k,k^{//}-k^{///},t)\,\psi_{\theta,l}(k^{//},k^{///},t)\,dk^{///}$$

$$+\int\psi_{\theta,l}(k,k^{///},t)\,.\,\psi_{\theta,k}(k,k^{//}-k^{///},t)\,dk^{///}+\psi_{\theta,\theta}(k,k^{/},t)\,\psi_{kl}(k^{//},t)$$

.....(6.3.36).

Again when $p^{///}$coincides with p and $p^{//}$also coincides with $p^{/}$, the equation (6.3.33) becomes

$$\psi_{\theta l,\theta k}(k,k^{/},t)=\iint\psi_{\theta,\theta}(k-k^{///},k^{/}-k^{//},t)\,\psi_{k,l}(k^{///},k^{//},t)\,dk^{//}dk^{///}$$

$$+\iint\psi_{\theta,k}(k-k^{///},k^{//},t)\,\psi_{\theta,l}(k^{/}-k^{//},k^{///},t)\,dk^{//}dk^{///}+\psi_{\theta l}(k,t)\,\psi_{\theta k}(k^{/},t)$$

.....(6.3.37).

Now, taking δ/δt of equation (6.3.30), we have

$$\frac{\partial^2}{\partial t^2}\psi_{\theta,\theta}(k,k^{/},t)=k_n\frac{\partial}{\partial t}\psi_{\theta n,\theta}(k,k^{/},t)+k_l^{/}\frac{\partial}{\partial t}\psi_{\theta,\theta l}(k,k^{/},t) \qquad (6.3.38).$$

Now, we have from the relations (6.3.31) to (6.3.37)

$$\frac{\partial}{\partial t}\psi_{\theta n,\theta}(k,k^{/},t)=-k_l^{/}\iint[\psi_{\theta,\theta}(k^{//}-k^{///},k^{/}-k^{//},t)\,\psi_{n,l}(k^{//},k^{///},t)$$

$$+\psi_{\theta,l}(k,k^{///},t)\,\psi_{\theta,n}(k-k^{///},k^{//},t)\,dk^{//}dk^{///}-k_l^{/}\psi_{\theta,n}(k,k^{///},t)\,\psi_{\theta l}(k^{/},t)$$

$$-\int k_t [\int \psi_{\theta,\theta}(k-k'', k', t)\, \psi_{n,t}(k'', k''', t)\, dk''' + \int \psi_{\theta,n}(k-k'', k', t)$$

$$\cdot \psi_{\theta,t}(k', k'', t)\, dk''' + \psi_{\theta,n}(k, k''', t)\, \psi_{\theta t}(k, t)\,]\, dk'' - \int k_j'' [\psi_{\theta,\theta}(k, k', t)$$

$$\cdot \psi_{nj}(k'', t) + \int \psi_{\theta,n}(k, k''-k''', t)\, \psi_{\theta,j}(k, k''', t)\, dk''' + \int \psi_{\theta,j}(k, k''', t)$$

$$\cdot \psi_{\theta,n}(k', k''-k'', t)\, dk''']\, dk'' + \int k_j [\phi_{\theta,\theta}(k, k', t)\, \phi_{nj}(k'', t)$$

$$+\int \phi_{\theta,n}(k, k''-k'', t)\, .\, \phi_{\theta,j}(k, k'', t)\, dk''' + \int \phi_{\theta,j}(k, k''', t)$$

$$\cdot \phi_{\theta,n}(k', k''-k''', t)\, dk''']\, dk'' + \int \frac{k_n'' k_r'' k_j''}{k''^2} [\psi_{\theta,\theta}(k, k', t)\, \psi_{rj}(k'', t)$$

$$+\int \psi_{\theta,r}(k, k''-k''', t)\, \psi_{\theta,j}(k, k''', t)\, dk''' + \int \psi_{\theta,j}(k, k''', t)$$

$$\psi_{\theta,r}(k', k''-k''', t)\, dk''']\, dk'' - \int \frac{k_n'' k_r' k_j''}{k''^2} \cdot [\phi_{\theta,\theta}(k, k', t)\, \phi_{rj}(k'', t)$$

$$+\int\phi_{\theta,r}(k,k''-k''',t)\,\phi_{\theta,j}(k,k''',t)\,dk'''+\int\phi_{\theta,j}(k,k''',t)$$

$$.\phi_{\theta,r}(k,k''-k''',t)\,dk''']\,dk''+2\varepsilon_{mjk}\omega_m\psi_{\theta j,\theta}(k,k',t)$$

$$-2\varepsilon_{mjk}\omega_m\frac{k_r''k_j''}{k''^2}\psi_{\theta j,\theta}(k,k',t)+f\{R\beta_{\theta n,\theta}(k,k',t)+\psi_{\theta n,\theta}(k,k',t)\} \quad (6.3.39),$$

where,

$$R=\frac{k_r''k_k''}{k''^2}-1$$

and

$$\frac{\partial}{\partial t}\psi_{\theta,\theta l}(k,k',t)=-k_n\iint[\psi_{\theta,\theta}(k-k''',k-k'',t)\,\psi_{n,l}(k'',k',t)$$

$$+\psi_{\theta,l}(k-k''',kk'',t)\,.\,\psi_{\theta,n}(k-k'',k''',t)]\,dk''dk'''-k_n\psi_{\theta,n}(k,k'',t)$$

$$.\,\psi_{\theta l}(k',t)-\int k_s'[\int\psi_{\theta,\theta}(k,k'-k'',t)\,\psi_{l,s}(k'',k''',t)\,dk'''+\int\psi_{\theta,s}(k,k''',t)$$

$$.\,\psi_{\theta,l}(k-k'',k''',t)\,dk'''+\psi_{\theta,l}(k,k'',t)\,.\,\psi_{\theta s}(k',t)]\,dk''-\int k_s''[\psi_{\theta,\theta}(k,k',t)$$

$$\cdot\psi_{1s}(k'',t)+\int\psi_{\theta,1}(k,k''-k''',t)\,\psi_{\theta,s}(k,k''',t)\,dk'''+\int\psi_{\theta,s}(k,k''',t)$$

$$\cdot\psi_{\theta,1}(k,k''-k''',t)\,dk''']\,dk''+\int k_s''\,[\phi_{\theta,\theta}(k,k',t)\,\phi_{1s}(k'',t)$$

$$+\int\phi_{\theta,1}(k,k''-k''',t)\,\phi_{\theta,s}(k,k''',t)\,dk'''+\int\phi_{\theta,s}(k,k''',t)$$

$$\cdot\phi_{\theta,1}(k,k''-k''',t)\,dk''']\,dk''+\int\frac{k_1''k_r''k_s''}{k''}\,[\int\psi_{\theta,r}(k,k''-k''',t)\,\psi_{\theta,s}(k,k''',t)$$

$$+\int\psi_{\theta,s}(k,k''',t)\,\psi_{\theta,r}(k,k''-k''',t)\,dk'''+\psi_{\theta,\theta}(k,k',t)\,\psi_{rs}(k'',t)]\,dk''$$

$$-\int\frac{k_1''k_r''k_s''}{k''^2}\,[\phi_{\theta,\theta}(k,k',t)\,\phi_{rs}(k'',t)+\int\phi_{\theta,r}(k,k''-k''',t)\,\phi_{\theta,s}(k,k''',t)\,dl$$

$$+\int\phi_{\theta,s}(k,k''',t)\,\phi_{\theta,r}(k,k''-k''',t)\,dk''']\,dk''-2\epsilon_{ms1}\omega_m\psi_{\theta,\theta s}(k,k',t)$$

$$-2\epsilon_{ms1}\omega_m\frac{k_r''k_1''}{k''^2}\psi_{\theta,\theta s}(k,k',t)+f\{R\beta_{\theta,\theta 1}(k,k',t)+\psi_{\theta,\theta 1}(k,k',t)\}\quad(6.3.40).$$

Wtih the help of equations (6.3.39) and (6.3.40), equation (6.3.38) can be written as

$$\frac{\partial^2}{\partial t^2}\psi_{\theta,\theta}(k,k',t)=-2k_nk_l'\iint[\psi_{\theta,\theta}(k-k''',k'-k'',t)\psi_{k}n,l(k'',k''',t)$$

$$+\psi_{\theta,l}(k-k''',k'',t)\psi_{\theta,n}(k-k'',k''',t)]dk''dk'''-2k_nk_l'\psi_{\theta,n}(k,k//,t)\psi_{\theta l}(k',t)$$

$$-\int k_nk_t[\int\psi_{\theta,\theta}(k-k'',k',t)\psi_{n,t}(k,k''',t)dk'''$$

$$+\int\psi_{\theta,n}(k-k''',k'',t)\psi_{\theta,t}(k',k'',t)dk'''+\psi_{\theta,n}(k,k''',t)\psi_{\theta t}(k',t)]dk''$$

$$-\int k_l'k_s'[\int\psi_{\theta,\theta}(k,k'-k''',t)\psi_{l,s}(k'',k''',t)dk+\int\psi_{\theta,s}(k,k''',t)$$

$$\cdot\psi_{\theta,l}(k-k'',k'''.t)dk'''+\psi_{\theta,l}(k',k''',t)\psi_{\theta s}(k',t)]dk''$$

$$-\int k_nk_j''\{\partial_{nr}-\frac{k_r''k_n''}{k''^2}\}[\int\psi_{\theta,r}(k,k''-k''',t)\cdot\psi_{\theta,j}(k,k''',t)dk'''$$

$$+\int\psi_{\theta,j}(k,k''',t)\psi_{\theta,r}(k,k''-k''',t)dk'''+\psi_{\theta,\theta}(k,k',t)\psi_{rj}(k'',t)]dk$$

$$+f[k_n\{R\beta_{\theta n,\theta}(k,k',t)+\psi_{\theta n,\theta}(k,k',t)\}+k_l'\{R\beta_{\theta,\theta l}(k,k',t)+\psi_{\theta,\theta l}(k,k',t)\}]$$

.....(6.3.41).

This is the required early period decay equation for the temperature spectrum $\psi_{\theta,\theta}(k,k',t)$ in presence of dust particles in a rotating system.

6.4 CONCLUSION

The last term of the right hand side of the equation (6.3.41) occurs only for the dust particles in the thermal decay process of MHD turbulence. The nineth and tenth terms of the right side of equation (6.3.41) occur only for the rotation in the thermal decay process of MHD turbulence. These two terms display the effect of coriolis force on the thermal decay process. The sixth and eighth terms of right hand side of the equation show the effect of magnetic field. If the dust particles is absent the derived result is reduced to that obtained early by sarker [55].

If the system is non rotating, the coriolis force will be absent and derived result is reduced to that obtained by Sinha [57].

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