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A Study on Turbulence and MHD Turbulence

Aziz, Md. Abdul

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A STUDY ON TURBULENCE AND MHD TURBULENCE

Ph.D. THESIS

By

MD. ABDUL AZIZ

B.Sc.(HONOURS) M.Sc.

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A

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BY

MD. ABDUL AZIZ

B.Sc. (HONOURS) M.Sc.

UNDER THE SUPERVISION OF

PROFESSOR DR. M. SHAMSUL ALAM SARKER

&

DR. M. ABUL KALAM AZAD

DEPARTMENT OF APPLIED MATHEMATICS

FACULTY OF SCIENCE

UNIVERSITY OF RAJSHAHI

RAJSHAHI-6205, BANGLADESH.

Dedicated to my Parents

Md. Bahar uddin

and

Kanchan Begum

Dr.M. Abul Kalam Azad
Assistant professor
Department of Applied Mathematics
University of Rajshahi
Rajshahi-6205, Bangladesh
Phone : (Off) 750041-49/4108
(Res.) (0721)751375
E-mail: azad267@gmail.com

Professor M. Shamsul Alam Sarker
M.Sc. (Raj.), Ph.D (Banaras)
Department of Applied Mathematics
University of Rajshahi
Rajshahi-6205, Bangladesh
Phone : (Off) 750041-49/4108
(Res.) (0721)750745
Fax : 88-0721-750064
E-mail: sasmathbd@yahoo.com

Date: 25.08.10

Declaration

Certified that the Thesis entitled **"A Study on turbulence and MHD turbulence"** submitted by Mr. Md. Abdul Aziz in fulfillment of the requirement for the degree of Doctor of Philosophy in Applied Mathematics, faculty of Science, University of Rajshahi, Rajshahi-6205, Bangladesh has been completed under our supervision. We believe that this research work is an original one and it has not been submitted elsewhere for any degree.

We wish him every success in life.

Dr. Md. Abul Kalam Azad
(Dr.M.Abul Kalam Azad)
Co- Supervisor

(Professor Dr. M.Shamsul Alam Sarker)
Supervisor

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Md. Abdul Aziz

(**Md. Abdul Aziz**)

Department of Applied Mathematics
University of Rajshahi ,
Rajshahi-6205

PREFACE

The thesis entitled "A Study on Turbulence and MHD Turbulence" is being presented for the award of the degree of Doctor of Philosophy in Applied Mathematics. It is the out come of my researches conducted in the Department of Applied Mathematics, Rajshahi University, Rajshahi, Bangladesh under the guidance of Professor Dr. M. Shamsul Alam Sarker and Dr. M. Abul Kalam Azad, Department of Applied Mathematics, Rajshahi University, Rajshahi- 6205, Bangladesh.

The whole thesis has been divided into six chapters.

The first chapter is a general introductory chapter and gives the general idea of Turbulence and Magneto hydrodynamic (MHD) Turbulence and its principal concepts. Some results and theories, which are needed in the subsequent Chapters, have been included in this chapter. The first order reaction, rotating system, equation of motion of dust particles, decay law of turbulence before the final period and in the final period, statistical theory of distribution functions in turbulence, Fourier transformations of Navier-stokes equation and their principal concepts and lastly, a brief review of the past researches related to this thesis have also been studied in this chapter. Throughout the work we have considered the flow of fluids to be isotropic and homogeneous. The notions generally adopted are those used by Batchelor, Chandrasekhar, Deissler, Kumar and Patel, Jain and Lundgren in their research papers. Number inside brackets [ ] refer to the references which are arranged alphabetically at the end of the thesis.

In chapter II-A, we have studied the first order reactant in Magneto-hydrodynamic Turbulence before the final period of decay in a rotating system. In this part we have studied the magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction in MHD turbulence before the final period of decay in a rotating system. Here, we have considered the two-point and three-point correlation equations and solved these equations after neglecting the fourth-order correlation terms.

In chapter II-B, we have considered the first order reactant in Magneto-hydrodynamic Turbulence before the final period of decay in presence of dust particle. Here, we have considered the two-point and three-point correlation equations and solved these equations after neglecting the fourth-order correlation terms.

In chapter II-C, we have studied the first order reactant in Magneto-hydrodynamic Turbulence before the final period of decay under the effect of rotation with an angular velocity Ω_m in presence of dust particles and we obtained the equation (2.17.18). This equation indicates that the decay law for magnetic energy fluctuation of dusty fluid MHD turbulence governing the concentration of a dilute contaminant undergoing a first order chemical reaction before the final period in a rotating system more rapidly. It is an extension work of the part-A and part-B of this chapter.

In chapter-III-A, the statistical theory of certain distribution function for simultaneous velocity, magnetic, temperature and concentration fields undergoing a first order reaction has been studied in MHD turbulence in a rotating system.

In chapter-III-B, we have studied the statistical theory of certain distribution function for simultaneous velocity, magnetic, temperature and concentration fields undergoing a first order reaction in MHD turbulence in presence of dust particles.

In chapter-III-C, we also have studied the statistical theory of certain distribution function for simultaneous velocity, magnetic, temperature and concentration fields undergoing a first order reaction in MHD turbulence in a rotating system in presence of dust particles. It is an extension work of the part-A and part-B of this chapter.

In chapter IV-A, we have considered the first order chemical reaction in MHD turbulence before the final period of decay for the case of multi-point and multi-time in a rotating system.

In chapter IV-B, we have considered the magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction in MHD turbulence before the final period of decay for the case of multi-point and multi-time in presence of dust particles.

In chapter IV-C, we have considered the first order chemical reaction in MHD turbulence before the final period of decay for the case of multi-point and multi-time in a rotating system in presence of dust particle. It is an extension work of the part-A and part-B of this chapter.

In chapter V, , we have studied the MHD flow of a dusty viscous incompressible fluid in a rotating frame between two parallel flat plates in presence of a uniform transverse magnetic field with pressure gradient. The velocities of the fluid and the dust particles for rotating frame are obtained and the effect of magnetic field on these velocities has been investigated.

In chapter VI, an over all review of the works with conclusions based on the findings of the thesis has been discussed.

The following research papers, which are extracted from this thesis, have been published, accepted and submitted for publication in different national and international journals:

(1) First Order Reactant in Magneto-Hydrodynamic (MHD) Turbulence before the Final Period of Decay in a Rotating System.
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(2) First Order Reactant in Magneto-Hydrodynamic Turbulence before the Final Period of Decay in presence of dust particles in a Rotating System.
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(3) First Order Reactant in Magneto-Hydrodynamic Turbulence before the Final Period of Decay for the Case of Multi-Point and Multi-Time in a Rotating System.
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(4) First Order Reactant in MHD Turbulence before the Final Period of Decay for the Case of Multi-Point and Multi-Time in Presence of Dust Particles.
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(5) First Order Reactant in Magneto-Hydrodynamic Turbulence before the Final Period of Decay in presence of dust particles.
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(6) Statistical Theory of Distribution Functions in Magneto-hudrodynamic Turbulence in a Rotating System Undergoing a First Order Reaction in Presence of Dust Particles.
(Published in the international journal "Research Journal of Mathematics and Statistics ", Vol- 2(2), 37-55, 2010).

(7) First Order Reactant in MHD Turbulence before the Final Period of Decay for the Case of Multi-Point and Multi-Time in a Rotating System in presence of dust particles. (Published in the international journal " Research Journal of Mathematics and Statistics , vol-2(2), 56-68, 2010).

(8) Statistical Theory of Certain Distribution Functions in MHD Turbulent flow Undergoing a First Order Reaction in Presence of Dust Particles. (Published in the international journal "journal of Modern Mathematics and Statistics". Vol-4(1), 11-21, 2010).

(9) Statistical Theory of Certain Distribution Functions in MHD Turbulence for Velocity and Concentration Undergoing a First Order Reaction in a Rotating System. (Accepted for publication in the international journal "Bangladesh Journal of Scientific & Industrial Research", 2010).

(10) Effect of Coriolis Force on Dusty Viscous Fluid Between Two Parallel Plates in MHD Flow. (Submitted for publication in the international journal "Journal of Physical Sciences.)

Md. Abdul Aziz

(Md. Abdul Aziz)

Department of Applied Mathematics
University of Rajshahi
Rajshahi-6205

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CHAPTER-I

General Introduction

1.1. Basic Concept of Turbulence:

Turbulent motions are very common in nature. The theory of turbulent motion has received considerable attention in recent developments of high-speed jet aircraft, plasma physics and chemical engineering. The formation of a turbulent boundary layer is one of the most frequently encountered phenomena in high-speed aerodynamics. Turbulence occurs nearly everywhere; in the oceans, in the atmosphere, in rivers even in stars and galaxies. It occurs when an airplane hits an air pocket. Much like there are currents in the ocean, there are currents in the air. Winds disturbed by thunderstorms or mountains are just one of the many causes of turbulence.

In turbulent flow, the motion of the fluid is steady so far as the temporal mean values of velocities and the pressures are concerned where as actually both velocities and the pressures are irregularly fluctuating. The velocity and pressure distributions in turbulent flows as well as the energy losses are determined mainly by turbulent fluctuations. The essential characteristic of turbulent flows is that the turbulent fluctuations are random in nature. It is common experience that the flow observed in nature such as rivers and winds usually differ from stream flow or laminar flow of a viscous fluid. The mean motion of such flows does not satisfy the Navier-Stokes equations for a viscous fluid. Such flows, which occur at high Reynolds numbers, are often termed turbulent flows.

Atmospheric scientists define "turbulence" as "a state of fluid flow in which the instantaneous velocities exhibit irregular and apparently random fluctuations." Those "irregular fluctuations" of the flow create the bumps. With sufficient disturbances the result is known as turbulence. The instability of laminar flow at a high Reynolds numbers, are causes disruption of the laminar pattern of fluid motion. In fluid dynamics, turbulence or turbulent flow is a fluid regime characterized by chaotic, stochastic property changes. Turbulence is one of those few things that many don't understand. It's not a hard concept at all. At least, the technical people understand the meaning of turbulence. The use of the word "Turbulence" to characterize a certain type of flow, namely, the counterpart of streamline motion is comparatively recent. Reynolds, O. [112] made the first systematic experimental investigation of turbulent flow. The turbulent motion of fluid was described by Reynolds [112], one of the pioneers in the study of turbulent flows as "sinuous motion" because fluid particles in turbulent flow appeared to follow sinusoidal or irregular paths.

The word "Turbulence" means: agitation, commotion, disturbance etc. Turbulence is rather a familiar notion; yet it is not easy to define in such a way as to cover the detailed characteristic comprehended in it and to make the definition agree with the modern view of it held by professionals in this field of applied science. Taylor and Vonkarman [146] suggested that, "Turbulence is an irregular motion which in general makes its appearance in fluids, gaseous or liquid, when they flow past solid surface or even when neighbouring streams of the same fluid flow past or over one another". According to this definition, the flow has to satisfy the condition of irregularity. But this irregularity is a very important feature. Because of irregularity, it is impossible to describe the motion in all details as a function of time and space co-ordinates. But fortunately turbulent motion is irregular in the sense that it is possible to describe it by laws of probability. It appears possible to indicate distinct average values of various quantities, such as velocity, pressure, temperature, etc and this is very important. It is not sufficient just to say that turbulence is an irregular motion yet we do not have clear-cut definition of turbulence.

In 1975, Hinze [51] gave the definition, "Turbulent fluid motion is an irregular condition of flow in which various quantities show a random variation with time and space co-ordinates, so that statistically distinct average values can be discerned".

Turbulence is a form of movement which is characterized by an irregular or agitated motion. Both liquids and gases can exhibit turbulence, and a number of factors can contribute to the formation of turbulence. The addition "with time and space co-ordinates'' is necessary; it is not sufficient to define turbulent motion as irregular in time alone. For instance, the case in which a given quantity of a fluid is moved bodily in an irregular way; the motion of each part of the fluid is then irregular with respect to time to a stationary observer, but not to an observer moving with the fluid. Nor is turbulent motion, a motion that is irregular in space alone, became a steady flow with an irregular flow pattern might then come under the definition of turbulence.

Different Types of Turbulence:

According to the definition of Taylor and Von Karman, "Turbulence can be generated by the friction forces at fixed walls (fluid flow through conduits, fluid flow past solid surfaces) or by the flow of layers of fluids with different velocities past or over one another".

The above definition indicates that there are two distinct types of turbulence.

(i) Wall turbulence

(ii) Free turbulence

(i) Wall turbulence: Turbulence which is generated by the viscous effect due to presence of a solid is called wall turbulence.

(ii) Free turbulence: Turbulence in the absence of walls, generated by the flow of layers of fluids at different velocities is called free turbulence.

In the case of real viscous fluids, viscosity effects will result in the conversion of kinetic energy of flow into heat; thus turbulent flow, like all flow of such fluids, is dissipative in nature. If there is no continuous external source of energy for the continuous generation of the turbulent motion, the motion will decay. Other effects of viscosity are to make the turbulence more homogeneous and to make it less dependent on direction.

Isotropic Turbulence:

The turbulence is called isotropic if its statistical features have no preference for any specific direction and minimum number of quantities and relations are required to describe its structure and behavior. No average shear stress can occur and consequently, no velocity gradient of the mean velocity. This mean velocity, if it occurs, is constant throughout the field.

Since it is very complicated problem and it is also a hypothetical type of turbulence, because no actual turbulent flow shows true isotropy, though conditions may be made such that isotropy is more or less closely approached. In order to bring out the essential features of the turbulence problem we have to study the simplest type of turbulence. In isotropic turbulence the mean value of any function of the velocity components and their derivatives is unaltered by any rotation or reflection of the axes of references. Thus in particular,

$$\overline{u^2} = \overline{v^2} = \overline{w^2}$$

and $\overline{uv} = \overline{vw} = \overline{wu} = 0$

Isotropy introduce a great simplicity into the calculations. The study of isotropic turbulence may also be of practical importance, since far from solid boundaries it has been observed that $\overline{u_1^2}$, $\overline{u_2^2}$, $\overline{u_3^2}$ tend to become equal to one another, e.g. in the natural winds at a sufficient height above the ground and in a pipe flow the axis.

From theoretical considerations and experimental evidence it is known that the fine structure of most actual non-isotropic turbulent flows is nearly isotropic (local isotropy). Hence many features of isotropic turbulence may apply to phenomena in actual turbulence that is determined mainly by the fine-scale structure, where local isotropy prevails.

Homogeneous Turbulence:

The turbulence which has quantitatively the same structure in all parts of the flow field is called homogeneous turbulence. In a homogeneous turbulent flow field the statistical characteristic are invariant for any translation in the space occupied by the fluid. Most of the theoretical works in turbulence and MHD turbulence in homogeneous and isotropic field in an incompressible fluid at rest.

The conception of homogeneous turbulence is idealized, in that there is no known method of realizing such a motion exactly. The various method of producing turbulent motion in a laboratory or in nature all involves discrimination between different parts of the fluid, so that the average properties of the motion depend on position. However, in certain circumstances this departure from exact independence of position can be made very small, and it is possible to get a close approximation to homogeneous turbulence.

Non-isotropic Turbulence:

In all other cases where the mean-velocity shows a gradient, the turbulence will be non-isotropic or an isotropic.

1.2. Method of Averages:

To describe a turbulent flow mathematically, it is necessary to consider an instantaneous velocity such as u is the sum of the time average part $\overline{u}$ and momentary fluctuation (fluctuating velocity) u′ i,e

$$u = \overline{u} + u' \qquad \text{------------------ (1.2.1)}$$

where $\overline{u}$ is average value or mean value, u' is fluctuating velocity and u is velocity of motion

In a steady flow $\overline{u}$ does not change with time. In talking the average of a turbulent quantity, the result depends not only on the scale used but also on the method of averaging. These are four different kinds of averaging procedure introduced by Pai [100] that are found to be useful for the study of turbulent flows. These are

(i) time average, (ii) space average, (iii) space-time average and (iv) ensemble average or the statistical average.

If the turbulent flow field is quasi-steady, time average can be used. For a homogeneous turbulence flow field, space average can be considered. If the flow field is steady and homogeneous, space-time average is used. Lastly, if the flow field is neither steady nor homogeneous, we assume that averaging is taken over a large number of experiments that have initial and boundary conditions. This type of average is called ensemble average or statistical average. Ensemble average is more general than the time and space averages and very useful for the study of in homogeneous, non-stationary turbulent flow. This type of averaging can be applied to any flow. Most of the modern theories have used the ensemble averaging procedure for describing the statistical properties of turbulence. However, like the time and space averages, the physical interpretation of the ensemble average is not so simple. In general any turbulent field is completely determined by the hierarchy of correlations.

$\langle u_i(r,t)\rangle$, $\langle u_i(r,t)u_j(r',t)\rangle$, $\langle u_i(r,t)u_j(r',t)u_m(r'',t)\rangle$, where, $\langle \quad \rangle$ denote the ensemble average defined by Leslie [88]

In homogeneous isotropic turbulence the first correlation represents the mean velocity, and is simply zero, the pair correlation $\langle u_i(r)u_j(r')\rangle$ is often considered to be a sufficient description of turbulent flow. The higher order correlations are assumed to give less and less information so that only a finite number of correlations are required to determine the statistical properties of turbulence. This is a possible method of reducing the infinite hierarchy of equations into a closed set.

The double correlation tensor $R_{ij}(\hat{r},\hat{x},t)$ for two points separated by the space vector $\hat{r}$ is defined by

$$R_{ij}(\hat{r},\hat{x},t)=\langle\ (\ \hat{x}-\frac{1}{2}(\hat{r},t\ )u_j(\ \hat{x}+\frac{1}{2}\hat{r},t\ )\rangle \qquad \text{------------(1.2.2)}$$

Similarly, the triple correlation tensor T_{ijk} or higher correlation tensors can be introduced.

The Fourier transform of R_{ij} with respect to $\hat{r}$ defined by

$$\phi_{ij}(\hat{k},\hat{x},t)=\frac{1}{(2\pi)^3}\int\int\int_{-\infty}^{\infty}\overline{e}^{i(\hat{k},\hat{r})}R_{ij}(\hat{r},\hat{x},t)d\hat{r}, \qquad \text{--------------(1.2.3)}$$

represents the energy spectrum function $E(\hat{k},t)$ in the sense that it describes the distribution of kinetic energy over the various wave number component of turbulent flows and where $\hat{k}$ is wave vector.The Fourier transform defined above can be treated as generalized functions or distributions in the sense of Lighthill [80]. It follows from the inverse Fourier transform that

$$\frac{1}{2}\langle u^2\rangle=\frac{1}{2}\langle u_i(\hat{x})u_i(\hat{x})\rangle=\frac{1}{2}R_{ij}(o,\hat{x},t)=\int_0^{\infty}E(\hat{k},t)d\hat{k}. \qquad \text{-------------(1.2.4)}$$

So that $E(\hat{k},t)$ represents the density of contributions to the kinetic energy in the wave numbers of space k, thus the investigation of the energy spectrum function $E(\hat{k},t)$ is the central problem of the dynamics of turbulence.

Expressed in mathematical form the four methods of averaging applied for instance.

(a) Time average for a stationary turbulence

$$\overset{t}{\overline{u}}(x,t)=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{+T}u(x,t)ds \qquad \text{-------------(1.2.5)}$$

In practice the scale used in the averaging process determines the value of the period 2T.

(b) Space average in which we take the average over all the space at given time, i.e

$$\overset{s}{\bar{u}}(x,t) = \lim_{v_b \to \infty} \frac{1}{v_b} \int_{v_b} u(s,t)ds \qquad \text{-------------(1.2.6)}$$

In practice the volume of space the scale used in the averaging process determines Vb.

(c) Space time average in which we take the average over a long period of time and over the space i.e.,

$$\overset{s,t}{\bar{u}}(x,t) = \lim_{T \to \infty,\ V_b \to \infty} \frac{1}{2T.V_b} \int_{-TV_b}^{+T} \int^{t} u(s,y)dsdy \qquad \text{-------------(1.2.7)}$$

In practice the scale used determines both the values of T and of Vb.

(d) Statistical average in which we take the average over the whole collection of sample turbulent functions for a fixed time, i.e.

$$\overset{\omega}{\bar{u}}(x,t,\omega) = \int_{\Omega} u(x,t,\omega)d\mu(\omega) \qquad \text{-------------(1.2.8)}$$

over the whole Ω space of ω, the random parameter. The measure is

$$\int_{\Omega} d\mu(\omega) = 1 \qquad \text{-------------(1.2.9)}$$

A random scalar function *u(x,t,w)* is a function of the spatial coordinates x and time t, which depends on a parameter *w*. The parameter *w* is chosen at random according to some probability law in a space Ω.

1.3. Reynolds rules of Averages:

At first Osborn Reynolds [112] introduced elementary statistical motion into the consideration of turbulent flow. In the theoretical investigation of turbulence, he assumed that instantaneous fluid velocity satisfies the Navier-Stokes equations for a viscous fluid and that the instantaneous velocity may be separated into a mean velocity and a turbulent fluctuating velocity. u, P, T and ρ be respectively the instantaneous velocity, pressure time and density, then the process of averaging we write

$$u = \bar{u} + u', \quad P = \bar{P} + P', \quad \rho = \bar{\rho} + \rho', \quad T = \bar{T} + T' \text{ etc} \qquad \text{-----------(1.3.1)}$$

In these expressions the quantities with bars denote mean variables and the quantities with prime denote the fluctuating variables.

Further more $\overline{u'} = \overline{P'} = \overline{T'} = 0$ -----------(1.3.2)

In the study of turbulence we often have to carry out an averaging procedure not only on single quantities but also on products of quantities. Here the over scores have the following properties.

Let $A = \bar{A} + A'$ and $B = \bar{B} + B'$ -----------(1.3.3)

In any further averaging procedure we can show that

$$\bar{A} = \overline{\bar{A} + A'} = \bar{\bar{A}} + \overline{A'} = \bar{A} \qquad \text{whence} \quad \overline{A'} = 0 \qquad \text{------------- (1.3.4)}$$

$$\bar{B} = \overline{\bar{B} + B'} = \bar{\bar{B}} + \overline{B'} = \bar{B} \qquad \text{whence} \quad \overline{B'} = 0 \qquad \text{------------- (1.3.5)}$$

In the above relations we used the properties that the average of the sum is equal to the sum of the averages and the average of a constant times B is equal to the constant times the average of B.

Next

$$\overline{\bar{A}\bar{B}} = \bar{\bar{A}}\bar{\bar{B}} = \bar{A}\bar{B} \qquad \text{-------------- (1.3.6)}$$

$$\overline{\bar{A}B'} = \bar{\bar{A}}\overline{B'} = \bar{A}\overline{B'} = 0 \qquad \therefore \overline{B'} = 0 \qquad \text{-------------- (1.3.7)}$$

$$\overline{\bar{B}A'} = \bar{\bar{B}}\overline{A'} = \bar{B}\overline{A'} = 0 \qquad \therefore \overline{A'} = 0 \qquad \text{-------------- (1.3.8)}$$

Similarly,

$$\overline{AB} = \overline{(\bar{A} + A')(\bar{B} + B')} = \overline{\bar{A}\bar{B}} + \overline{\bar{A}B'} + \overline{A'\bar{B}} + \overline{A'B'} = \bar{A}\bar{B} + \overline{A'B'} \qquad \text{-------------- (1.3.9)}$$

Note that the average of a product is not equal to the product of the averages. Terms such as $\overline{A'B'}$ are called correlations.

1.4. Spectral Representation of the Turbulence:

Theoretical treatment of the turbulence is merely related to the solution of the Navier-Stokes equations. These equations, however, contain more unknowns than number of equations and therefore additional assumptions must be made. This is known as " Closure problem". An alternative approach is based on the spectral form of the dynamical Navier-Stokes equation. The spectral form of the turbulence is still under-determined, but it has a simple physical interpretation and is more convenient. The spectral approach is, however, almost exclusively used for the description of homogeneous turbulence [94, 95]. The principal concepts of spectral representation in the study of turbulence are described below:

If we neglect the body forces from the Navier-Stokes equation (1.5.2) and multiply the x_i-component of Navier-Stokes equation written for the point P by u'_j and multiply the x'_j component of the equation written for the point P′ by u'_i adding and taking ensemble averages we get.

$$\frac{\partial}{\partial t}\overline{u_i u'_j} + \overline{u'_j u_l \frac{\partial u_i}{\partial x_l}} + \overline{u_i u'_l \frac{\partial u'_j}{\partial x'_l}} = -\frac{1}{\rho}\left[\overline{u'_j \frac{\partial p}{\partial x_i}} + \overline{u_i \frac{\partial p'}{\partial x'_j}}\right] + v\left[\overline{u'_j \frac{\partial^2 u_i}{\partial x^2{}_l}} + \overline{u_i \frac{\partial^2 u'_j}{\partial x^2{}_l}}\right] \quad \text{-----------} (1.4.1)$$

Since in homogeneous turbulence the statistical quantities are independent of position in space and considering the point P and P′. Separated by a distance vector $\bar{r}$ and applying the laws of spatial covariances, a simplified form of equation (1.4.1) is obtained as:

$$\frac{\partial}{\partial t}\overline{u_i u'_j} = -\frac{\partial}{\partial r_1}\left(\overline{u_i u'_j u_l} - \overline{u_i u'_j u'_l}\right) + \frac{1}{\rho}\left[\frac{\partial \overline{p u'_j}}{\partial r_l} - \frac{\partial \overline{p' u_i}}{\partial r_j}\right] + 2v\frac{\partial^2 \overline{u_i u'_j}}{\partial r_l^2} \quad \text{---------} (1.4.2)$$

The covariance $\overline{u_i u'_j}$ is not suitable for direct analysis of quantitative estimate of the turbulent flows and it is better to use the three-dimensional Fourier transforms of $\overline{u_i u'_j}$ with respect to $\bar{r}$. The variable that corresponds to $\bar{r}$ in the three dimensional wave-number space is a vector $\bar{K} = (K_1, K_2, K_3)$. We define the wave number spectral density as:

$$\phi_{ij}(\vec{K}) = \frac{1}{(2\pi)^3}\int \overline{u_i u'_j}\exp(-i\bar{K}.\bar{r})d\bar{r} = \frac{1}{(2\pi)^3}\iiint \overline{u_i u_j}\exp\{-i(K_1 r_1 + K_2 r_2 + K_3 r_3)\}dr_1 dr_2 dr_3$$

---------(1.4.3)

It can be shown that if $\overline{u_i u'_j}$ has a continuous range of wavelength, $\phi_{ij}(\vec{K})$has a continuous distribution in wave number space. We can rigorously regard $\phi_{ij}(\vec{K})dK_1dK_2dK_3$ as the contribution of elementary volume $dK_1dK_2dK_3$ (centred at wave number $\hat{K}$ and therefore representing a wave number of length $\frac{2\pi}{|\vec{K}|}$, in the direction of vector $\vec{K}$) to the value of $\overline{u_i u'_j}$ hence the name "Spectral density". This is consistent with the behaviour of the inverse transform

$$\overline{u_i u'_j(r)} = \int_{-\infty}^{\infty} \phi_{ij}(\vec{K}) \exp(i\vec{K}.r) d\vec{K} \qquad \text{---------- (1.4.4)}$$

The one dimensional wave number spectrum of $\overline{u_i u'_j}$ for a wave number component in the x_1 direction is

$$\phi_{ij}(K_i) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{u_i u'_j}(r_l) \exp(-i\vec{K}_l.r_l) dr_l \qquad \text{---------- (1.4.5)}$$

whose inverse is

$$\overline{u_i u'_j(r)} = \int_{-\infty}^{\infty} \phi_{ij}(K_l) \exp(ik_l, r_l).dK_l \qquad \text{----------- (1.4.6)}$$

The equation (1.4.2) for unstrained homogeneous turbulence becomes on Fourier transforming as:

$$\frac{\partial \phi_{ij}(\vec{K})}{\partial t} = \Gamma_{ij}(\vec{K}) + \Pi_{ij}(\vec{K}) - 2vK_l^2.\phi_{ij}(\vec{K}) \qquad \text{------------- (1.4.7)}$$

where Γ and Π are the transforms of the triple product and pressure terms respectively.

1.5. Correlation Functions:

Taylor, G. I. [144] introduced new notions into the study of the statistical theory of Turbulence, Taylor successfully developed a statistical theory of turbulence which is applicable to continuous movements and satisfies the equation of motion.The first important new notion was that of studying the correlation or coefficient of correlation between two fluctuating quantities in turbulent flow. In his theory, Taylor makes much use of the correlation between the components of the fluctuations neighbouring points.

The statistical property of a random variable may be described by the correlation function, which is defined as follows:

Consider the fluctuating variables u_i and u_j and assume that there exists certain correlation between them. The correlation function is defined as

$$P_{ij} = \overline{u_i u_j} \qquad \text{------------ (1.5.1)}$$

where the bar denotes the average by certain process. Some times it is convenient to use the correlation coefficient such as

$$R_{ij} = \frac{\overline{u_i u_j}}{\sqrt{u_i^2}\sqrt{u_j^2}} \qquad \text{------------ (1.5.2)}$$

By Cauchy inequality, we have

$$\overline{u_i u_j} - \sqrt{u_i^2}\sqrt{u_j^2} \le 0 \qquad \text{------------ (1.5.3)}$$

hence $-1 \le R_{ij} \le 1$

If we consider u_i and u_j as the velocity components in a flow field, the correlation of Equation (1.5.1) as a tensor of second rank.

By a different process of averaging we obtain different kinds of correlation functions. If we consider u_i and u_j as the velocity components at a given point in space, u_i and u_j are functions of time; hence, we should take the time average in equation (1.5.1) to get the correlation function P_{ij}. If we consider u_i and u_j as the velocity components at a given time, u_i and u_j are functions of space co-ordinates $x(x_1, x_2, x_3)$; hence, we should take the space average in equation to get the correlation function.

More generally if we consider u_i and u_j as functions of both time t and spatial co-ordinates $x(x_1, x_2, x_3)$, we take a space-time average in equation (1.7.1) to get the correlation function.The correlation function between the components of the fluctuating velocity at the same time at two different points of the fluid, first introduce by Taylor, G. I. [144], has been investigated extensively in the isotropic turbulence.

The correlation function between two fluctuating velocity components at the same point and at the same time gives the Reynolds Stress. The correlation function between two fluctuating quantities may also be defined in a manner similar to above.

1.6. Historical Back Ground of Early Work of Turbulence:

Osborne Reynolds [112] first made the systematic investigations and gave the experimental results to understanding the facts of turbulent flow. He made the remarkable difference between laminar and turbulent flows by proposing the Reynolds number and gave the Reynolds stresses to describe the turbulent phenomena. Reynolds averaged the Navier-Stokes equations for an incompressible fluid. Thus he established the so-called Reynolds equations for the mean values. His technique followed closely that used by Maxwell in 1850 when Maxwell deduced the Navier-Stokes equation from the Kinetic theory of gases. Therefore, the theory of turbulence was based on analogies with the discontinuous collisions between the discrete entities studied in Kinetic gas theory. Prandtl [102] developed His "mixing length" theory based on the problems of practical importance such as pipe flows over boundaries of specific shapes. Prandtl's theory was successfully applied to the turbulent flow of a liquid in a circular pipe and also to the meteorological problem of wind distribution in the layer of air adjacent to the ground.

The origin of the idea of statistical approach to the problem of turbulence may be traced back to Taylor's [143] in which he has advanced the concept of the Lagrangian correlation coefficient that provides a theoretical basis for turbulent diffusion. Tailor, G. I. [144,145] and Von Karman, T. [152,153] broke away from the concept, which described turbulence in terms of collisions between discrete entities and instead introduce the concept of velocity correlation at two or more points, as one of the parameters involved in describing turbulent motion. Taylor, G. I. introduced the so-called " energy spectrum" method to describe the probability density function for energy in the turbulent flow field. Von Karman proved that the correlation of velocities at two points is a tensorial character. He introduced the "correlation tensor" method. Taylor, G. I. [144] introduces the idea that the velocity of the fluid of turbulent motion is a random continuous function of position and time. To make the turbulent motion amenable to mathematical treatment, he assumes the turbulent fluid to be homogeneous and isotropic. In its supports, he describes the measurements showing that the turbulence generated downstream from a regular array to rods in a wind tunnel is approximately homogeneous and isotropic. In spite of the fact that the turbulence in nature is not always exactly homogeneous and isotropic, it is essential to study homogeneous and isotropic turbulence as a first step to understand the more complicated phenomenon of non-homogeneous turbulence.

Taylor [148] took into account the non-linearity of the dynamical equations and showed that it results in the skew ness of the probability distribution of the difference between the velocity components at two points. He showed that the non-linearity of the dynamical equation is also responsible for the existence of the interaction between components of the turbulent having different fluctuations. Kolmogoroff's [76, 77] work contributed significantly to understanding the physics of turbulence. His outstanding works in the theory of local homogeneous and local isotropic turbulent flow resulted in the "2/3 Kolmogoroff law", the analog of which in the language of spectra is the 5/3 law.

1.7. First-order Reactions:

Chemical Reaction:

A chemical reaction is a process that is usually characterized by a chemical change in which the starting materials (reactants) are different from the products. **Chemical reactions tend to involve the motion of electrons, leading to the formation and breaking of chemical bonds**. There are several different types of chemical reactions and more than one way of classifying them. It is a process that always results in the inter-conversion of chemical substances.

First Order reaction:

A first order reaction (order = 1) has a rate proportional to the concentration of one of the reactants. A common example of a first-order reaction is the phenomenon of radioactive decay. In this case, reaction rate is directly proportional to amount of reactant. A first-order reaction depends on the concentration of only one reactant. Other reactants can be present, but each will be zero-order. The sum of concentration exponents in the rate law for a first order reaction is one.

A first-order reaction depends on the concentration of only one reactant (a unimolecular reaction). Other reactants can be present, but each will be zero-order. The rate of law for an elementary reaction that is first order with respect to a reactant A is

$$r = -\frac{d[A]}{dt} = k[A]$$ ----------(1.7.1)

k is the first order rate of constant, which has units of 1/time.

Reactant:

The substance or substances initially involved in a chemical reaction are called reactants which are different from the products.

Reactant Concentrations:

In chemistry, concentration is the measure of how much of a given substance there is mixed with another substance. This can apply to any sort of chemical mixture. Concentration is a way of describing mixture composition.

It usually makes the reaction happen at a faster rate if raised through increased collisions per unit time.

If the concentration of one of the reactants remains constant (because it is a catalyst or it is in great excess with respect to the other reactants) its concentration can be included in the rate constant.

Rate and order of Reaction:

The rate of a chemical reaction is the amount of substances reacted or produced per unit time. The Order of reaction, in chemical kinetics, with respect to a certain reactant, is defined as the power to which its concentration term in the rate equation is raised.

For example, given a chemical reaction A + B → C with a rate equation

$r = k[A]^2[B]^1$ --------------(1.7.2)

the reaction order with respect to A would be 2 and with respect to B would be 1, the total reaction order would be 2+1=3. It is not necessary that the order of a reaction is a whole number - zero and fractional values of order are possible - but they tend to be integers. Reaction orders can be determined only by experiments.

According to Bansal [22(a)] the general reaction equation in which A and B are transformed to give P

$aA + bB \rightarrow cP$, -------------(1.7.3) the reaction rate can be written as

$$-\frac{1}{a}\frac{d[A]}{dt}, \quad -\frac{1}{b}\frac{d[B]}{dt}, \quad +\frac{1}{c}\frac{d[P]}{dt}$$

and the rate of law be written in the form of the equation

$$-\frac{1}{a}\frac{d[A]}{dt} = k[A]^n\,[B]^m \qquad \text{--------------(1.7.4)}$$

where [A], [B] and [P] denote the active concentrations in moles/litre of species A, B and P, t represent the time, n and m are integers, k is the proportionality constant referred to as the reaction rate constant or specific rate of constant and a, b, c are the stoichiometric coefficients.

Since the concentrations of A and B are diminishing , $-\frac{1}{a}\frac{d[A]}{dt}$, $-\frac{1}{b}\frac{d[B]}{dt}$ are negative number while $+\frac{1}{c}\frac{d[P]}{dt}$ is positive. Any of these derivatives may be used to express the rate of the reaction.

The order of a reaction is the algebraic sum of the exponents of all the concentration terms, which appear in the rate law (1.9.4).

For the reaction given in equation (1.9.3) the rate law may be expressed as

$$-\frac{1}{a}\frac{d[A]}{dt} = k[A]^n\,[B]^m$$

where n is the order of the reaction with respect to A and m is the order of the reaction with respect to B. The over all order of the reaction is given by the sum (n+m).

The Rate Constant:

The rate constant isn't actually a true constant! It varies, for example, if we change the temperature of the reaction, add a catalyst, or change the catalyst. The rate constant is constant for a given reaction only if all we are changing is the concentration of the reactants. A first order reaction has a rate constant of 1.00 s^{-1}.

1.8. Rotating System:

The essential characteristic of turbulent flows is that turbulent fluctuations are random in nature. The system is usually rotating with a constant angular velocity in geophysical flows. Such large-scale flows are generally turbulent. When the motion is referred to axes, which rotate steadily with the bulk of the fluid, the coriolis and centrifugal force must be supposed to act on the fluid. The coriolis force due to rotation plays an important role in a rotating system of turbulent flow, while the centrifugal force with the potential is incorporated into the pressure.

Turbulence in the presence of Coriolis force is an interesting tropic in astrophysics as well as in fluid mechanics. Ohji [98] considered the effect of Coreolis force on turbulent motion in the presence of strong magnetic field with the assumption that Coriolis force term ($-2\Omega\times U$) is balanced by $\nabla\langle x\rangle$ (the geostropic wind approximation) where x represents the generalized pressure.

Kishore and Dixit [61], Kishore and Singh [63], Dixit and Upadhyay [39], Kishore and Golsefied [66] discussed the effect of coriolis force on acceleration covariance in ordinary and MHD turbulent flow. Funada, Tuitiya and Ohji [47] considered the effect of coriolis force on turbulent motion in presence of strong magnetic field. Kishore and Sarker [71] studied the rate of change of vorticity covariance in MHD turbulence in a rotating system. Sarker [123] studied the thermal decay process of MHD turbulence in a rotating system.

1.9. Equation of Motion of Dust Particles:

The influence of dust particles on viscous flows has a great importance in petroleum industry and in the purification of crude oil. Other important applications of dust particles in boundary layer, include soil solvation by natural winds and dust entrainment in a cloud during nuclear explosion. Knowledge of the behaviour of discrete particles in a turbulent flow is of great interest to many branches of technology, particularly if there is a substantial difference between particles and the fluid. Saffman P.G. [118] derived an equation that described the motion of a fluid containing small dust particles, which is applicable to laminar flows as well as turbulent flow. Sinha [134] studied the effect of dust particles in addition to the magnetic field fluctuation on the turbulent flow of an incompressible fluid.

The relative motion of dust particles and the air will dissipate energy because of the drag between dust and air, and extract energy from turbulent fluctuations. If as certainly seems possible, the turbulent intensity is reduced than the Reynolds stresses will be decreased and the force required to maintain a given flow rate will likewise be reduced.

In order to formulate the problem in a reasonably simple manner and to bring out the essential features, we shall make simplifying assumption about the motion of dust particles. It will be supposed that their velocity and number density can be described by fields $u(\vec{x},t)$ and $N(\vec{x},t)$. We also assume that the bulk concentration (i.e. concentration of volume) of dust is very small so that the effect of dust particles on the gas is equivalent to an extra force $KN(\vec{v}-\vec{u})$ per unit volume, where $\vec{u}(\vec{x},t)$ is the velocity of the gas and K is constant. It is also supposed that the Reynolds number of the relative motion of dust and gas is small compared with unity, so that the force between the dust and gas is proportional to the relative velocity. Then with small bulk concentration and the neglect of the compressibility of the gas, the equations of motion and continuity of the gas are:

$$\rho\left[\frac{\partial\vec{u}}{\partial t}+(\vec{u}.\nabla)\vec{u}\right]=-\nabla p+\nu\nabla^2\vec{u}+KN(\vec{v}-\vec{u}) \qquad \text{----------} (1.9.1)$$

$$\text{div } \vec{u}=0 \qquad \text{----------} (1.9.2)$$

where p, ρ and μ are the pressure, density, and viscosity of the clean gas respectively. f, dimension frequency; N, constant number of density of dust particle. K,the Stokes's resistance coefficient which for spherical particle of radius r is 6πμr.

As will be seen below, the effect of the dust is measured by the mass concentration. The bulk concentration is $f\frac{\rho}{\rho_1}$ where ρ_1 is the density of the material in the dust particles.

For common materials $\frac{\rho}{\rho_1}$ will be of the order of several thousand or more, so that the mass concentration may be significant fraction of unity, while the bulk concentration is small. it is to be noted that for suspension in liquids, the bulk and mass concentration will roughly be the same. So that the qualitative differences in the motion of dusty gases and the suspensions in the liquids may be expected. For spherical particles, the Einstein increase in the viscosity is

$\frac{5}{2}\mu f\frac{\rho}{\rho_1}$, which is negligible for a dusty gas but may be significant for a liquid suspension.

The force exerted on the dust by the gas is equal and opposite to the force exerted on the gas by dust, so that the equation of motion of the dust is:

$$mN\left[\frac{\partial\vec{v}}{\partial t}+(\vec{v}.\nabla)\vec{v}\right]=mN\vec{g}+KN(\vec{v}-\vec{u}) \qquad \text{------------ (1.9.3)}$$

where mN the mass of the dust per unit volume and $\vec{g}$ is the acceleration due to gravity. The buoyancy force is neglected since $\frac{\rho}{\rho_1}$ is small.

The equation of continuity of the dust is:

$$\frac{\partial N}{\partial t}+div(N\vec{v})=0 \qquad \text{------------- (1.9.4)}$$

Here, $\nu=\frac{\mu}{\rho}$ is kinetic viscosity of the clean gas and $\tau=\frac{M}{K}$ is called the relaxation time of the dust particles. It is measure of the time for the dust to adjust to changes in the gas velocity. For spherical particles of radius ε,

$$\tau=\frac{\frac{4}{3}\pi\varepsilon^3\rho}{6\pi\varepsilon\mu}$$

Or $$\tau=\frac{2}{9}\frac{\varepsilon^2}{\nu}\frac{\rho_1}{\rho}, \qquad \text{------------- (1.9.5)}$$

K= $6\pi\mu\epsilon$, Stokes drag formula, M= (4/3) $\pi\epsilon^3\rho_1$, mass of the particle of radius ϵ . ρ_1 is the density of the dust particles.

The effect of dust is described in two parameters f and τ. The former describes how much dust is present and the latter is determined by the size of individual particles. Making the dust fine, will decreaseτ, and making coarse, will increase τ in a manner proportional to the surface area of the particles.

1.10. Decay of Turbulence before the final period and in the final period:

The turbulent flows, in the absence of external agencies always decay. In considering the dynamic equations for the velocity correlation and for the energy spectrum, it has been shown that these correlations and spectra change with time and the turbulence decays if no energy sources are present to sustain it. As in all fluid flows, an important parameter is the Reynolds number and the character of the turbulence may vary appreciably whether the Reynolds number of turbulence is large or small. Batchelor and Townsend [8,9,10] have made many measurements of the decay of an isotropic turbulence produced by grids. From the results of these measurements Batchelor [9] arrives at the conclusion that different periods of decay may actually be distinguished; an initial period, a final period and a transition period.

Townsend's experiments have shown that the final period seems to apply to distances greater than 500M. Of course, this value too should depend on the initial Reynolds number of turbulence. In Townsend's experiments the Reynolds number $Re_M = \overline{U}_1 M/\nu$ was about 650. Where, $Re_M \rightarrow$mesh Reynolds number; M, Mesh of a grid; $\overline{U}_1$, speed; ν, kinematic viscocity.
In the initial period the decay is determined predominantly by the decay of the energy containing eddies; in the final period the viscous effects predominate over inertial effects. Thus, in the final period, where the Reynolds number of turbulence is very small, the inertial terms in the dynamic equations may be neglected.

According to Deissler [36], in the final period of decay the inertia terms (triple correlations) in the two point correlation equation obtained from the momentum and continuity equations can be neglected because the Reynolds number of the eddies is small, and a solution can be obtained. However, at earlier times the inertia terms in the two-point correlation equation can't be neglected. So that in order to obtain a solution, an intuitive assumption is generally introduced to relate the triple correlations to the double correlations. The situation in homogeneous turbulence is therefore analogous to that in turbulent shear flow where intuitive assumptions have been introduced to relate the Reynolds stress or the eddy diffusivity to the mean flow; although one case of homogeneous turbulence, the turbulence in the final period, has been solved without introducing intuitive hypothesis.

1.11. Statistical Theory of Distribution Functions in Turbulence and it's Properties:

In the statistical theory, the distribution function are discussed by several authors in the past, but the dynamical equations describing the time evolution of the finite dimensional probability distributions in turbulence were first proposed by Lundgren [84] and Monin [93,94]. Lundgren [84] considered a large ensemble of identical fluid system in turbulent state. In his consideration each number of the ensemble is an incompressible fluid in an infinite space with velocity $\hat{u}(\hat{r},t)$ satisfying the continuity and Navier-Stokes equations. The only difference in the members of ensemble is the initial conditions that vary from member to member. He considered a function $F(\hat{u}(\hat{r}_1,t),\hat{u}(\hat{r}_2,t)---)$ whose ensemble is given as $\langle F(\hat{u}(r_1,t),\hat{u}(r_2,t)---)\rangle$ and defined one point distribution function $f_1(\hat{r}_1.\hat{v}_1,t)$ such that $\int f_1(\hat{r}_1.\hat{v}_1,t)\,d\hat{v}_1$ is the probability that the velocity at a point $\hat{r}_1$ at time t is in element $d\hat{v}_1$ about $\hat{v}_1$ and is given by $f_1(\hat{r}_1,\hat{v}_1,t) = < \delta(\hat{u}(\hat{r}_1,t)-\hat{v}_1)\ \rangle$

and two points distribution function is given by

$$f_2(\hat{r}_1,\hat{v}_1,\hat{r}_2,\hat{v}_2,t) = \langle \delta(\hat{u}(r_1,t)-\hat{v}_1)\delta(\hat{u}(r_2,t)-\hat{v}_2)\rangle$$

In short one and two point distribution functions are denoted as $f_1^{(1)}$ and $f_2^{(1,2)}$. Here δ is the dirac-delta function, which is defined as

$$\int \delta(\overline{u}-\overline{v})d\overline{v} = \begin{cases} 1 & \text{at the point } \overline{u}=\overline{v} \\ 0 & \text{elsewhere} \end{cases}.$$

and $\langle \quad \rangle$ denote the ensemble average.

1.12. Fourier Transformation of the Navier-Stokes Equation:

The principal reason for using Fourier transformation is that they convert differential operators into multipliers. The equations are so complicated in configuration (or coordinate) space that very little can be done with them, and the transformation to wave number (or Fourier) space simplifies them very considerably. Another and more mathematical argument shows that these transforms are right method of treating a homogeneous problem. Associated with any correlation function, $\phi(\vec{x},\vec{x}')$ is a sequence of eigen functions $\phi(\vec{n},\vec{x}')$ and their associated eigen-values $\lambda(\vec{n})$. These quantities satisfy the value equation.

$$\int \phi(\vec{x},\vec{x}')\Psi(\vec{n},\vec{x})d^3\vec{x}' = \lambda(\vec{n})\Psi(\vec{n},\vec{x}) \qquad \text{-------------- (1.12.1)}$$

and the orthonormalization relation

$$\int \Psi(\vec{n},\vec{x})\Psi^*(\vec{m},\vec{n})d^3\vec{x} = 1, \quad \text{if } m=n \qquad \text{-------------- (1.12.2)}$$

$$=0 \quad \text{otherwise}$$

These equations imply that ϕ is a scalar. Actually it is a tensor of order two, but this complicates the argument without introducing anything essentially new. The index $\vec{n}$ is in general a complex variable and ψ^* denotes the complex conjugate of ψ (strictly, ψ^* is the adjoint of ψ, but since ϕ is real and symmetric the adjoint is simply the complex conjugate). The integrations in equations (1.12.1) and (1.12.2) are over all space, which may be finite or infinite. If the space is finite $\vec{n}$ is usually an infinite but countable sequence, while if space is infinite, $\vec{n}$ will be a continuous variable. Here the eigen functions all have real eigen-values. If follows from (1.12.1) and (1.12.2) that.

$$\phi(\vec{x}.\vec{x}) = \sum_{all\ n} \lambda(\vec{n}).\psi(\vec{n}.\vec{x})\psi^*(\vec{n},\vec{x}') \qquad \text{------------- (1.12.3)}$$

and this is the diagonal representation of the correlation function in terms of its eigen functions. Evidently these functions are only defined "within a phase" that is, a factor exp(iγ) can be added to $\psi(\vec{n},\vec{x})$ without altering $\phi(\vec{x},\vec{x}')$ provided γ is real and independent of x. For a homogeneous field, ϕ is a function of $\vec{x},\vec{x}'$ only and the problem is to find the eigen functions which are also homogeneous within a phase in the sense that

$$\psi(\vec{n},\vec{x}') = \exp(i\gamma)\psi(\vec{n},\vec{x}+\vec{a})$$

This equation is satisfied by the Fourier equation

$$\psi(\vec{n},\vec{x}) = \exp(i\vec{n}.\vec{x}) = \exp(i\vec{n}_j\vec{x}_j)$$

with $\gamma = -\vec{n}.\vec{a}$. In this situation (instance), therefore, "the index", $\vec{n}$ is a wave number. Equation (11.3) becomes.

$$\phi(\vec{x},\vec{x}') = \sum \lambda(\vec{n})\exp\{in(\vec{x}-\vec{x}')\}$$

so that $\lambda(\vec{n})$ may be identified with $\phi(\vec{n})$, the Fourier transform of the correlation function.

Since we are considering homogeneous isotropic turbulence, the turbulent field must be infinite in extent. This produces, mathematical difficulties, which can only be resolved by using functional calculus. This difficulty is avoided by supposing that the turbulence is confined to the inside of a large box with sides (a_1, a_2, a_3) and that it obeys cyclic boundary conditions on the sides of this box. The a_i is allowed to tend to infinity at an appropriate point in the analysis. Thus the Fourier transform is defined by

$$U_i(\vec{x}) = (2\pi)^3 (a_1, a_2, a_3)^{-1} \sum_K u_i(\vec{K}) \exp(i\vec{K}.\vec{x}) \qquad \text{--------------(1.12.4)}$$

Here $\vec{K}$ is limited to wave vectors of the from

$$\frac{2n_1\pi}{a_1}, \frac{2n_2\pi}{a_2}, \frac{2n_3\pi}{a_3}$$

where n_i are integers while the a_i are, as before the sides of the elementary box. As these sides become infinitely large, equation (1.12.4) goes over into standard form,

$$U_i(\vec{x}) = \int u_i(\vec{K}) \exp(i\vec{K}.\vec{x}) d^3\vec{K} \qquad \text{-------------- (1.12.5)}$$

The inverse of (1.12.5) is,

$$u_i(\vec{K}) = (2\pi)^{-3} \int_{box} u_i(\vec{x}) \exp(-i\vec{K}\vec{x}) d^3x \qquad \text{------------- (1.12.6)}$$

The Fourier transform of Navier-Stokes equation may be written as

$$\left[\frac{d}{dt} + vK^2\right] u_i(\vec{K}) = M_{ijm}(\vec{K}) \sum^{\Delta} u_j(\vec{P}) U_m(\vec{r}) \qquad \text{-------------- (1.12.7)}$$

where $\sum^{\Delta}$ is a short notation for the integral operator in

$$\iint U_j(\vec{K}) U_m(\vec{r}).\delta(\vec{K} - \vec{P} - \vec{r})(d^3\vec{p})(d^3\vec{r}) \qquad \text{-------------- (1.12.8)}$$

where δ_K, p+r is the Kronecker delta symbol which is zero unless

$$\vec{K} = \vec{p} + \vec{r}$$

Here, $M_{ijm}\left(\vec{K}\right)$ is a simple algebraic multiplier and not a differential operator. We have

$$M_{ijm}\left(\vec{K}\right) = -\frac{1}{2}i.P_{ijm}\left(\vec{K}\right) \qquad \text{---------------- (1.12.9)}$$

where, $P_{ijm}\left(\vec{K}\right) = K_m P_{ij}\left(\vec{K}\right) + K_j P_{im}\left(\vec{K}\right)$ and $P_{ij} = \delta_{ij} - \frac{K_i K_j}{K^2}$

$P_{ij}\left(\vec{K}\right)$ is the Fourier transform of $P_{ij}(\nabla)$ but $P_{ijm}\left(\vec{K}\right)$ is not the transform of $P_{ijm}(\nabla)$.

As it is stands, equation (1.12.7) can't describe stationary turbulence since it contains no input of energy to balance the dissipative effect of viscosity. In real life this input is provided by effects, such as the interaction of mean velocity gradient with the Reynolds stress, which are incompatible with the ideas of homogeneity and isotropy. To avoid this difficulty, we introduce in to the right hand side of equation (1.12.7) a hypothetical homogeneous isotropic stirring force f_i. The equation then reads.

$$\left[\frac{d}{dt} + \nu K^2\right] u_i\left(\vec{K}\right) = M_{ijm}\left(\vec{K}\right) \sum^{\Delta} u_j\left(\vec{P}\right) u_m\left(\vec{r}\right) + \partial_i\left(\vec{K}\right) \qquad \text{-------------- (1.12.10)}$$

1.13. Magneto-hydrodynamic (MHD) Turbulence:

The magneto-hydrodynamic turbulence is the study of the interaction between a magnetic field and the turbulent motions of an electrically conducting fluid. The interaction between the velocity and the magnetic fields results in a transfer of energy between the Kinetic and magnetic spectra, and it is thought that the interstellar magnetic field is maintained by a "dynamo" action from turbulence in the interstellar gas.

Magneto-hydrodynamic (MHD) is an important branch of fluid dynamics. MHD is the science which deals with the motion of highly conducting fluids in the presence of a magnetic field the motion of the conducting fluid across the magnetic field generates electric current which changes the magnetic field and the action of the magnetic field on these currents gives rise to mechanical force which modifies the flow of the field. From historical point of view it seems that the first attempt to study the problem of MHD is due to Faraday. Later on in 1937 Hartmann took up Faraday's idea in understood conditions. Hartmann carried out experiments, which demonstrated the influence of a very intense magnetic field on the motion of mercury.

Modern applications of magneto-hydrodynamics in the fields of propulsion, nuclear fission and electrical power generation make the problem of magneto-hydrodynamic turbulence one of considerable interest to engineers, since turbulent phenomena seen to be inherent in almost all type of flow problems. There are two basic approaches to the problem, the macroscopic fluid continuum model known as MHD, and the microscopic statistical model known as plasma dynamics. We shall be concerned here only with the MHD, that is electrically conducting fluids, and study the problems of MHD turbulent flow.

The theory of turbulence in an incompressible viscous and electrically conducting fluid is formulated probabilistically through the use of the joint characteristic functional and the functional calculus. The use of the joint characteristic functional approach relies upon the fact that the velocity and magnetic fields are both solenoid, and hence, in the probabilistic sense, are jointly distributed over the phase space consisting of the set of all solenoid vector fields. The formulation of the problem in phase space is completely carried out. The full space-time functional formulation of the problem as developed by Lewis and Kraichnan [82] for "ordinary turbulence" is extended to magneto-hydrodynamic turbulence. This approach enables us to generate space-time correlation between the velocity and magnetic field components rather than merely spatial correlations as were used in the original [53] Hopf presentation. Dynamical equation for various order space-time correlations between velocity and magnetic field components are derived from the joint characteristic functional by its expansion in a Taylor series.The concept of Kolomogoroff's [77] equilibrium hypothesis for ordinary turbulence is extended to magneto-hydrodynamic turbulence. The problem of predicting the form of the energy spectra in the equilibrium range is taken up.

The fundamental equations of magneto-hydrodynamics for an incompressible fluid are

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u} = -\frac{1}{\rho}\nabla p + \frac{\rho e}{\rho}\vec{E} + \frac{\mu}{\rho}\vec{j} \times \vec{H} + \nu\nabla^2\vec{u} + \vec{F} \qquad \text{------------ (1.13.1)}$$

$$\nabla.\vec{u} = 0 \qquad \text{------------ (1.13.2)}$$

$$\frac{K}{c}\frac{\partial \vec{E}}{\partial t} = curl\vec{H} - 4\pi\vec{J} \qquad \text{----------- (1.13.3)}$$

$$\frac{\mu_e}{c}\frac{\partial \vec{H}}{\partial t} = -curl\vec{E} \qquad \text{----------- (1.13.4)}$$

$\nabla.\vec{H} = 0$ ----------- (1.13.5)

$J = \sigma\left(c\vec{E} + \mu_e \vec{u} \times \vec{H}\right) + \rho_e \dfrac{\vec{u}}{c}$ ----------- (1.13.6)

where $\vec{u}$, the velocity vector; $\vec{F}$, the body force; P, the pressure; ρ, the density of the fluid which is constant; ρ_e, the excess electric charge; $\vec{E}$, the electric field strength; μ_e, the magnetic permeability; $\vec{J}$, the electric current density; $\hat{H}$, the magnetic field strength; ν, the coefficient of kinematic viscosity; k, the dielectric constant; c, the speed of light; σ, the electrical conductivity; ∇, the gradient operator, $\nabla.\nabla = \nabla^2$ and t is the time.

When conductivity σ of the fluid tends to infinity the electric field strength $\vec{E}$, at each point must tend to the value $\frac{\mu_e \vec{u} \times \vec{H}}{c}$, otherwise the current $\vec{J}$ given by equation (1.15.6) becomes very large even when very slightest mass motion is present. Hence when σ is large we may assume that

$$\vec{E} = -\mu_e \frac{\vec{u} \times \vec{H}}{c} \quad \text{------------- (1.13.7)}$$

a relation which is increasingly valid as $\sigma \to \infty$

An important consequence of relation (1.13.7) is that under the circumstances in which this is a good approximation the energy in the electric field is of the order of $|\vec{u}|^2 / c^2$ of the energy in the magnetic field and can, therefore, be neglected. This approximation is known as the approximation of Magneto-hydrodynamics. We have to consider only the interaction between the two fields $\vec{u}$ and $\vec{H}$.

In the MHD approximation, Maxwell equation (1.13.3) becomes,

$$\vec{J} = \frac{1}{4\pi} curl\vec{H} \quad \text{---------- (1.13.8)}$$

In the framework of approximations (1.13.7) and (1.13.8) the Navier-Stokes equation are modified to take into account the electromagnetic body force (assuming that there is no body force $\vec{F}$) and equation (1.13.1) becomes

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u} = \frac{\mu_e}{4\pi\rho} curl\vec{H} \times \vec{H} - \frac{1}{\rho}\nabla P + \nu\nabla^2\vec{u} \quad \text{---------- (1.13.9)}$$

Again, in the approximation (1.13.7), Maxwell equation (1.13.4) becomes

$$\frac{\partial \vec{H}}{\partial t} = curl(\vec{u} \times \vec{H}) \quad \text{-------- (1.13.10)}$$

In a higher approximation in which the loss of energy by Joule heat is allowed for the equation (1.13.10) is modified to [12]

$$\frac{\partial \vec{H}}{\partial t} - curl(\vec{u} \times \vec{H}) = \lambda \nabla^2 \vec{H} \quad \text{--------- (1.13.11)}$$

where $\lambda = (4\pi\mu_e\sigma)^{-1}$ is the magnetic diffusivity

Now the magnetic field intensity $\vec{H}$ is a solenoidal vector, and in an incompressible fluid the velocity $\vec{u}$ is also a solenoidal vector. When we use this property of $\vec{u}$ and $\vec{H}$ equations (1.13.9) and (1.13.11) can be written in the form [11] as

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i \partial u_k}{\partial x_k} - \frac{\mu_e}{4\pi\rho}\frac{\partial}{\partial x_k}(H_i H_k) = -\frac{1}{\rho}\frac{\partial}{\partial x_k}(P + \mu_e \frac{|\vec{H}|^2}{8\pi}) + \nu\nabla^2 u_i \quad \text{---------- (1.13.12)}$$

and

$$\frac{\partial H_i}{\partial t} + \frac{\partial}{\partial x_k}(H_i u_k - u_i H_k) = \lambda \nabla^2 H_i \quad \text{----------- (1.13.13)}$$

where, here and in the sequel, summation over the repeated indices is to be understood. Equations (1.13.12) and (1.13.13) form the basis of Batchelor's [12] discussion. Chandrasekhar [24] extended the invariant theory of turbulence to the case of magneto-hydrodynamics. He introduced the new variable as

$$\bar{h} = \sqrt{\frac{\mu_e}{4\pi\rho}}.\vec{H} \quad \text{----------- (1.13.14)}$$

for H. It has the dimension of velocity (known as Alfven's velocity) but behaves as vorticity.

In terms of $\bar{h}$ the equations (1.13.12) and (1.13.13) can be expressed as

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_k}(u_i u_k - h_i h_k) = -\frac{\partial P_n}{\partial x_i} + \nu\nabla^2 u_i \quad \text{----------- (1.13.15)}$$

and $\frac{\partial h_i}{\partial t}+\frac{\partial}{\partial x_k}\left(h_i u_k - u_i h_k\right)=\lambda\nabla^2 h_i$ ----------- (1.13.16)

where $P_n=\frac{P}{\rho}+\frac{1}{2}\left|\vec{h}\right|^2$, is the total MHD pressure and $\lambda=\left(4\pi\mu_e\sigma\right)^{-1}$ is the magnetic diffusivity. Chandrasekhar [29, 30] in his theory, considered the correlation's between $\vec{u}$ and $\vec{h}$ at two points p and p′ in the field of isotropic turbulence in the same manner as in the ordinary turbulence. Here, we have the double correlation, $\overline{u_i u_j'}$, $\overline{h_i h_j'}$ and $\overline{u_i h_j'}$, and the triple correlation,

$$\overline{u_i u_j u_k'}\ ,\ \overline{h_i h_j u_k'}\ ,\ \overline{u_i u_j h_k'}\ ,\ \overline{h_i h_j h_k'}\ ,\ \overline{\left(h_i u_j - u_i h_j\right)h_k'}\ \text{, and}\ \overline{\left(h_j' u_k' - h_k' u_j'\right)u_i}\ ,$$

where the subscripts refer to the components of the vectors i,j,k=1,2,3. Each of these double and triple correlations depends on one scalar function in the case of isotropic turbulence because the divergence of both $\vec{u}$ and $\vec{h}$ is zero.

Equations (1.13.15) and (1.13.16) are derived by coupling Maxwell's equations for the electromagnetic field and Navier-Stokes equations for the velocity field. The Maxwell equations are modified to include the induced electric field due to the fluid motion and the Navier-Stokes equations are modified to include the Lorentz force on fluid elements due to the magnetic field. The so-called "Magneto-hydrodynamic approximation" is made, in which displacement currents are neglected in Maxwell's equations. This approximation is well founded provided we are not dealing with very rapid oscillations of the electromagnetic field quantities, as is the case in the propagation of electromagnetic waves. Under this approximation, the energy in the electric field is of the order of $\frac{1}{c^2}$ times the energy in the magnetic field, where c is the speed of light and hence may be neglected. Therefore, we have only to consider the interaction between the velocity field $\vec{u}$ and the magnetic field $\vec{h}$.

1.14. A Brief Description of Past Researches Relevant to this Thesis work:

The system is usually rotating with a constant angular velocity in geophysical flows. Such large-scale flows are generally turbulent. When the motion is referred to axes, which rotate steadily with the bulk of the fluid, the coriolis and centrifugal force must be supposed to act on the fluid. The coriolis force due to rotation plays an important role in a rotating system of turbulent flow, while the centrifugal force with the potential is incorporated into the pressure. Turbulence in the presence of coriolis force is an interesting topic in astrophysics as well as in fluid mechanics.

Ohji [98] considered the effect of coriolis force on turbulent motion in the presence of strong magnetic field with the assumption that Coriolis force term ($-2\Omega\times U$) is balanced by $\nabla\langle x\rangle$ (the geostropic wind approximation) where x represents the generalized pressure. Kishore and Dixit [61], Kishore and Singh [63], Dixit and Upadhyay [39], Kishore and Golsefied [66] discussed the effect of coriolis force on acceleration covariance in ordinary and MHD turbulent flow. Funada, Tuitiya and Ohji [47] considered the effect of coriolis force on turbulent motion in presence of strong magnetic field. Kishore and Sarker [71] studied the rate of change of vorticity covariance in MHD turbulence in a rotating system. Sarkar [121] discussed the vorticity covariance of dusty fluid turbulence in a rotating frame. Shimomura and Yoshizawa [131], Shimomura [132] and [133] also discussed the statistical analysis of turbulent viscosity, turbulent scalar flux and turbulent shear flows respectively in a rotating system by two-scale direct interaction approach. Sarker [123] studied the Thermal decay process of MHD turbulence in a rotating system. Saffman [118] derived an equation that described the motion of a fluid containing small dust particles, which is applicable to laminar flows as well as turbulent flow. Using the Saffman's equation Michael and Miller [92] discussed the motion of dusty gas occupying the semi-infinite space above a rigid plane boundary. Sinha [134], Sarker [122], Sarker and Rahman [124] considered dust particle on their won works.

The essential characteristic of turbulent flows is that turbulent fluctuations are random in nature and therefore, by the application of statistical laws, it has been possible to give the idea of turbulent fluctuations. The turbulent flows, in the absence of external agencies always decay. Millionshtchikov [90], Batchelor and Townsend [10], Proudman and Reid [110], Tatsumi [142], Deissler [36,37], Ghosh [48,49] had given various analytical theories for the decay process of turbulence so far. Further Monin and Yaglom [94] gave the spectral approach for the decay process of turbulence. Although, MHD turbulent fluctuations are random in nature but exhibit the characteristic structure likewise the hydrodynamic turbulence, hence the statistical laws can also be applied in MHD turbulence. Mazumdar [96] derived an early period decay equation for general type of turbulence for an incompressible fluid. Also Sinha [134] discussed the decay process of MHD turbulence and derived an early period decay equation. Sarker and Kishore [120] discussed the decay of MHD turbulence before the final period.

The approach is phenomenological in the sense that they considered the region where the variations of the mean temperature and mean velocity may be neglected because of the transportation of the thermal energy from place to place is very rapid. Deissler [36,37] developed a theory for homogeneous turbulence, which was valid for times before the final period. Using Deissler's theory Loeffler and Deissler [81] studied the temperature fluctuations in homogeneous turbulence before the final period. Using Deissler same theory Kumar and Patel [73] studied the first order reactants in homogeneous turbulence before the final period for the case of multi-point and single time. The problem [73] also extended to the case of multi-point and multi-time concentration correlation in homogeneous turbulence by Kumar and Patel [74]. The numerical result of [74] carried out by Patel [106].

Following Deissler's theory, Sarker and Islam [128] also studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time. Sarker and Rahman [125] studied the decay of temperature fluctuations in MHD turbulence before the final period, Sarker and Islam [129] considered the decay of dusty fluid turbulence before the final in a rotating system. Sarker and Rahman [124] discussed the decay of turbulence before the final period in presence of dust particles. Sarker and Islam [130] studied the effect of very strong magnetic field on acceleration covariance in MHD turbulence of dusty fluid turbulence in a rotating system. Further Rahman and Sarker [115] studied the decay of dusty fluid MHD turbulence before the final period. In their approach they considered the two and three point correlation equations and solved these equations after neglecting the fourth and higher order correlation terms compared to the second and third order correlation terms.

Azad and Sarker [1] studied the Decay of MHD turbulence before the final period for the case of multi-point and multi-time in presence of dust particle. Sarker and Islam [127] studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time. Islam and Sarker [56] discussed the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time. Sarker and Islam [129] also studied the decay of dusty fluid turbulence before the final period in a rotating system. Reddy [116] studied about the flow of dusty viscous liquid through rectangular channel. Hazem Attia [54] studied unsteady flow of a dusty conducting fluid between parallel porous plates with temperature dependent viscosity. Bhargava and Takhar [21] studied the effect of Hall currents on the MHD flow and heat transfer of a second order fluid between two parallel porous plates. Varma and Mathur [150] studied in this field. Sreeharireddy, Nagarajan and Sivaiah [139] also studied on MHD flow of a viscous conducting liquid between two parallel plates.

By analyzing the above all theories, we have studied the **Chapter II, Chapter IV and Chapter V**.

Hopf [53], Kraichnan [78], Edward [41] and Hering [50] have given various analytical theories in the statistical of turbulence. But the dynamical equations describe the time evolution of the finite dimensional probability distribution of turbulent quantities were first derived by Lundgren [83]. Lundgren [83] derived a hierarchy of coupled equations for multi-point turbulence velocity distribution functions, which resemble with BBGKY hierarchy of equations of Ta-Yu-Wu [141] in the kinetic theory of gases. Further Lundgren [84] considered a similar problem for non-homogeneous turbulence. The basic difficulty is that the above theories faced to closure problem. Some general approaches to closure problem for multi dimensional probability density equations those were made by Lyubimov and Ulinch [86 , 87]. Two other closure hypotheses for the probability distribution equation of single time values were investigated by Fox [45], Lundgren [85], Bray and Moss [20] considered the probability density function of a progress variable in a idealized premixed turbulent flow. Bigler [19] gave the hypothesis that in turbulent flow, the thermo-chemical quantities can be related locally a few scalars. Further Pope [107] gave a more suitable model for the probability density functions of scalars in turbulent reacting flows.

Also Kishore [60] studied the distributions functions in the statistical theory of MHD turbulence of an incompressible fluid. Pope [109] derived the transport equation for the joint probability density function of velocity and scalars in turbulent flow. Kishore and Singh [62] derived the transport equation for the bivariate joint distribution function of velocity and temperature in turbulent flow. Kishore and Singh [64] have been derived the transport equation for the joint distribution function of velocity, temperature and concentration in convective turbulent flow. Dixit and Upadhyay [40] considered the distribution functions in the statistical theory of MHD turbulence of an incompressible fluid in the presence of the coriolis force. Kollman and Janicka [75] derived the transport equation for the probability density function of a scalar in turbulent shear flow and considered a closure model based on gradient – flux model.

At this stage, one is met with the difficulty that the N-point distribution function depends upon the N+1-point distribution function and thus result is an unclosed system. This so-called "closer problem" is encountered in turbulence, kinetic theory and other non-linear system. Sarker and Kishore [119] discussed the distribution functions in the statistical theory of convective MHD turbulence of an incompressible fluid. Further Sarker and Kishore [126] discussed the distribution functions in the statistical theory of convective MHD turbulence of mixture of a miscible incompressible fluid.

Following the above theories, in **Chapter III**, an attempt is made to define the statistical theory of distribution function for simultaneous velocity, magnetic, temperature and concentration fields undergoing a first order reaction in MHD turbulence in a rotating system, in presence of dust particle and for the both. Finally, the transport equations for evolution of distribution functions have been derived and various properties of the distribution function have also been discussed. The resulting one-point equation is compared with the first equation of BBGKY hierarchy of equations and the closure difficulty is to be removed in the case of ordinary turbulence.

CHAPTER- II

PART-A

FIRST ORDER REACTANT IN MAGNETO-HYDRODYNAMIC TURBULENCE BEFORE THE FINAL PERIOD OF DECAY IN A ROTATING SYSTEM

2.1. Introduction:

The system is usually rotating with a constant angular velocity in geophysical flows. Such large-scale flows are generally turbulent. When the motion is referred to axes, which rotate steadily with the bulk of the fluid, the coriolis and centrifugal force must be supposed to act on the fluid. The coriolis force due to rotation plays an important role in a rotating system of turbulent flow, while the centrifugal force with the potential is incorporated into the pressure. Kishore and Dixit [61], Kishore and Singh [63], Dixit and Upadhyay [39], Kishore and Golsefied [66] discussed the effect of coriolis force on acceleration covariance in ordinary and MHD turbulent flow. Funada, Tuitiya and Ohji [47] considered the effect of coriolis force on turbulent motion in presence of strong magnetic field. Kishore and Sarker [71] studied the rate of change of vorticity covariance in MHD turbulence in a rotating system. Sarker [123] studied the thermal decay process of MHD turbulence in a rotating system.

Turbulence in the presence of Coriolis force is an interesting topic in astrophysics as well as in fluid mechanics. Ohji [98] considered the effect of coreolis force on turbulent motion in presence of strong magnetic field with the assumption that the Coriolis force term ($-2\Omega\times U$) is balanced by the geostropic wind approximation. Deissler [36, 37] developed a theory "decay of homogeneous turbulence for times before the final period". Using Deissler theory, Kumar and Patel [73] studied the first-order reactant in homogeneous turbulence before the final period of decay for the case of multi-point and single-time correlation. Kumar and Patel [74] extended their problem [73] for the case of multi-point and multi-time concentration correlation. Patel [106] also studied in detail the same problem to carry out the numerical results. Sarker and Kishore [120] studied the decay of MHD turbulence at time before the final period using Chandrasekhar's relation [27]. Sarker and Islam [127] studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time. Sarker and.Azad [137] studied the Decay of MHD turbulence before the final period for the case of multi-point and multi-time in a rotating system. Islam and Sarker [56] studied the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time. Sarker and Islam [128] also studied the first order reactant in MHD turbulence before the final period of decay. In their approach they considered the two and three-point correlation equations and solved these equations after neglecting fourth and higher order correlation terms.

In this chapter following Deissler's theory[36, 37], we have studied the magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction in MHD turbulence before the final period of decay in a rotating system. Here, we have considered the two-point and three-point correlation equations and solved these equations after neglecting the fourth-order correlation terms. Finally we obtained the decay law for magnetic field energy fluctuation of concentration of dilute contaminant undergoing a first order chemical reaction in MHD turbulence in a rotating system comes out to be

$$\langle h^2\rangle = \exp[-R(t-t_0)\left[A(t-t_0)^{-3/2} + \exp[-\{2\in_{mkl}\Omega_m\}]B(t-t_0)^{-5}\right],$$

where $\langle h^2\rangle$ denotes the total energy (mean square of the magnetic field fluctuations of concentration), t is the time and A, B and t_0 are constants.

2.2. Basic Equations:

The equation of motion and the equation of continuity for viscous, incompressible MHD turbulent flow in a rotating system are given by the equation Chandrasekhar [27] as

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_k}\left(u_i u_k - h_i h_k\right) = -\frac{\partial w}{\partial x_i} + \nu\frac{\partial^2 u_i}{\partial x_k \partial x_k} - 2\in_{mki}\Omega_m u_i\ , \quad \text{------- (2.2.1)}$$

$$\frac{\partial h_i}{\partial t} + \frac{\partial}{\partial x_k}(h_i u_k - u_i h_k) = \lambda\frac{\partial^2 h_i}{\partial x_k \partial x_k}, \quad \text{---------(2.2.2)}$$

$$\frac{\partial v_i}{\partial t} + v_k\frac{\partial v_i}{\partial x_k} = -\frac{k}{m_s}(v_i - u_i)\,, \quad \text{--------- (2.2.3)}$$

with $\frac{\partial u_i}{\partial x_i} = \frac{\partial v_i}{\partial x_i} = \frac{\partial h_i}{\partial x_i} = 0$. --------- (2.2.4)

Here, u_i, turbulent velocity component;

h_i, magnetic field fluctuation component.

$w(\hat{x},t) = \frac{p}{\rho} + \frac{1}{2}\langle h^2 \rangle + \frac{1}{2}\left|\hat{\Omega}\times\hat{x}\right|^2$, total MHD pressure inclusive of potential and centrifugal force;

$p(\hat{x},t)$ = hydrodynamic pressure,

ρ = fluid density,

$p_M = \frac{\nu}{\lambda}$, magnetic prandtl number,

$p_r = \frac{\nu}{\gamma}$, prandtl number,

ν = kinematic viscosity,

$\gamma = \frac{K}{\rho c_p}$, thermal diffusivity,

K= Stokes's resistance coefficient which for spherical particle of radius r is 6πμr.

$\lambda = (4\pi\mu\sigma)^{-1}$, magnetic diffusivity,

c_p = heat capacity at constant pressure,

Ω_m = constant angular velocity components,

$\in_{mki}$ = alternating tensor,

$m_s = \frac{4}{3}\pi R_s^{\,3}\rho_s$, mass of single spherical dust particle of radius R_s,

x_k = Space co-ordinate.

The subscripts can take on the values 1, 2 or 3.

2.3. Two-point Correlation and Spectral Equations:

The induction equation of a magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction at the points p and p' separated by the vector $\hat{r}$ could be written as

$$\frac{\partial h_i}{\partial t}+u_k\frac{\partial h_i}{\partial x_k}-h_k\frac{\partial u_i}{\partial x_k}=\lambda\frac{\partial^2 h_i}{\partial x_k\partial x_k}-Rh_i \qquad \text{---------- (2.3.1)}$$

and $$\frac{\partial h_j'}{\partial t}+u_k'\frac{\partial h_j'}{\partial x_k'}-h_k'\frac{\partial u_j'}{\partial x_k'}=\lambda\frac{\partial^2 h_j'}{\partial x_k'\partial x_k'}-Rh_j', \qquad \text{---------- (2.3.2)}$$

where, R is the constant reaction rate.

Multiplying equation (2.3.1) by h_j' and (2.3.2) by h_i, adding and taking ensemble average, we get

$$\frac{\partial\langle h_ih_j'\rangle}{\partial t}+\frac{\partial}{\partial x}[\langle u_kh_ih_j'\rangle-\langle h_ku_ih_j'\rangle]+\frac{\partial}{\partial x_k'}[\langle u_k'h_ih_j'\rangle-\langle h_k'u_j'h_i\rangle].$$

$$=\lambda[\frac{\partial^2\langle h_ih_j'\rangle}{\partial x_k\partial x_k}+\frac{\partial^2\langle h_ih_j'\rangle}{\partial x_k'\partial x_k'}]-2R\langle h_ih_j'\rangle\ . \qquad \text{-------------(2.3.3)}$$

Angular bracket $\langle..........\rangle$ is used to denote an ensemble average.

Using the transformations

$$\frac{\partial}{\partial r_k}=-\frac{\partial}{\partial x_k}=\frac{\partial}{\partial x_k'} \qquad \text{-------------(2.3.4)}$$

and the Chandrasekhar relations [27]

$$\langle u_kh_ih_j'\rangle=-\langle u_k'h_ih_j'\rangle,\langle u_j'h_ih_k'\rangle=-\langle u_ih_kh_j'\rangle, \qquad \text{-------------(2.3.5)}$$

equation (2.3.3) becomes

$$\left[\frac{\partial\langle h_ih_j'\rangle}{\partial t}+2\frac{\partial}{\partial r_k}[\langle u_k'h_ih_j'\rangle-\langle u_ih_kh_j'\rangle]\right].\quad =2\lambda\frac{\partial^2\langle h_ih_j'\rangle}{\partial r_k\partial r_k}-2R\langle h_ih_j'\rangle. \qquad \text{---------(2.3.6)}$$

Now we write equation (2.3.6) in spectral form in order to reduce it to an ordinary differential equation by use of the following three-dimensional Fourier transforms.

$$\langle h_ih_j'(\hat r)\rangle=\int_{-\infty}^{\infty}\langle\psi_i\psi_j'(\hat k)\rangle\exp\left[\hat i(\hat k.\hat r)\right]d\hat k, \qquad \text{---------- (2.3.7)}$$

$$\langle u_ih_kh_j'(\hat r)\rangle=\int_{-\infty}^{\infty}\langle\alpha_i\psi_k\psi_j'(\hat k)\rangle\exp\left[\hat i(\hat k.\hat r)\right]d\hat K. \qquad \text{---------- (2.3.8)}$$

Interchanging i and j, points p and p' then,

$$\langle u_k' h_i h_j'(\hat{r})\rangle = \langle u_k h_i h_j'(-\hat{r})\rangle = \int_{-\infty}^{\infty} \langle \alpha_k \psi_i \psi_j'(-\hat{k})\rangle \exp\left[\hat{i}(\hat{k}.\hat{r})\right] d\hat{k}, \qquad \text{----------- (2.3.9)}$$

where, $\hat{k}$ is known as wave number vector and d$\hat{k}$ = $dk_1dk_2dk_3$.

Substituting of equation (2.3.7) to (2.3.9) in to equation (2.3.6) leads to the Spectral equation

$$\frac{\partial\langle \psi_i \psi_j'(\hat{k})\rangle}{\partial t} + 2\lambda K^2 \langle \psi_i \psi_j'(\hat{k})\rangle + 2R\langle \psi_i \psi_j'(\hat{k})\rangle = 2ik[\ \langle \alpha_i \psi_i \psi_j'(\hat{k})\rangle - \langle \alpha_k \psi_i \psi_j'(-\hat{k})\rangle]$$

--------- (2.3.10)

$$\Rightarrow \frac{\partial\langle \psi_i \psi_j'(\hat{k})\rangle}{\partial t} + 2\lambda[K^2 + \frac{R}{\lambda}]\langle \psi_i \psi_j'(\hat{k})\rangle = 2ik[\ \langle \alpha_i \psi_i \psi_j'(\hat{k})\rangle - \langle \alpha_k \psi_i \psi_j'(-\hat{k})\rangle]\ . \text{-----(2.3.11)}$$

The tensor equation (2.3.11) becomes a scalar equation by contraction of the indices i and j

$$\frac{\partial\langle \psi_i \psi_i'(\hat{k})\rangle}{\partial t} + 2\lambda[K^2 + \frac{R}{\lambda}]\langle \psi_i \psi_i'(\hat{k})\rangle = 2ik[\ \langle \alpha_i \psi_i \psi_i'(\hat{k})\rangle - \langle \alpha_k \psi_i \psi_i'(-\hat{k})\rangle]. \qquad \text{------(2.3.12)}$$

The term on the right hand side of equation (2.3.12) is called energy transfer term while the second term on the left hand side is the dissipation term.

2.4. Three-point Correlation and Spectral Equations:

Similar Procedure can be used to find the three-point correlation equation. For this purpose we take the momentum equation of MHD turbulence in a rotating system at the point P and the induction equations of magnetic field fluctuation, governing the concentration of a dilute contaminant undergoing a first order chemical reaction at P' and P'' separated by the vectors $\hat{r}$ and $\hat{r}'$ as

$$\frac{\partial u_l}{\partial t} + u_k \frac{\partial u_l}{\partial k_k} - h_k \frac{\partial h_l}{\partial x_k} = -\frac{\partial w}{\partial x_l} + \nu \frac{\partial^2 u_l}{\partial x_k \partial x_k} - 2 \in_{mkl} \Omega_m u_l, \qquad \text{----------- (2.4.1)}$$

$$\frac{\partial h_i'}{\partial t} + u_k' \frac{\partial h_i'}{\partial x_k'} - h_k' \frac{\partial u_i'}{\partial x_k'} = \lambda \frac{\partial^2 h_i'}{\partial x_k' \partial x_k'} - Rh_i', \qquad \text{---------- (2.4.2)}$$

$$\frac{\partial h_j''}{\partial t} + u_k'' \frac{\partial h_j''}{\partial x_k''} - h_k'' \frac{\partial u_j''}{\partial x_k''} = \lambda \frac{\partial^2 h_j''}{\partial x_k'' \partial x_k''} - Rh_j'', \qquad \text{-----------(2.4.3)}$$

where, $w(\hat{x},t)=\frac{P}{\rho}+\frac{1}{2}\langle h^2\rangle+\frac{1}{2}\left|\hat{\Omega}\times\hat{x}\right|^2$, total MHD pressure inclusive of potential and centrifugal force $P(\hat{x},t)$, hydrodynamic pressure; Ωm, constant angular velocity components; $\in_{mki}$, alternating tensor.

Multiplying equation (2.4.1) by $h_i'h_j''$ (2.4.2) by $u_l h_j''$ and (2.4.3) by $u_l h_i'$, adding and taking ensemble average, we obtain

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t}+\frac{\partial}{\partial x_k}[\langle u_k u_l h_i' h_j''\rangle-\langle h_k h_l h_i' h_j''\rangle]+\frac{\partial}{\partial x_k'}[\langle u_l u_k' h_i' h_j''\rangle-\langle u_l u_i' h_k' h_j''\rangle]$$

$$+\frac{\partial}{\partial x_k''}[\langle u_l u_k'' h_i' h_j''\rangle-\langle u_l u_j'' h_i' h_j''\rangle]$$

$$=-\frac{\partial\langle w h_i' h_j''\rangle}{\partial x_l}+\nu\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k\partial x_k}+\lambda\left[\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k'\partial x_k'}+\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k''\partial x_k''}\right]$$

$$-2R\langle u_l h_i' h_j''\rangle-2\in_{mkl}\Omega_m\langle u_l h_i' h_j''\rangle. \qquad \text{---------- (2.4.4)}$$

Using the transformations

$$\frac{\partial}{\partial x_k}=-\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'}\right),\ \frac{\partial}{\partial x_k'}=\frac{\partial}{\partial r_k},\ \frac{\partial}{\partial x_k''}=\frac{\partial}{\partial r_k'} \qquad \text{----------(2.4.5)}$$

into equation (2.4.4)

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t}-\lambda\left[(1+P_M)\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k\partial r_k}+(1+P_M)\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k'\partial r_k'}+2P_M\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k\partial r_k}\right]$$

$$=\frac{\partial}{\partial r_k}\langle u_l u_k h_i' h_j''\rangle+\frac{\partial}{\partial r_k'}\langle u_l u_k h_i' h_j''\rangle-\frac{\partial}{\partial r_k'}\langle h_l h_k h_i' h_j''\rangle-\frac{\partial}{\partial r_k'}\langle h_l h_k h_i' h_j''\rangle$$

$$-\frac{\partial}{\partial r_k}\langle u_l u_k' h_i' h_j''\rangle+\frac{\partial}{\partial r_k}\langle u_l u_i' h_k' h_j''\rangle-\frac{\partial}{\partial r_k'}\langle u_l u_k'' h_i' h_j''\rangle+\frac{\partial}{\partial r_k'}\langle u_l u_j'' h_i' h_j''\rangle$$

$$+\frac{\partial}{\partial r_l}\langle w h_i' h_j''\rangle+\frac{\partial}{\partial r_l'}\langle w h_i' h_j''\rangle-2R\langle u h_i' h_j''\rangle-2\in_{mkl}\Omega_m\langle u_l h_i' h_j''\rangle. \qquad \text{---------(2.4.6)}$$

In order to write the equation (2.4.6) to spectral form, we can define the following six dimensional Fourier transforms:

$$\langle u_l h_i'(\hat{r}) h_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_l \beta_i'(\hat{k})\beta_j''(\hat{k})\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'\hat{r}')\right] d\hat{k} d\hat{k}' \qquad \text{---------} (2.4.7)$$

$$\langle u_l u_k' h_i'(\hat{r}) h_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_l \phi_k'(\hat{k})\beta_i'(\hat{k})\beta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')\right] d\hat{k} d\hat{k}' \qquad \text{--------} (2.4.8)$$

$$\langle u_l u_i'(\hat{r}) h_k'(\hat{r}) h_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_l \phi_i'(\hat{k})\beta_k'(\hat{k})\beta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'\hat{r}')\right] d\hat{k} d\hat{k}' \qquad \text{--------} (2.4.9)$$

$$\langle u_l u_k h_i'(\hat{r}) h_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_l \phi_k \beta_i'(\hat{k})\beta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'\hat{r}')\right] d\hat{k} d\hat{k}' \qquad \text{---------} (2.4.10)$$

$$\langle h_l h_k h_i'(\hat{r}) h_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \beta_l \beta_k \beta_i'(\hat{k})\beta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')\right] d\hat{k} d\hat{k}' \qquad \text{--------} (2.4.11)$$

$$\langle w h_i'(\hat{r}) h_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \gamma \beta_i'(\hat{k})\beta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'\hat{r}')\right] d\hat{k} d\hat{k}'. \qquad \text{-------} (2.4.12)$$

Interchanging the points p' and p'' along with the indices i and j, result in the relations

$$\langle u_l u_k'' h_j'' h_i'\rangle = \langle u_l u_k' h_i' h_j''\rangle, \quad \langle u_l u_j'' h_i' h_j''\rangle = \langle u_l u_i' h_k' h_j''\rangle, \quad \langle u_l u_j'' h_i'\rangle = \langle u_l u_i' h_j''\rangle \qquad \text{--------}(2.4.13)$$

By use of this facts(2.4.13) and equations (2.4.7)-(2.4.12), the equation (2.4.6) may be transformed as

$$\frac{\partial\langle \phi_l \beta_i' \beta_j''\rangle}{\partial t} + \lambda\left[(1+P_M)k^2 + (1+P_M)k'^2 + 2P_M k_k k_k' + 2\tfrac{R}{\lambda} + \frac{2\in_{mkl}\Omega_m}{\lambda}\right]\langle \phi_l \beta_i' \beta_j''\rangle$$

$$= i(k_k + k_k')\langle \phi_l \phi_k \beta_i' \beta_j''\rangle - i(k_k + k_k')\langle \beta_l \beta_k \beta_i' \beta_j''\rangle - i(k_k + k_k')\langle \phi_l \phi_k' \beta_i' \beta_j''\rangle + i(k_k + k')\langle \phi_l \phi_i' \beta_i' \beta_j''\rangle$$

$$+ i(k_l + k_l')\langle \gamma \beta_i' \beta_j''\rangle . \qquad \text{------------} (2.4.14)$$

The tensor equation (2.4.14) can be converted to scalar equation by contraction of the indices i and j

$$\frac{\partial\langle\phi_l\beta_i'\beta_i''\rangle}{\partial t}+\lambda\left[(1+P_M)(k^2+k'^2)+2P_Mk_kk_k'+2\frac{R}{\lambda}+\frac{2\in_{mkl}\Omega_m}{\lambda}\right]\langle\phi_l\beta_i'\beta_i''\rangle$$

$$=i(k_k+k_k')\langle\phi_l\phi_k\beta_i'\beta_i''\rangle-i(k_k+k_k')\langle\beta_l\beta_k\beta_i'\beta_i''\rangle-i(k_k+k_k')\langle\phi_l\phi_k'\beta_i'\beta_i''\rangle+i(k_k+k')\langle\phi_l\phi_i'\beta_i'\beta_i''\rangle$$

$$+i(k_l+k_l')\langle\gamma\beta_i'\beta_i''\rangle. \qquad \text{----------- (2.4.15)}$$

To relate the terms on right hand side of equation (2.4.15) derived from the quadruple correlation terms and from the pressure force term in equation (2.4.6), we take the derivative with respect to x_l of the momentum equation (2.4.1) for the point p, and combine with the continuity equation to give

$$-\frac{\partial^2\langle w\rangle}{\partial x_l\partial x_l}=\frac{\partial^2}{\partial x_l\partial x_k}\left(u_lu_k-h_lh_k\right). \qquad \text{---------- (2.4.16)}$$

Multiplying equation (2.4.16) by $h_i'h_j''$ taking time averages and writing this equation in terms of the independent variables $\hat{r}$ and $\hat{r}'$

$$-\left[\frac{\partial^2}{\partial r_l\partial r_l}+2\frac{\partial^2}{\partial r_l\partial r_l'}+\frac{\partial^2}{\partial r_l'\partial r_l'}\right]\langle wh_i'h_j''\rangle=\left[\frac{\partial^2}{\partial r_l\partial r_k}+\frac{\partial^2}{\partial r_l'\partial r_k}+\frac{\partial^2}{\partial r_l\partial r_k'}+\frac{\partial^2}{\partial r_l'\partial r_k'}\right]$$

$$\times\left(\langle u_lu_kh_i'h_j''\rangle-\langle h_lh_kh_i'h_j''\rangle\right). \qquad \text{------------ (2.4.17)}$$

Now taking the Fourier transforms of equation (2.4.17) we get

$$-\langle\gamma\beta_i'\beta_j''\rangle=\frac{\left(k_lk_k+k_l'k_k+k_lk_k'+k_l'k_k'\right)\left(\langle\phi_l\phi_k\beta_i'\beta_j''\rangle-\langle\beta_l\beta_k\beta_i'\theta\beta_j''\rangle\right)}{k^2+2k_lk_l'+k'^2}.$$

$$\text{------------ (2.4.18)}$$

Thus the equations (2.4.17) and (2.4.18) are the spectral equation corresponding to the three-point correlation equations. Equation (2.4.18) can be used to eliminate $\langle\gamma\beta_i'\beta_j''\rangle$ from the equation (2.4.15).

2.5. Solution for Times Before the Final Period:

It is known that equation for final period of decay is obtained by considering the two-point correlations after neglecting the 3rd order correlation terms. To study the decay for times before the final period, the three point correlations are considered and the quadruple correlation terms are neglected because the quadruple correlation terms decays faster than the lower-order correlation terms. But to get a better picture of the MHD homogeneous turbulence decay from its initial period to its final period , three-point correlation equations are to be considered. Here, we neglect the quadruple correlation terms since the decay faster than the lower order correlation terms.

Putting the value of $\langle \gamma\beta_i'\beta_j''\rangle$ from equation (2.4.18) into equation (2.4.15) and neglecting all the quadruple correlation terms, we have

$$\frac{\partial\langle \phi_l\beta_i'\beta_i''\rangle}{\partial t} + \lambda\left[(1+P_M)(k^2+k'^2)+2P_M k_k k_k' + 2\frac{R}{\lambda} + \frac{2\in_{mkl}\Omega_m}{\lambda}\right]\langle \phi_l\beta_i'\beta_i''\rangle = 0$$

----------- (2.5.1)

Taking inner multiplication by k_l, we get

$$\frac{\partial(k_l\langle \phi_l\beta_i'\beta_i''\rangle)}{\partial t} + \lambda\left[(1+P_M)(k^2+k'^2)+2P_M k_k k_k' + \frac{2R}{\lambda} + \frac{2\in_{mkl}\Omega_m}{\lambda}\right](K_l\langle \phi_l\beta_i'\beta_i''\rangle) = 0$$

----------- (2.5.2)

Integrating the equation (2.5.2) between t_0 and t, and gives

$$k_l\langle \phi_l\beta_i'\beta_i''\rangle = k_l[\langle \phi_l\beta_i'\beta_i''\rangle]_0 \exp\{-\lambda[((1+P_M)(k^2+k'^2)+2P_M k_k k'\cos\theta + \frac{2R}{\lambda} + \frac{2\in_{mkl}\Omega_m}{\lambda}](t-t_0)\},$$

----------(2.5.3)

where θ is the angle between $\hat{k}$ and $\hat{k}'$ and $[\langle \phi_l\beta_i'\theta_i''\rangle]_0$ is the value of $\langle \phi_l\beta_i'\theta_i''\rangle$ at $t=t_0$.

Now, by letting $r'=0$ in equation (2.4.7) and comparing with equations (2.3.8) and (2.3.9), we get

$$\langle \alpha_i\psi_k\psi_i'(\hat{k})\rangle = \int_{-\infty}^{\infty}\langle \phi_l\beta_i'(\hat{k})\beta_i''(\hat{k}')\rangle d\hat{k}',$$

-------- (2.5.4)

$$\langle \alpha_k \psi_i \psi_i^{'}(-\hat{k})\rangle = \int_{-\infty}^{\infty} \langle \phi_l \beta_i'(-\hat{k})\beta_i^{''}(-\hat{k}^{'})\rangle d\hat{k}' . \qquad \text{-------- (2.5.5)}$$

Substituting equation (2.5.3) - (2.5.5) in equation (2.3.12), we get

$$\frac{\partial\langle \psi_i \psi_i^{'}(\hat{k})\rangle}{\partial t} + 2\lambda[k^2 + \frac{R}{\lambda}]\langle \psi_i \psi_i^{'}(\hat{k})\rangle = \int_{-\infty}^{\infty} 2ik_l \left[\langle \phi_l \beta_i'(\hat{k})\beta_i^{''}(\hat{k}')\rangle - \langle \phi_l \beta_i'(-\hat{k}')\beta_i^{''}(-\hat{k}')\rangle \right]$$

$$\times \exp\left[-\lambda\{(1+p_M)(k^2+k'^2)+2P_M kk'\cos\theta + \frac{2R}{\lambda} + 2\frac{\in_{mkl}\Omega_m}{\lambda}\}(t-t_0) \right] d\hat{K}' . \qquad \text{-------- (2.5.6)}$$

Now, $d\hat{k}'$ can be expressed in terms of k' and θ that is $d\hat{K}' = -2\pi k'^2 d(\cos\theta)dk'$ (cf Deissler [37]).

With the above relation, equation (2.5.6) to give

$$\frac{\partial\langle \psi_i \psi_i^{'}(\hat{k})\rangle}{\partial t} + 2\lambda[k^2 + \frac{R}{\lambda}]\langle \psi_i \psi_i^{'}(\hat{k})\rangle = 2\int_{-\infty}^{\infty} 2ik_l \left[\langle \phi_l \beta_i'(\hat{k})\beta_i^{''}(\hat{k}')\rangle - \langle \phi_l \beta_i'(-\hat{k}')\beta_i^{''}(-\hat{k}')\rangle \right]_0$$

$$\times k'^2 \left[\int_{-1}^{1} \exp\left\{ -\lambda(t-t_0)[(1+P_M)(k^2+k'^2)+2P_M kk'\cos\theta + \frac{2R}{\lambda} + 2\frac{\in_{mkl}\Omega_m}{\lambda}] \right\} d(\cos\theta) \right] d\hat{K}' .$$

--------- (2.5.7)

In order to find the solution completely and following Loeffler and Deissler [81] we assume that

$$ik_l \left[\langle \phi_l \beta_i'(\hat{K})\beta_i^{''}(\hat{K}')\rangle - \langle \phi_l \beta_i'(-\hat{k})\beta_i^{''}(-\hat{k}')\rangle \right]_0 = -\frac{\xi_0}{(2\pi)^2}\left[k^2 k'^4 - k^4 k'^2 \right], \qquad \text{----------- (2.5.8)}$$

where ξ_0 is a constant depending on the initial conditions. The negative sign is placed in front of ξ_0 in order to make the transfer of energy from small large wave numbers for positive value of ξ_0.

Substituting equation (2.5.8) into equation (2.5.7) and completing the integration with respect to $\cos\theta$, one obtains

$$\frac{\partial(2\pi\langle\psi_i\psi_i^{'}(\hat{K})\rangle}{\partial t}+2\lambda[k^2+\frac{R}{\lambda}](2\pi\langle\psi_i\psi_I^{'}(\hat{K})\rangle)=-\frac{\xi_0}{\nu(t-t_o)}\int_0^\infty\left(k^3k'^5-k^5k'^3\right)$$

$$\times\left[\exp\left\{-\lambda(t-t_o)[(1+P_M)(k^2+k'^2)-2P_Mkk'+\frac{2R}{\lambda}+\frac{2\in_{mkl}\Omega_m}{\lambda}]\right\}\right.$$

$$\left.-\exp\left\{-\lambda(t-t_o)[(1+P_M)(k^2+k'^2)+2P_Mkk^{'}+\frac{2R}{\lambda}+2\frac{\in_{mkl}\Omega_m}{\lambda}]\right\}\right]dK. \qquad \text{--------- (2.5.9)}$$

Multiplying both sides of equation (2.5.9) by k^2, we get

$$\frac{\partial H}{\partial t}+2\lambda\left(k^2+\frac{R}{\lambda}\right)H=G, \qquad \text{-------- (2.5.10)}$$

where, $H=2\pi k^2\langle\psi_i\psi_k^{'}(\hat{K})\rangle$ is the magnetic energy spectrum function and G is the magnetic energy transfer term is given by

$$G=-\frac{\xi_0}{\nu(t-t_0)}\int_0^\infty(k^3k'^5-k^5k'^3)\left[\exp\left\{-\lambda(t-t_0)[(1+P_M)(k^2+k'^2)-2P_Mkk'+\frac{2R}{\lambda}+\frac{2\in_{mkl}\Omega_m}{\lambda}]\right\}\right.$$

$$\left.-\exp\left\{-\lambda(t-t_0)[(1+P_M)(k^2+k'^2)+2P_Mkk'+\frac{2R}{\lambda}+\frac{2\in_{mkl}\Omega_m}{\lambda}]\right\}\right]dK'. \qquad \text{------- (2.5.11)}$$

Integrating equation (2.5.11) with respect to K', we have

$$G=\frac{-\xi_0P_M\sqrt{\pi}}{4\lambda^{3/2}(t-t_0)^{3/2}(1+P_M)^{5/2}}\exp[-\{2R+\frac{2\in_{mkl}\Omega_m}{\lambda}\}(t-t_0)]\exp\left[-\lambda(t-t_0)(\frac{1+2P_M}{1+P_M})k^2\right]$$

$$\times\left[\frac{15P_Mk^4}{4\nu^2(t-t_0)^2(1+P_M)}+\frac{1}{(t-t_0)}\left\{\frac{5P_M^2}{\nu(1+P_M)^2}-\frac{3}{2\nu}\right\}k^6+\frac{P_M}{(1+P_M)}\left\{\frac{P_M^2}{(1+P_M)^2}-1\right\}k^8\right].$$

-------------- (2.5.12)

The series of equation (2.5.7) contains only even powers of k and start with k^4 and the equation represents the transfer function arising owing to consideration of magnetic field at three points at a time.

It is interesting to note that if we integrate equation (2.5.12) over all wave numbers, we find that

$$\int_0^\infty G dk = 0 \qquad \text{-------- (2.5.13)}$$

which indicates that the expression for G satisfies the condition of continuity and homogeneity.

The linear equation (2.5.10) can be solved to give

$$H = \exp\left[-2\lambda(k^2 + \frac{R}{\lambda})(t - t_o)\right]\int G \exp\left[2\lambda(k^2 + \frac{R}{\lambda})(t - t_o)\right]dt$$

$$+ J(k)\exp\left[-2\lambda(k^2 + \frac{R}{\lambda})(t - t_o)\right], \qquad \text{----------- (2.5.14)}$$

where, J(K)= $\frac{N_0 k^2}{\pi}$ is a constant of integration and can be obtained as by Corrsin[32].

Substituting the values of G from equation (2.5.12) in to equation (2.5.14) and integrating with respect to t, we get

$$H = \frac{N_0 k^2}{\pi}\exp\left[-2\lambda(k^2 + \frac{R}{\lambda})(t - t_o)\right] + \frac{\xi_0 \sqrt{\pi}\, p_M}{4\lambda^{3/2}(1 + P_M)^{7/2}}$$

$$\times \exp[-\{2R + 2 \in_{mkl} \Omega_m\}(t - t_o)]\exp\left[-\lambda k^2\left\{\frac{1 + 2p_M}{(1 + p_M)}\right\}(t - t_o)\right]$$

$$\left[\frac{3p_{M'} k^4}{2\lambda^2 P_M (t - t_o)^{5/2}} + \frac{(7p_M - 6)k^6}{3\lambda(1 + p_M)(t - t_o)^{3/2}} - \frac{4(3p_M^2 - 2p_M + 3)k^8}{3(1 + p_M)^2 (t - t_o)^{1/2}}\right.$$

$$\left. + \frac{8\lambda^{1/2}(3p_M^2 - 2p_M + 3)k^9}{3(1 + p_M)^{5/2}} N(\omega)\right], \qquad \text{------ (2.5.15)}$$

where $N(\omega) = e^{-\omega^2}\int_o^\omega e^{x^2} dx, \qquad \omega = k\sqrt{\frac{\lambda(t - t_o)}{(1 + p_M)}}$.

By setting $\hat{r}=0$, $j=i$, $d\hat{K}=-2\pi k^2 d(\cos\theta)dk$ and $H=2\pi k^2\left\langle \psi_i \psi_j^{'}(\hat{K})\right\rangle$ in equation (2.3.7), we get the expression for magnetic energy decay with the fluctuating concentration as

$$\frac{\langle h^2\rangle}{2}=\frac{\langle h_i h_i^{'}\rangle}{2}=\int_0^{\infty} H d\hat{k}. \qquad \text{---------- (2.5.16)}$$

Substituting equation (2.5.15) in to (2.5.16) and after integration with respect to k, we get

$$\frac{\langle h^2\rangle}{2}=\exp[-2R(t-t_0)]\left[\frac{N_o(t-t_o)^{-3/2}}{8\lambda^{3/2}\sqrt{2\pi}}+\exp[-\{2\in_{mkl}\Omega_m\}]\times\frac{\xi_0\pi(t-t_o)^{-5}}{4\lambda^6(1+p_M)(1+2p_M)}\right.$$
$$\left.\times\left\{\frac{9}{16}+\frac{5p_M(7p_M-6)}{16(1+2p_M)}-\frac{35p_M(3p_M^2-2p_M+3)}{8(1+2p_M)^2}+\frac{8p_M(3p_M^2-2p_M+3)}{3.2^6.(1+2p_M)^3}\sum_{n=0}^{\infty}\frac{1.3.5....(2n+9)}{n!(2n+1)2^{2n}(1+p_M)^n}\right\}\right]$$

or $$\frac{\langle h^2\rangle}{2}=\exp[-2R(t-t_0)]\left[\frac{N_o(t-t_o)^{-3/2}}{8\lambda^{3/2}\sqrt{2\pi}}+\exp[-\{2\in_{mkl}\Omega_m\}]\xi_0 Z(t-t_o)^{-5}\right],$$

---------- (2.5.17)

where

$$Z=\frac{\pi}{(1+p_M)(1+2p_M)^{5/2}}\left[\frac{9}{16}+\frac{5p_M(7p_M-6)}{16(1+2p_M)}-\frac{35p_M(3p_M^2-2p_M+3)}{8(1+2p_M)^2}+\frac{8p_M(3p_M^2-2p_M+3)}{3.2^6.(1+2p_M)^3}+----\right]$$

Thus the decay law for magnetic energy fluctuation of MHD turbulence in a rotating system governing the concentration of a dilute contaminant undergoing a first order chemical reaction before the final period may be written as

$$\langle h^2\rangle=\exp[-2R(t-t_0)]\left[A(t-t_0)^{-3/2}+\exp[-\{2\in_{mkl}\Omega_m\}]B(t-t_0)^{-5}\right], \qquad \text{----------(2.5.18)}$$

where, $A=\frac{N_0}{8\lambda^{3/2}\sqrt{2\pi}}$, $B=\xi_0 Z$.

2.6. Results and Discussion:

In equation (2.5.18) we obtained the decay law for magnetic energy fluctuation of MHD turbulence governing the concentration of a dilute contaminant undergoing a first order chemical reaction before the final period in a rotating system considering three-point correlation after neglecting quadruple correlation terms.

If the system is non-rotating, i.e. $\Omega_m = 0$, then the equation (2.5.18) becomes

$$\langle h^2 \rangle = \exp[-2R(t-t_0)]\left[A(t-t_0)^{-3/2} + B(t-t_0)^{-5}\right], \qquad \text{----------} (2.6.1)$$

which was obtained earlier by Sarker and Islam [128]

In absence of chemical reaction, i.e, R=0 then the equation (2.6.1) becomes

$$\langle h^2 \rangle = \left[A(t-t_0)^{-3/2} + B(t-t_0)^{-5}\right], \qquad \text{-----------} (2.6.2)$$

which was obtained earlier by Sarker and Kishor [120].

This study shows that due to the effect of rotation of fluid in MHD turbulence in a rotating system with chemical reaction of the first order in the concentrarion the magnetic field fluctuation i,e.the turbulent energy decays more rapidly than the energy for non-rotating fluid and the faster rate is governed by $\exp[-\{2\in_{mkl}\Omega_m\}]$. Here the chemical reaction ($R\neq0$) in MHD turbulence causes the concentration to decay more they would for non-rotating system and it is governed by $\exp\left[-\{2RT_M + \in_{mkl}\Omega_m\}\right]$.

The first term of right hand side of equation (2.5.18) corresponds to the energy of magnetic field fluctuation of concentration for the two-point correlation and the second term represents magnetic energy for the three-point correlation. In equation (2.5.18), the term associated with the three-point correlation die out faster than the two-point correlation. For large times the last term in the equation (2.5.18) becomes negligible, leaving the -3/2 power decay law for the final period. If higher order correlations are considered in the analysis, it appears that more terms of higher power of time would be added to the equation (2.5.18).

CHAPTER- II

PART-B

FIRST ORDER REACTANT IN MAGNETO-HYDRODYNAMIC TURBULENCE BEFORE THE FINAL PERIOD OF DECAY IN PRESENCE OF DUST PARTICLES

2.7. Introduction:

The influence of dust particles on viscous flows has a great importance in petroleum industry and in the purification of crude oil. Other important applications of dust particles in boundary layer, include soil solvation by natural winds and dust entrainment in a cloud during nuclear explosion. Knowledge of the behaviour of discrete particles in a turbulent flow is of great interest to many branches of technology, particularly if there is a substantial difference between particles and the fluid. A dust particle in air, or in any other gas, has a much larger inertia than the equivalent volume of air and will not therefore participate readily in turbulent fluctuations. The relative motion of dust participate and the air will dissipate energy because of the drag between dust and air, and extract energy from turbulent intensity is reduced than the Reynolds stresses will be decreased and the force required to maintain a given flow rate will likewise be reduced.

Sinha [134] studied the effect of dust particles in addition to the magnetic field fluctuation on the turbulent flow of an incompressible fluid. Saffman [118] derived an equation that described the motion of a fluid containing small dust particles, which is applicable to laminar flows as well as turbulent flow. Sarker [121] discussed the vorticity covariance of dusty fluid turbulence in a rotating frame. Deissler [36,37] developed a theory "decay of homogeneous turbulence for times before the final period". Using Deissler's theory, Loeffler and Deissler [81] studied the decay of temperature fluctuations in homogeneous turbulence before the final period. In their approach they considered the two and three-point correlation equations and solved these equations after neglecting fourth and higher order correlation terms. Using Deissler theory, Kumar and Patel [73] studied the first-order reactant in homogeneous turbulence before the final period of decay for the case of multi-point and single-time correlation. Kumar and Patel [74] extended their problem [73] for the case of multi-point and multi-time concentration correlation. Patel [106] also studied in detail the same problem to carry out the numerical results. Sarker and Kishore [120] studied the decay of MHD turbulence at time before the final period using chandrasekher's relation [27]. Sarker and Islam [127] studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time. Azad and Sarker [1] studied the Decay of MHD turbulence before the final period for the case of multi-point and multi-time in presence of dust particle. Islam and Sarker [56] studied the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time. Sarker and Islam [128] also studied the first order reactant in MHD turbulence before the final period of decay.

In this chapter, we have studied the magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction in dusty fluid MHD turbulence before the final period of decay. Here, we have considered the two-point and three-point correlation equations and solved these equations after neglecting the fourth-order correlation terms. Finally we obtained the decay law for magnetic field energy fluctuation of concentration of dilute contaminant undergoing a first order chemical reaction in dusty fluid MHD turbulence comes out to be

$$\langle h^2\rangle = \exp[-R(t-t_0)\left[A(t-t_0)^{-3/2} + \exp[fs]B(t-t_0)^{-5}\right]$$

where, $\langle h^2\rangle$ denotes the total energy (mean square of the magnetic field fluctuations of concentration), t is the time and A,B and t_0 are constants.

2.8. Basic Equations:

The equation of motion and the equation of continuity for viscous, incompressible MHD dusty fluid turbulent flow are given by the equation Chandrasekhar [27] as

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_k}\left(u_i u_k - h_i h_k\right) = -\frac{\partial w}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_k \partial x_k} + f(u_i - v_i), \qquad \text{---------- (2.8.1)}$$

$$\frac{\partial h_i}{\partial t} + \frac{\partial}{\partial x_k}(h_i u_k - u_i h_k) = \lambda \frac{\partial^2 h_i}{\partial x_k \partial x_k}, \qquad \text{----------(2.8.2)}$$

$$\frac{\partial v_i}{\partial t} + v_k \frac{\partial v_i}{\partial x_k} = -\frac{k}{m_s}(v_i - u_i) \qquad \text{---------- (2.8.3)}$$

with

$$\frac{\partial u_i}{\partial x_i} = \frac{\partial v_i}{\partial x_i} = \frac{\partial h_i}{\partial x_i} = 0. \qquad \text{---------- (2.8.4)}$$

where, ρ_s is the constant density of the material in dust particle, N is the constant number of density of dust particle

The subscripts can take on the values 1, 2 or 3.

2.9. Three-point Correlation and Spectral Equations:

Similar Procedure can be used to find the three points correlation equation. For this purpose we take the momentum equation of dusty fluid MHD turbulence at the point P and the induction equations of magnetic field fluctuation, governing the concentration of a dilute contaminant undergoing a first order chemical reaction at P' and P'' separated by the vectors $\hat{r}$ and $\hat{r}'$ as

$$\frac{\partial u_l}{\partial t} + u_k \frac{\partial u_l}{\partial k_k} - h_k \frac{\partial h_l}{\partial x_k} = -\frac{\partial w}{\partial x_l} + \nu \frac{\partial^2 u_l}{\partial x_k \partial x_k} + f(u_l - v_l), \qquad \text{---------- (2.9.1)}$$

$$\frac{\partial h_i^{'}}{\partial t} + u_k^{'} \frac{\partial h_i^{'}}{\partial x_k^{'}} - h_k^{'} \frac{\partial u_i^{'}}{\partial x_k^{'}} = \lambda \frac{\partial^2 h_i^{'}}{\partial x_k^{'} \partial x_k^{'}} - R h_i^{'}, \qquad \text{---------- (2.9.2)}$$

$$\frac{\partial h_j^{''}}{\partial t} + u_k^{''} \frac{\partial h_j^{''}}{\partial x_k^{''}} - h_k^{''} \frac{\partial u_j^{''}}{\partial x_k^{''}} = \lambda \frac{\partial^2 h_j^{''}}{\partial x_k^{''} \partial x_k^{''}} - R h_j^{''}. \qquad \text{-----------(2.9.3)}$$

Multiplying equation (2.9.1) by $h_i' h_j^{''}$ (2.9.2) by $u_l h_j^{''}$ and (2.9.3) by $u_l h_i'$, adding and taking ensemble average, we obtain

$$\frac{\partial \langle u_l h_i^{'} h_j^{''} \rangle}{\partial t} + \frac{\partial}{\partial x_k}[\langle u_k u_l h_i^{'} h_j^{''} \rangle - \langle h_k h_l h_i^{'} h_j^{''} \rangle] + \frac{\partial}{\partial x_k^{'}}[\langle u_l u_k^{'} h_i^{'} h_j^{''} \rangle - \langle u_l u_i^{'} h_k^{'} h_j^{''} \rangle].$$

$$+ \frac{\partial}{\partial x_k^{''}}[\langle u_l u_k^{''} h_i^{'} h_j^{''} \rangle - \langle u_l u_j^{''} h_i^{'} h_j^{''} \rangle]$$

$$= -\frac{\partial\langle wh_i'h_j''\rangle}{\partial x_l} + \nu\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k \partial x_k} + \lambda\left[\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k' \partial x_k'} + \frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k'' \partial x_k''}\right]$$

$$-2R\langle u_l h_i' h_j''\rangle + f(\langle u_l h_i' h_j''\rangle - \langle v_l h_i' h_j''\rangle). \quad \text{--------- (2.9.4)}$$

Using the transformations

$$\frac{\partial}{\partial x_k} = -\left(\frac{\partial}{\partial r_k} + \frac{\partial}{\partial r_k'}\right), \quad \frac{\partial}{\partial x_k'} = \frac{\partial}{\partial r_k}, \quad \frac{\partial}{\partial x_k''} = \frac{\partial}{\partial r_k'} \quad \text{----------(2.9.5)}$$

into equation (2.9.4)

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t} - \lambda\left[(1+P_M)\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k \partial r_k} + (1+P_M)\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k' \partial r_k'} + 2P_M\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k \partial r_k}\right]$$

$$= \frac{\partial}{\partial r_k}\langle u_l u_k h_i' h_j''\rangle + \frac{\partial}{\partial r_k'}\langle u_l u_k h_i' h_j''\rangle - \frac{\partial}{\partial r_k'}\langle h_l h_k h_i' h_j''\rangle - \frac{\partial}{\partial r_k'}\langle h_l h_k h_i' h_j''\rangle$$

$$-\frac{\partial}{\partial r_k}\langle u_l u_k' h_i' h_j''\rangle + \frac{\partial}{\partial r_k}\langle u_l u_i' h_k' h_j''\rangle - \frac{\partial}{\partial r_k'}\langle u_l u_k'' h_i' h_j''\rangle + \frac{\partial}{\partial r_k'}\langle u_l u_j'' h_i' h_j''\rangle$$

$$+\frac{\partial}{\partial r_l}\langle wh_i' h_j''\rangle + \frac{\partial}{\partial r_l'}\langle wh_i' h_j''\rangle - 2R\langle uh_i' h_j''\rangle + f[\langle u_l h_i' h_j''\rangle - \langle v_l h_i' h_j''\rangle] \quad \text{-----------(2.9.6)}$$

In order to write the equation (2.9.6) to spectral form, we can use the six dimensional Fourier transforms (2.4.7) - (2.4.12) and the equation

$$\langle v_l h_i'(\hat{r}) h_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \delta_l \beta_i'(\hat{k})\beta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r} + \hat{k}'.\hat{r}')\right] d\hat{k}\, d\hat{k}'. \quad \text{------- (2.9.7)}$$

Interchanging the points p' and p'' along with the indices i and j, result in the relations

$$\langle u_l u_k'' h_j'' h_i'\rangle = \langle u_l u_k' h_i' h_j''\rangle, \quad \langle u_l u_j'' h_i' h_j''\rangle = \langle u_l u_i' h_k' h_j''\rangle, \quad \langle u_l u_j'' h_i'\rangle = \langle u_l u_i' h_j''\rangle \quad \text{--------(2.9.8)}$$

By use of the relation (2.9.8) and the equations (2.4.7)-(2.4.12) and (2.9.7) the equation (2.9.6) may be transformed as

$$\frac{\partial\langle\phi_l\beta_i'\beta_j''\rangle}{\partial t}+\lambda\left[(1+P_M)k^2+(1+P_M)k'^2+2P_Mk_kk_k'+2\frac{R}{\lambda}-\frac{f}{\lambda}\right]\langle\phi_l\beta_i'\beta_j''\rangle$$

$$=i(k_k+k_k')\langle\phi_l\phi_k\beta_i'\beta_j''\rangle-i(k_k+k_k')\langle\beta_l\beta_k\beta_i'\beta_j''\rangle-i(k_k+k_k')\langle\phi_l\phi_k'\beta_i'\beta_j''\rangle+i(k_k+k')\langle\phi_l\phi_i'\beta_i'\beta_j''\rangle$$

$$+i(k_l+k_l')\langle\gamma\beta_i'\beta_j''\rangle-f\langle\delta_l\beta_i'\beta_j''\rangle. \qquad \text{---------- (2.9.9)}$$

The tensor equation (2.9.9) can be converted to scalar equation by contraction of the indices i and j

$$\frac{\partial\langle\phi_l\beta_i'\beta_i''\rangle}{\partial t}+\lambda\left[(1+P_M)(k^2+k'^2)+2P_Mk_kk_k'+2\frac{R}{\lambda}-\frac{f}{\lambda}\right]\langle\phi_l\beta_i'\beta_i''\rangle$$

$$=i(k_k+k_k')\langle\phi_l\phi_k\beta_i'\beta_i''\rangle-i(k_k+k_k')\langle\beta_l\beta_k\beta_i'\beta_i''\rangle-i(k_k+k_k')\langle\phi_l\phi_k'\beta_i'\beta_i''\rangle+i(k_k+k')\langle\phi_l\phi_i'\beta_i'\beta_i''\rangle$$

$$+i(k_l+k_l')\langle\gamma\beta_i'\beta_i''\rangle-f\langle\delta_l\beta_i'\beta_i''\rangle. \qquad \text{------------(2.9.10)}$$

To relate the terms on right hand side of equation (2.9.10) derived from the quadruple correlation terms and from the pressure force term in equation (2.9.10), we take the derivative with respect to x_l of the momentum equation (2.9.1) for the point p, and combine with the continuity equation to give

$$-\frac{\partial^2\langle w\rangle}{\partial x_l\partial x_l}=\frac{\partial^2}{\partial x_l\partial x_k}\left(u_lu_k-h_lh_k\right). \qquad \text{------------- (2.9.11)}$$

Multiplying equation (2.9.11) by $h_i'h_j''$ taking time averages and writing this equation in terms of the independent variables $\hat{r}$ and $\hat{r}'$

$$-\left[\frac{\partial^2}{\partial r_l\partial r_l}+2\frac{\partial^2}{\partial r_l\partial r_l'}+\frac{\partial^2}{\partial r_l'\partial r_l'}\right]\langle wh_i'h_j''\rangle=\left[\frac{\partial^2}{\partial r_l\partial r_k}+\frac{\partial^2}{\partial r_l'\partial r_k}+\frac{\partial^2}{\partial r_l\partial r_k'}+\frac{\partial^2}{\partial r_l'\partial r_k'}\right]$$

$$\times\left(\langle u_lu_kh_i'h_j''\rangle-\langle h_lh_kh_i'h_j''\rangle\right). \qquad \text{------------ (2.9.12)}$$

Now taking the Fourier transforms of equation (2.9.12) we get

$$-\langle \gamma\beta_i'\beta_j''\rangle = \frac{(k_l k_k + k_l' k_k + k_l k_k' + k_l' k_k')(\langle \phi_l \phi_k \beta_i' \beta_j''\rangle - \langle \beta_l \beta_k \beta_i' \theta \beta_j''\rangle)}{k^2 + 2k_l k_l' + k'^2}$$

-------------- (2.9.13)

Thus the equations (2.9.12) and (2.9.13) are the spectral equation corresponding to the three-point correlation equations. Equation (2.9.13) can be used to eliminate $\langle \gamma\beta_i'\beta_j''\rangle$ from the equation (2.9.10).

2.10. Solution for Times Before the Final Period:

By using the similar procedure of the Art.(2.5) we can neglect the quadruple correlation terms because the decay faster than the lower order correlation terms.

Substituting the value of $\langle \gamma\beta_i'\beta_j''\rangle$ from equation (2.9.13) into equation (2.9.10) and neglecting all the quadruple correlation terms, we have

$$\frac{\partial\langle \phi_l\beta_i'\beta_i''\rangle}{\partial t} + \lambda\left[(1+P_M)(k^2+k'^2) + 2P_M k_k k_k' + 2\frac{R}{\lambda} - \frac{f}{\lambda}\right]\langle \phi_l\beta_i'\beta_i''\rangle + f\langle \delta_l\beta_i'\beta_i''\rangle = 0$$

$$\Rightarrow \frac{\partial\langle \phi_l\beta_i'\beta_i''\rangle}{\partial t} + \lambda\left[(1+P_M)(k^2+k'^2) + 2P_M k_k k_k' + 2\frac{R}{\lambda} - \frac{fs}{\lambda}\right]\langle \phi_l\beta_i'\beta_i''\rangle = 0 \quad \text{-------(2.10.1)}$$

where $\langle \delta_l\beta_i'\beta_i''\rangle = C\langle \phi_l\beta_i'\beta_i''\rangle$ and 1-C=S, C and S are arbitrary constants.

Taking inner multiplication by k_l, we get

$$\frac{\partial(k_l\langle \phi_l\beta_i'\beta_i''\rangle)}{\partial t} + \lambda\left[(1+P_M)(k^2+k'^2) + 2P_M k_k k_k' + \frac{2R}{\lambda} - \frac{fs}{\lambda}\right](K_l\langle \phi_l\beta_i'\beta_i''\rangle) = 0 \quad \text{-----(2.10.2)}$$

Integrating the equation (2.10.2) between t_0 and t, and gives

$$k_l\langle \phi_l\beta_i'\beta_i''\rangle = k_l[\langle \phi_l\beta_i'\beta_i''\rangle_0]\exp\{-\lambda[((1+P_M)(k^2+k'^2) + 2P_M k_k k'\cos\theta$$
$$+\frac{2R}{\lambda} - \frac{fs}{\lambda}](t-t0)\}, \quad \text{--------(2.10.3)}$$

where θ is the angle between $\hat{k}$ and $\hat{k}'$ and $\langle \phi_l\beta_i'\theta_i''\rangle_0$ is the value of $\langle \phi_l\beta_i'\theta_i''\rangle$ at t= t_0.

Now, by letting $r' = 0$ in equation (2.4.7) and comparing with equations (2.3.8) and (2.3.9), we get

$$\langle \alpha_i \psi_k \psi_i^{'}(\hat{k})\rangle = \int_{-\infty}^{\infty} \langle \phi_l \beta_i'(\hat{k})\beta_i^{''}(\hat{k}')\rangle d\hat{k}', \quad \text{-------- (2.10.4)}$$

$$\langle \alpha_k \psi_i \psi_i^{'}(-\hat{k})\rangle = \int_{-\infty}^{\infty} \langle \phi_l \beta_i'(-\hat{k})\beta_i^{''}(-\hat{k}')\rangle d\hat{k}' \quad \text{-------- (2.10.5)}$$

Substituting equation (2.10.3) - (2.10.5) in equation (2.3.12), we get

$$\frac{\partial \langle \psi_i \psi_i^{'}(\hat{k})\rangle}{\partial t} + 2\lambda[k^2 + \frac{R}{\lambda}]\langle \psi_i \psi_i^{'}(\hat{k})\rangle = \int_{-\infty}^{\infty} 2ik_l \left[\langle \phi_l \beta'_i(\hat{k})\beta_i^{''}(\hat{k}')\rangle - \langle \phi_l \beta'_i(-\hat{k}')\beta_i^{''}(-\hat{k}')\rangle \right]$$

$$\times \exp\left[-\lambda\{(1+p_M)(k^2 + k'^2) + 2P_M kk' \cos\theta + \frac{2R}{\lambda} - \frac{fs}{\lambda}\}(t - t_0) \right] d\hat{K}' \quad \text{------ (2.10.6)}$$

Now, $d\hat{k}'$ can be expressed in terms of k' and θ that is $d\hat{K}' = -2\pi k'^2 d(\cos\theta) dk'$ (cf. Deissler [37]).

With the above relation, equation (2.10.6) to give

$$\frac{\partial \langle \psi_i \psi_i^{'}(\hat{k})\rangle}{\partial t} + 2\lambda[k^2 + \frac{R}{\lambda}]\langle \psi_i \psi_i^{'}(\hat{k})\rangle = 2\int_{-\infty}^{\infty} 2ik_l \left[\langle \phi_l \beta'_i(\hat{k})\beta_i^{''}(\hat{k}')\rangle - \langle \phi_l \beta'_i(-\hat{k}')\beta_i^{''}(-\hat{k}')\rangle \right]_0$$

$$\times k'^2 \left[\int_{-1}^{1} \exp\left\{ -\lambda(t - t_0)[(1+P_M)(k^2 + k'^2) + 2P_M kk' \cos\theta + \frac{2R}{\lambda} - \frac{fS}{\lambda}] \right\} d(\cos\theta) \right] d\hat{K}'$$

$$\text{---------(2.10.7)}$$

Following Loeffler and Deissler [81] in order to find the solution completely, we assume that

$$ik_l \left[\langle \phi_l \beta_i'(\hat{K})\beta_i^{''}(\hat{K}')\rangle - \langle \phi_l \beta_i'(-\hat{k})\beta_i^{''}(-\hat{k}')\rangle \right]_0 = -\frac{\xi_0}{(2\pi)^2}\left[k^2 k'^4 - k^4 k'^2\right], \quad \text{----------- (2.10.8)}$$

where, ξ_0 is a constant depending on the initial conditions. The negative sign is placed in front of ξ_0 in order to make the transfer of energy from small large wave numbers for positive value of ξ_0.

Substituting equation (2.10.8) into equation (2.10.7) and completing the integration with respect to $\cos\theta$, one obtains

$$\frac{\partial(2\pi\langle\psi_i\psi_i^{'}(\hat{K})\rangle}{\partial t}+2\lambda[k^2+\frac{R}{\lambda}](2\pi\langle\psi_i\psi_I^{'}(\hat{K})\rangle)=-\frac{\xi_0}{\nu(t-t_o)}\int_0^{\infty}\left(k^3k'^5-k^5k'^3\right)$$

$$\times\left[\exp\left\{-\lambda(t-t_o)[(1+P_M)(k^2+k'^2)-2P_M kk'+\frac{2R}{\lambda}-\frac{fS}{\lambda}]\right\}\right.$$

$$\left.-\exp\left\{-\lambda(t-t_o)[(1+P_M)(k^2+k'^2)+2P_M kk'+\frac{2R}{\lambda}-\frac{fS}{\lambda}]\right\}\right]dK'. \qquad \text{----------} (2.10.9)$$

Multiplying both sides of equation (2.10.9) by k^2, we get

$$\frac{\partial H}{\partial t}+2\lambda\left(k^2+\frac{R}{\lambda}\right)H=G, \qquad \text{--------} (2.10.10)$$

where, $H=2\pi k^2\langle\psi_i\psi_k^{'}(\hat{K})\rangle$ is the magnetic energy spectrum function and G is the magnetic energy transfer term is given by

$$G=-\frac{\xi_0}{\nu(t-t_0)}\int_0^{\infty}(k^3k'^5-k^5k'^3)\left[\exp\left\{-\lambda(t-t_0)[(1+P_M)(k^2+k'^2)-2P_M kk'+\frac{2R}{\lambda}-\frac{fS}{\lambda}]\right\}\right.$$

$$\left.-\exp\left\{-\lambda(t-t_0)[(1+P_M)(k^2+k'^2)+2P_M kk'+\frac{2R}{\lambda}-\frac{fS}{\lambda}]\right\}\right]dK' \qquad \text{---------}(2.10.11)$$

Integrating equation (2.10.11) with respect to K', we have

$$G=\frac{-\xi_0 P_M\sqrt{\pi}}{4\lambda^{3/2}(t-t_0)^{3/2}(1+P_M)^{5/2}}\exp[-\{2R-\frac{fS}{\lambda}\}(t-t_0)]\exp\left[-\lambda(t-t_0)(\frac{1+2P_M}{1+P_M})k^2\right]$$

$$\times\left[\frac{15P_M k^4}{4\nu^2(t-t_0)^2(1+P_M)}+\frac{1}{(t-t_0)}\left\{\frac{5P_M^2}{\nu(1+P_M)^2}-\frac{3}{2\nu}\right\}k^6+\frac{P_M}{(1+P_M)}\left\{\frac{P_M^2}{(1+P_M)^2}-1\right\}k^8\right]$$

$$\text{-------------}(2.10.12)$$

The series of equation (2.10.7) contains only even powers of k and start with k^4 and the equation represents the transfer function arising owing to consideration of magnetic field at three points at a time.

By integrating the equation (2.10.12) over all wave numbers, we find that

$$\int_0^\infty G dk = 0 \qquad \text{-------- (2.10.13)}$$

which indicates that the expression for G satisfies the condition of continuity and homogeneity.

The linear equation (2.10.10) can be solved to give

$$H = \exp\left[-2\lambda(k^2 + \frac{R}{\lambda})(t-t_o)\right]\int G \exp\left[2\lambda(k^2 + \frac{R}{\lambda})(t-t_o)\right]dt$$

$$+ J(k)\exp\left[-2\lambda(k^2 + \frac{R}{\lambda})(t-t_o)\right] \qquad \text{---------- (2.10.14)}$$

where, J(K)= $\frac{N_0 k^2}{\pi}$ is a constant of integration.

Substituting the values of G from equation (2.10.12) in to equation (2.10.14) and integrating with respect to t, we get

$$H = \frac{N_0 k^2}{\pi}\exp\left[-2\lambda(k^2 + \frac{R}{\lambda})(t-t_o)\right] + \frac{\xi_0\sqrt{\pi}\,p_M}{4\lambda^{3/2}(1+P_M)^{7/2}}$$

$$\times \exp[-\{2R - fS\}(t-t_o)]\exp\left[-\lambda k^2\left\{\frac{1+2p_M}{(1+p_M)}\right\}(t-t_o)\right]\times$$

$$\left[\frac{3p_{M'}k^4}{2\lambda^2 P_M(t-t_o)^{5/2}} + \frac{(7p_M-6)k^6}{3\lambda(1+p_M)(t-t_o)^{3/2}} - \frac{4(3p_M^2-2p_M+3)k^8}{3(1+p_M)^2(t-t_o)^{1/2}}\right.$$

$$\left.+\frac{8\lambda^{1/2}(3p_M^2-2p_M+3)k^9}{3(1+p_M)^{5/2}}N(\omega)\right] \qquad \text{--------- (2.10.15)}$$

where $N(\omega) = e^{-\omega^2}\int_o^\omega e^{x^2}dx, \qquad \omega = k\sqrt{\frac{\lambda(t-t_o)}{(1+p_M)}}.$

By setting $\hat{r}=0,\ j=i,\ d\hat{K} = -2\pi k^2 d(\cos\theta)dk$ and $H = 2\pi k^2\left\langle \psi_i \psi_j^{'}(\hat{K})\right\rangle$ in equation (2.3.7), we get the expression for magnetic energy decay with the fluctuating concentration as

$$\frac{\langle h^2\rangle}{2} = \frac{\langle h_i h_i^{'}\rangle}{2} = \int_0^\infty H d\hat{k}. \qquad \text{--------- (2.10.16)}$$

Substituting equation (2.10.15) in to (2.10.16) and after integration with respect to k, we get

$$\frac{\langle h^2\rangle}{2}=\exp[-2R(t-t_0)]\left[\frac{N_o(t-t_o)^{-3/2}}{8\lambda^{3/2}\sqrt{2\pi}}+\exp[fS\}]\times\frac{\xi_0\pi(t-t_o)^{-5}}{4\lambda^6(1+p_M)(1+2p_M)}\right.$$

$$\left.\times\left\{\frac{9}{16}+\frac{5p_M(7p_M-6)}{16(1+2p_M)}-\frac{35p_M(3p_M^2-2p_M+3)}{8(1+2p_M)^2}+\frac{8p_M(3p_M^2-2p_M+3)}{3.2^6.(1+2p_M)^3}\sum_{n=0}^{\infty}\frac{1.3.5....(2n+9)}{n!(2n+1)2^{2n}(1+p_M)^n}\right\}\right]$$

or $$\frac{\langle h^2\rangle}{2}=\exp[-2R(t-t_0)]\left[\frac{N_o(t-t_o)^{-3/2}}{8\lambda^{3/2}\sqrt{2\pi}}+\exp[fS\}]\xi_0 Z(t-t_o)^{-5}\right] \qquad \text{----------(2.10.17)}$$

where

$$Z=\frac{\pi}{(1+p_M)(1+2p_M)^{5/2}}\left[\frac{9}{16}+\frac{5p_M(7p_M-6)}{16(1+2p_M)}-\frac{35p_M(3p_M^2-2p_M+3)}{8(1+2p_M)^2}+\frac{8p_M(3p_M^2-2p_M+3)}{3.2^6.(1+2p_M)^3}+----\right]$$

Thus the decay law for magnetic energy fluctuation of dusty fluid MHD turbulence governing the concentration of a dilute contaminant undergoing a first order chemical reaction before the final period may be written as

$$\langle h^2\rangle=\exp[-2R(t-t_0)]\left[A(t-t_0)^{-3/2}+\exp[fs\}]B(t-t_0)^{-5}\right] \qquad \text{--------- (2.10.18)}$$

where, $A=\frac{N_0}{8\lambda^{3/2}\sqrt{2\pi}}$, $B=\xi_0 Z$

2.11. Results and Discussion:

In equation (2.10.18) we obtained the decay law for magnetic energy fluctuation of dusty MHD turbulence governing the concentration of a dilute contaminant undergoing a first order chemical reaction before the final period considering three-point correlation after neglecting quadruple correlation terms.

If the fluid is clean, i.e. f=0 then the equation (2.10.18) becomes

$$\langle h^2 \rangle = \exp[-2R(t-t_0)]\left[A(t-t_0)^{-3/2} + B(t-t_0)^{-5}\right] \qquad \text{----------(2.11.1)}$$

which was obtained earlier by Sarker and Islam[128].

In absence of chemical reaction, i.e, R=0 then the equation (2.11.1) becomes

$$\langle h^2 \rangle = \left[A(t-t_0)^{-3/2} + B(t-t_0)^{-5}\right] \qquad \text{-----------(2.11.2)}$$

which was obtained earlier by Sarker and Kishor [120].

This study shows that due to the effect of dust particles in the magnetic field with chemical reaction of the first order in the concentrarion the magnetic field fluctuation i,e.the turbulent energy decays more rapidly than the energy for clean fluid and the faster rate is governed by $\exp[fs]$. Here the chemical reaction ($R \neq 0$) in dusty fluid MHD turbulence causes the concentration to decay more they would for clean fluid and it is governed by $\exp[-\{2RT_M - fs\}]$

The first term of right hand side of equation (2.10.18) corresponds to the energy of magnetic field fluctuation of concentration for the two-point correlation and the second term represents magnetic energy for the three-point correlation. In equation (2.10.18), the term associated with the three-point correlation die out faster than the two-point correlation. For large times the last term in the equation (2.10.18) becomes negligible, leaving the -3/2 power decay law for the final period.

CHAPTER- II

PART-C

FIRST ORDER REACTANT IN MAGNETO-HYDRODYNAMIC TURBULENCE BEFORE THE FINAL PERIOD OF DECAY IN PRESENCE OF DUST PARTICLE IN A ROTATING SYSTEM

2.12. Introduction:

Deissler [36, 37] developed a theory "decay of homogeneous turbulence for times before the final period". Using Deissler's theory, Loeffler and Deissler [81] studied the decay of temperature fluctuations in homogeneous turbulence before the final period. In their approach they considered the two and three-point correlation equations and solved these equations after neglecting fourth and higher order correlation terms. Using Deissler theory, Kumar and Patel [73] studied the first-order reactant in homogeneous turbulence before the final period of decay for the case of multi-point and single-time correlation. Kumar and Patel [74] extended their problem [73] for the case of multi-point and multi-time concentration correlation. Patel [106] also studied in detail the same problem to carry out the numerical results. Sarker and Kishore [120] studied the decay of MHD turbulence at time before the final period using chandrasekher's relation [27]. Sarker and Islam [127] studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time. Azad and Sarker[2] studied the Decay of dusty fluid MHD turbulence before the final period in a rotating system for the case of multi-point and multi-time. Sarker and Islam[129] studied the Decay of dusty fluid turbulence before the final period in a rotating system. Islam and Sarker [56] studied the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time. Sarker and Islam [128] also studied the first order reactant in MHD turbulence before the final period of decay.

Here, we have studied the magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction in dusty fluid MHD turbulence before the final period of decay in a rotating system. Here, we have considered the two-point and three-point correlation equations and solved these equations after neglecting the fourth-order correlation terms. Finally we obtained the decay law for magnetic field energy fluctuation of concentration of dilute contaminant undergoing a first order chemical reaction in dusty fluid MHD turbulence in a rotating system comes out to be

$$\langle h^2 \rangle = \exp[-R(t-t_0)]\left[A(t-t_0)^{-3/2} + \exp[-\{2 \in_{mkl} \Omega_m - fs\}]B(t-t_0)^{-5}\right]$$

where $\langle h^2 \rangle$ denotes the total energy (mean square of the magnetic field fluctuations of concentration), t is the time and A,B and t_0 are constants.

2.13. Basic Equations:

The equation of motion and the equation of continuity for viscous, incompressible MHD dusty fluid turbulent flow in a rotating system are given by Chandrasekhar [27] as

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_k}(u_i u_k - h_i h_k) = -\frac{\partial w}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_k \partial x_k} - 2 \in_{mki} \Omega_m u_i + f(u_i - v_i), \quad \text{----------- (2.13.1)}$$

$$\frac{\partial h_i}{\partial t} + \frac{\partial}{\partial x_k}(h_i u_k - u_i h_k) = \lambda \frac{\partial^2 h_i}{\partial x_k \partial x_k}, \quad \text{--------- (2.13.2)}$$

$$\frac{\partial v_i}{\partial t} + v_k \frac{\partial v_i}{\partial x_k} = -\frac{k}{m_s}(v_i - u_i), \quad \text{--------- (2.13.3)}$$

with

$$\frac{\partial u_i}{\partial x_i} = \frac{\partial v_i}{\partial x_i} = \frac{\partial h_i}{\partial x_i} = 0 \quad \text{---------- (2.13.4)}$$

where, Ω_m is the constant angular velocity components.

The subscripts can take on the values 1, 2 or 3.

2.14. Two-point Correlation and Spectral Equations:

Under the condition that (i) the turbulence and the concentration magnetic field are homogeneous (ii) the chemical reaction has no effect on the velocity field and (iii) the reaction rate and the magnetic diffusivity are constant, the induction equation of a magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction at the points p and p' separated by the vector $\hat{r}$ could be written as

$$\frac{\partial h_i}{\partial t}+u_k\frac{\partial h_i}{\partial x_k}-h_k\frac{\partial u_i}{\partial x_k}=\lambda\frac{\partial^2 h_i}{\partial x_k\partial x_k}-Rh_i \qquad \text{---------- (2.14.1)}$$

and $$\frac{\partial h_j^{'}}{\partial t}+u_k^{'}\frac{\partial h_j^{'}}{\partial x_k^{'}}-h_k^{'}\frac{\partial u_j^{'}}{\partial x_k^{'}}=\lambda\frac{\partial^2 h_j^{'}}{\partial x_k^{'}\partial x_k^{'}}-Rh_j^{'}\,, \qquad \text{------- (2.14.2)}$$

where, R is the constant reaction rate.

Multiplying equation (2.14.1) by $h_j^{'}$ and (2.14.2) by h_i, adding and taking ensemble average, we get

$$\frac{\partial\langle h_ih_j^{'}\rangle}{\partial t}+\frac{\partial}{\partial x}[\langle u_kh_ih_j^{'}\rangle-\langle h_ku_ih_j^{'}\rangle]+\frac{\partial}{\partial x_k^{'}}[\langle u_k^{'}h_ih_j^{'}\rangle-\langle h_k^{'}u_j^{'}h_i\rangle].$$

$$=\lambda[\frac{\partial^2\langle h_ih_j^{'}\rangle}{\partial x_k\partial x_k}+\frac{\partial^2\langle h_ih_j^{'}\rangle}{\partial x_k^{'}\partial x_k^{'}}]-2R\langle h_ih_j^{'}\rangle. \qquad \text{-------------(2.14.3)}$$

Angular bracket $\langle ....\rangle$ is used to denote an ensemble average.

Using the transformations,

$$\frac{\partial}{\partial r_k}=-\frac{\partial}{\partial x_k}=\frac{\partial}{\partial x_k'} \qquad \text{-------------(2.14.4)}$$

and the Chandrasekhar relations [27]

$$\langle u_kh_ih_j^{'}\rangle=-\langle u_k^{'}h_ih_j^{'}\rangle,\langle u_j^{'}h_ih_k^{'}\rangle=-\langle u_ih_kh_j^{'}\rangle. \qquad \text{--------------(2.14.5)}$$

Equation (2.14.3) becomes

$$\left[\frac{\partial\langle h_ih_j^{'}\rangle}{\partial t}+2\frac{\partial}{\partial r_k}[\langle u_k^{'}h_ih_j^{'}\rangle-\langle u_ih_kh_j^{'}\rangle]\right].$$

$$=2\lambda\frac{\partial^2\langle h_ih_j^{'}\rangle}{\partial r_k\partial r_k}-2R\langle h_ih_j^{'}\rangle \qquad \text{---------------(2.14.6)}$$

Now we write equation (2.14.6) in spectral form in order to reduce it to an ordinary differential equation by use of the following three-dimensional Fourier transforms.

$$\langle h_ih_j^{'}(\hat{r})\rangle=\int_{-\infty}^{\infty}\langle\psi_i\psi_j^{'}(\hat{k})\rangle\exp\left[\hat{i}(\hat{k}.\hat{r})\right]d\hat{k}\,, \qquad \text{---------- (2.14.7)}$$

$$\langle u_i h_k h'_j(\hat{r})\rangle = \int_{-\infty}^{\infty} \langle \alpha_i \psi_k \psi'_j(\hat{k})\rangle \exp\left[\hat{i}(\hat{k}.\hat{r})\right] d\hat{K}. \quad \text{---------- (2.14.8)}$$

Interchanging i and j, points p and p' then,

$$\langle u'_k h_i h'_j(\hat{r})\rangle = \langle u_k h_i h'_j(-\hat{r})\rangle = \int_{-\infty}^{\infty} \langle \alpha_k \psi_i \psi'_j(-\hat{k})\rangle \exp\left[\hat{i}(\hat{k}.\hat{r})\right] d\hat{k}. \quad \text{----------- (2.14.9)}$$

where $\hat{k}$ is known as wave number vector and d$\hat{k}$ = $dk_1 dk_2 dk_3$.

Substituting of equation (2.14.7) to (2.14.9) in to equation (2.14.6) leads to the Spectral equation

$$\frac{\partial\langle \psi_i \psi'_j(\hat{k})\rangle}{\partial t} + 2\lambda K^2 \langle \psi_i \psi'_j(\hat{k})\rangle + 2R\langle \psi_i \psi'_j(\hat{k})\rangle$$
$$= 2ik[\langle \alpha_i \psi_i \psi'_j(\hat{k})\rangle - \langle \alpha_k \psi_i \psi'_j(-\hat{k})\rangle] \quad \text{------------(2.14.10)}$$

$$\Rightarrow \quad \frac{\partial\langle \psi_i \psi'_j(\hat{k})\rangle}{\partial t} + 2\lambda[K^2 + \frac{R}{\lambda}]\langle \psi_i \psi'_j(\hat{k})\rangle$$
$$= 2ik[\langle \alpha_i \psi_i \psi'_j(\hat{k})\rangle - \langle \alpha_k \psi_i \psi'_j(-\hat{k})\rangle]. \quad \text{----------(2.14.11)}$$

The tensor equation (2.14.11) becomes a scalar equation by contraction of the indices i and j

$$\frac{\partial\langle \psi_i \psi'_i(\hat{k})\rangle}{\partial t} + 2\lambda[K^2 + \frac{R}{\lambda}]\langle \psi_i \psi'_i(\hat{k})\rangle$$
$$= 2ik[\langle \alpha_i \psi_i \psi'_i(\hat{k})\rangle - \langle \alpha_k \psi_i \psi'_i(-\hat{k})\rangle]. \quad \text{-------------(2.14.12)}$$

The term on the right hand side of equation (2.14.12) is called energy transfer term while the second term on the left hand side is the dissipation term.

2.15. Three-point Correlation and Spectral Equations:

To find the three points correlation equations similar procedure can be used. For this purpose we take the momentum equation of dusty fluid MHD turbulence in a rotating system at the point P and the induction equations of magnetic field fluctuation, governing the concentration of a dilute contaminant undergoing a first order chemical reaction at P' and P'' separated by the vectors $\hat{r}$ and $\hat{r}'$ as

$$\frac{\partial u_l}{\partial t}+u_k\frac{\partial u_l}{\partial k_k}-h_k\frac{\partial h_l}{\partial x_k}=-\frac{\partial w}{\partial x_l}+\nu\frac{\partial^2 u_l}{\partial x_k\partial x_k}-2\in_{mkl}\Omega_m u_l+f(u_l-v_l), \quad \text{------- (2.15.1)}$$

$$\frac{\partial h_i^{'}}{\partial t}+u_k^{'}\frac{\partial h_i^{'}}{\partial x_k^{'}}-h_k^{'}\frac{\partial u_i^{'}}{\partial x_k^{'}}=\lambda\frac{\partial^2 h_i^{'}}{\partial x_k^{'}\partial x_k^{'}}-Rh_i^{'}, \quad \text{------- (2.15.2)}$$

$$\frac{\partial h_j^{''}}{\partial t}+u_k^{''}\frac{\partial h_j^{''}}{\partial x_k^{''}}-h_k^{''}\frac{\partial u_j^{''}}{\partial x_k^{''}}=\lambda\frac{\partial^2 h_j^{''}}{\partial x_k^{''}\partial x_k^{''}}-Rh_j^{''}, \quad \text{------------(2.15.3)}$$

where, $w(\hat{x},t)=\frac{P}{\rho}+\frac{1}{2}\langle h^2\rangle+\frac{1}{2}\left|\hat{\Omega}\times\hat{x}\right|^2$, total MHD pressure inclusive of potential and centrifugal force $P(\hat{x},t)$, hydrodynamic pressure; Ωm, constant angular velocity components; $\in_{mkl}$, alternating tensor, $f=\frac{kN}{\rho}$, dimension frequency; N, constant number density of dust particle.

Multiplying equation (2.15.1) by $h_i'h_j^{''}$ (2.15.2) by $u_l h_j^{''}$ and (2.15.3) by $u_l h_i'$, adding and taking ensemble average, we obtain

$$\frac{\partial\langle u_l h_i^{'}h_j^{''}\rangle}{\partial t}+\frac{\partial}{\partial x_k}[\langle u_k u_l h_i^{'}h_j^{''}\rangle-\langle h_k h_l h_i^{'}h_j^{''}\rangle]+\frac{\partial}{\partial x_k^{'}}[\langle u_l u_k^{'} h_i^{'}h_j^{''}\rangle-\langle u_l u_i^{'} h_k^{'}h_j^{''}\rangle].$$

$$+\frac{\partial}{\partial x_k^{''}}[\langle u_l u_k^{''} h_i^{'}h_j^{''}\rangle-\langle u_l u_j^{''} h_i^{'}h_j^{''}\rangle]$$

$$=-\frac{\partial\langle w h_i' h_j^{''}\rangle}{\partial x_l}+\nu\frac{\partial^2\langle u_l h_i' h_j^{''}\rangle}{\partial x_k\partial x_k}+\lambda\left[\frac{\partial^2\langle u_l h_i' h_j^{''}\rangle}{\partial x_k'\partial x_k'}+\frac{\partial^2\langle u_l h_i' h_j^{''}\rangle}{\partial x_k''\partial x_k''}\right]$$

$$-2R\langle u_l h_i^{'} h_j^{''}\rangle-2\in_{mki}\Omega_m\langle u_l h_i' h_j^{''}\rangle+f(\langle u_l h_i^{'} h_j^{''}\rangle-\langle v_l h_i' h_j^{''}\rangle) \quad \text{------ (2.15.4)}$$

Using the transformations

$$\frac{\partial}{\partial x_k}=-(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'}),\quad \frac{\partial}{\partial x_k'}=\frac{\partial}{\partial r_k},\quad \frac{\partial}{\partial x_k''}=\frac{\partial}{\partial r_k'} \quad \text{----------(2.15.5)}$$

into equation (2.15.4)

$$\frac{\partial\langle u_l h_i' h_j^{''}\rangle}{\partial t}-\lambda\left[(1+P_M)\frac{\partial^2\langle u_l h_i' h_j^{''}\rangle}{\partial r_k\partial r_k}+(1+P_M)\frac{\partial^2\langle u_l h_i' h_j^{''}\rangle}{\partial r_k'\partial r_k'}+2P_M\frac{\partial^2\langle u_l h_i' h_j^{''}\rangle}{\partial r_k\partial r_k}\right]$$

$$=\frac{\partial}{\partial r_k}\langle u_l u_k h_i^{'} h_j^{''}\rangle+\frac{\partial}{\partial r_k^{'}}\langle u_l u_k h_i^{'} h_j^{''}\rangle-\frac{\partial}{\partial r_k^{'}}\langle h_l h_k h_i^{'} h_j^{''}\rangle-\frac{\partial}{\partial r_k^{'}}\langle h_l h_k h_i^{'} h_j^{''}\rangle$$

$$-\frac{\partial}{\partial r_k}\langle u_l u_k^{'} h_i^{'} h_j^{''}\rangle+\frac{\partial}{\partial r_k}\langle u_l u_i^{'} h_k^{'} h_j^{''}\rangle-\frac{\partial}{\partial r_k^{'}}\langle u_l u_k^{''} h_i^{'} h_j^{''}\rangle+\frac{\partial}{\partial r_k^{'}}\langle u_l u_j^{''} h_i^{'} h_j^{''}\rangle$$

$$+\frac{\partial}{\partial r_l}\langle w h_i^{'} h_j^{''}\rangle+\frac{\partial}{\partial r_l^{'}}\langle w h_i^{'} h_j^{''}\rangle-2R\langle u h_i^{'} h_j^{''}\rangle-2\in_{mkl}\Omega_m\langle u_l h_i^{'} h_j^{''}\rangle$$

$$+f[\langle u_l h_i^{'} h_j^{''}\rangle-\langle v_l h_i^{'} h_j^{''}\rangle] \qquad \text{-----------(2.15.6)}$$

In order to write the equation (2.15.6) to spectral form, we can use the six dimensional Fourier transforms (2.4.7) – (2.4.12) and the equation

$$\langle v_l h_i'(\hat{r}) h_j^{''}(\hat{r}')\rangle=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle\delta_l\beta_i'(\hat{k})\beta_j^{''}(\hat{k}')\rangle\exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')\right]d\hat{k}d\hat{k}' . \qquad \text{------- (2.15.7)}$$

Interchanging the points p' and p'' along with the indices i and j, result in the relations

$$\langle u_l u_k^{''} h_j^{''} h_i^{'}\rangle=\langle u_l u_k^{'} h_i^{'} h_j^{''}\rangle,\quad \langle u_l u_j^{''} h_i^{'} h_j^{''}\rangle=\langle u_l u_i^{'} h_k^{'} h_j^{''}\rangle,\quad \langle u_l u_j^{''} h_i^{'}\rangle=\langle u_l u_i^{'} h_j^{''}\rangle \qquad \text{--------(2.15.8)}$$

By use of these facts (2.15.8) and the equations (2.4.7) - (2.4.12) and (2.15.7), the equation (2.15.6) may be transformed as

$$\frac{\partial\langle\phi_l\beta_i'\beta_j^{''}\rangle}{\partial t}+\lambda\left[(1+P_M)k^2+(1+P_M)k'^2+2P_M k_k k_k'+2\frac{R}{\lambda}+\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{f}{\lambda}\right]\langle\phi_l\beta_i'\beta_j^{''}\rangle$$

$$=i(k_k+k_k')\langle\phi_l\phi_k\beta_i'\beta_j^{''}\rangle-i(k_k+k_k')\langle\beta_l\beta_k\beta_i'\beta_j^{''}\rangle-i(k_k+k_k')\langle\phi_l\phi_k'\beta_i'\beta_j^{''}\rangle+i(k_k+k')\langle\phi_l\phi_i'\beta_i'\beta_j^{''}\rangle$$

$$+i(k_l+k_l')\langle\gamma\beta_i'\beta_j^{''}\rangle-f\langle\delta_l\beta_i'\beta_j^{''}\rangle . \qquad \text{------------ (2.15.9)}$$

The tensor equation (2.15.9) can be converted to scalar equation by contraction of the indices i and j

$$\frac{\partial\langle\phi_l\beta_i'\beta_i^{''}\rangle}{\partial t}+\lambda\left[(1+P_M)(k^2+k'^2)+2P_M k_k k_k'+2\frac{R}{\lambda}+\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{f}{\lambda}\right]\langle\phi_l\beta_i'\beta_i^{''}\rangle$$

$$= i(k_k + k'_k)\langle\phi_l\phi_k\beta'_i\beta''_i\rangle - i(k_k + k'_k)\langle\beta_l\beta_k\beta'_i\beta''_i\rangle - i(k_k + k'_k)\langle\phi_l\phi'_k\beta'_i\beta''_i\rangle + i(k_k + k')\langle\phi_l\phi'_i\beta'_i\beta''_i\rangle$$

$$+ i(k_l + k'_l)\langle\gamma\beta'_i\beta''_i\rangle - f\langle\delta_l\beta'_i\beta''_i\rangle . \quad \text{------------(2.15.10)}$$

To relate the terms on right hand side of equation (2.15.10) derived from the quadruple correlation terms and from the pressure force term in equation (2.15.6), we take the derivative with respect to x_l of the momentum equation (2.15.1) for the point p, and combine with the continuity equation to give

$$-\frac{\partial^2\langle w\rangle}{\partial x_l \partial x_l} = \frac{\partial^2}{\partial x_l \partial x_k}\left(u_l u_k - h_l h_k\right). \quad \text{------------- (2.15.11)}$$

Multiplying equation (2.15.11) by $h'_i h''_j$ taking time averages and writing this equation in terms of the independent variables $\hat{r}$ and $\hat{r}'$

$$-\left[\frac{\partial^2}{\partial r_l \partial r_l} + 2\frac{\partial^2}{\partial r_l \partial r'_l} + \frac{\partial^2}{\partial r'_l \partial r'_l}\right]\langle w h'_i h''_j\rangle = \left[\frac{\partial^2}{\partial r_l \partial r_k} + \frac{\partial^2}{\partial r'_l \partial r_k} + \frac{\partial^2}{\partial r_l \partial r'_k} + \frac{\partial^2}{\partial r'_l \partial r'_k}\right]$$

$$\times\left(\langle u_l u_k h'_i h''_j\rangle - \langle h_l h_k h'_i h''_j\rangle\right).. \quad \text{------------ (2.15.12)}$$

Now taking the Fourier transforms of equation (2.15.12) we get

$$-\langle\gamma\beta'_i\beta''_j\rangle = \frac{\left(k_l k_k + k'_l k_k + k_l k'_k + k'_l k'_k\right)\left(\langle\phi_l\phi_k\beta'_i\beta''_j\rangle - \langle\beta_l\beta_k\beta'_i\theta\beta''_j\rangle\right)}{k^2 + 2k_l k'_l + k'^2}$$

$$\text{-------------(2.15.13)}$$

Thus the equations (2.15.12) and (2.15.13) are the spectral equation corresponding to the three-point correlation equations. Equation (2.15.13) can be used to eliminate $\langle\gamma\beta'_i\beta''_j\rangle$ from the equation (2.5.10).

2.16. Solution for Times Before the Final Period:

As Art.(2.5) we neglect the quadruple correlation terms since the decay faster than the lower order correlation terms.

Putting the value of $\langle\gamma\beta'_i\beta''_j\rangle$ from equation (2.15.13) into equation (2.15.10) and neglecting all the quadruple correlation terms, we have

$$\frac{\partial\langle\phi_l\beta_i'\beta_i''\rangle}{\partial t}+\lambda\left[(1+P_M)(k^2+k'^2)+2P_M k_k k_k'+2\tfrac{R}{\lambda}+\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{f}{\lambda}\right]\langle\phi_l\beta_i'\beta_i''\rangle$$

$$+f\langle\delta_l\beta_i'\beta_i''\rangle=0$$

$$\Rightarrow\frac{\partial\langle\phi_l\beta_i'\beta_i''\rangle}{\partial t}+\lambda\left[(1+P_M)(k^2+k'^2)+2P_M k_k k_k'+2\tfrac{R}{\lambda}+\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{fs}{\lambda}\right]\langle\phi_l\beta_i'\beta_i''\rangle=0$$

------------- (2.16.1)

where, $\langle\delta_l\beta_i'\beta_i''\rangle$=C$\langle\phi_l\beta_i'\beta_i''\rangle$ and 1-C=S, C and S are arbitrary constants.

Taking inner multiplication by k_l, we get

$$\frac{\partial(k_l\langle\phi_l\beta_i'\beta_i''\rangle)}{\partial t}+\lambda\left[(1+P_M)(k^2+k'^2)+2P_M k_k k_k'+\frac{2R}{\lambda}+\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{fs}{\lambda}\right](K_l\langle\phi_l\beta_i'\beta_i''\rangle)=0$$

------------(2.16.2)

Integrating the equation (2.16.2) between t_0 and t, and gives

$$k_l\langle\phi_l\beta_i'\beta_i''\rangle=k_l[\langle\phi_l\beta_i'\beta_i''\rangle_0]\exp\{-\lambda[((1+P_M)(k^2+k'^2)+2P_M k_k k'\cos\theta$$

$$+\frac{2R}{\lambda}+\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{fs}{\lambda}](t-t_0)\}$$

--------(2.16.3)

where, θ is the angle between $\hat{k}$ and $\hat{k}'$ and $\langle\phi_l\beta_i'\theta_i''\rangle_0$ is the value of $\langle\phi_l\beta_i'\theta_i''\rangle$ at t= t_o.

Now, by letting $r'=0$ in equation (2.4.7) and comparing with equations (2.14.8) and (2.14.9), we get

$$\langle\alpha_i\psi_k\psi_i'(\hat{k})\rangle=\int_{-\infty}^{\infty}\langle\phi_l\beta_i'(\hat{k})\beta_i''(\hat{k}')\rangle d\hat{k}',$$

-------- (2.16.4)

$$\langle\alpha_k\psi_i\psi_i'(-\hat{k})\rangle=\int_{-\infty}^{\infty}\langle\phi_l\beta_i'(-\hat{k})\beta_i''(-\hat{k}')\rangle d\hat{k}'$$

-------- (2.16.5)

Substituting equation (2.16.3) - (2.16.5) in equation (2.14.12), we get

$$\frac{\partial\langle\psi_i\psi_i'(\hat{k})\rangle}{\partial t}+2\lambda[k^2+\frac{R}{\lambda}]\langle\psi_i\psi_i'(\hat{k})\rangle=\int_{-\infty}^{\infty}2ik_l\left[\langle\phi_l\beta_i'(\hat{k})\beta_i''(\hat{k}')\rangle-\langle\phi_l\beta_i'(-\hat{k}')\beta_i''(-\hat{k}')\rangle\right]$$

$$\times \exp\left[-\lambda\{(1+p_M)(k^2+k'^2)+2P_M kk'\cos\theta+\frac{2R}{\lambda}+2\frac{\in_{mkl}\Omega_m}{\lambda}-\frac{fs}{\lambda}\}(t-t_0)\right]d\hat{K}'$$

----- (2.16.6)

Now, $d\hat{k}'$ can be expressed in terms of k' and θ that is $d\hat{K}'=-2\pi k'^2 d(\cos\theta)dk'$ (cf. Deissler [37]).

With the above relation, equation (2.16.6) to give

$$\frac{\partial\langle\psi_i\psi_i'(\hat{k})\rangle}{\partial t}+2\lambda[k^2+\frac{R}{\lambda}]\langle\psi_i\psi_i'(\hat{k})\rangle=2\int_{-\infty}^{\infty}2ik_l\left[\langle\phi_l\beta_i'(\hat{k})\beta_i''(\hat{k}')\rangle-\langle\phi_l\beta_i'(-\hat{k}')\beta_i''(-\hat{k}')\rangle\right]_0$$

$$\times k'^2\left[\int_{-1}^{1}\exp\left\{-\lambda(t-t_0)[(1+P_M)(k^2+k'^2)+2P_M kk'\cos\theta+\frac{2R}{\lambda}+2\frac{\in_{mkl}\Omega_m}{\lambda}-\frac{fS}{\lambda}]\right\}d(\cos\theta)\right]d\hat{K}'$$

---------(2.16.7)

In order to find the solution completely and following Loeffler and Deissler [81] we assume that

$$ik_l\left[\langle\phi_l\beta_i'(\hat{K})\beta_i''(\hat{K}')\rangle-\langle\phi_l\beta_i'(-\hat{k})\beta_i''(-\hat{k}')\rangle\right]_0=-\frac{\xi_0}{(2\pi)^2}\left[k^2k'^4-k^4k'^2\right]$$

----------- (2.16.8)

where, ξ_0 is a constant. The negative sign is placed in front of ξ_0 in order to make the transfer of energy from small large wave numbers for positive value of ξ_0.

Substituting equation (2.16.8) into equation (2.16.7) and completing the integration with respect to $\cos\theta$, one obtains

$$\frac{\partial(2\pi\langle\psi_i\psi_i'(\hat{K})\rangle}{\partial t}+2\lambda[k^2+\frac{R}{\lambda}](2\pi\langle\psi_i\psi_I'(\hat{K})\rangle)=-\frac{\xi_0}{\nu(t-t_o)}\int_0^{\infty}\left(k^3k'^5-k^5k'^3\right)$$

$$\times\left[\exp\left\{-\lambda(t-t_o)[(1+P_M)(k^2+k'^2)-2P_M kk'+\frac{2R}{\lambda}+\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{fS}{\lambda}]\right\}\right.$$

$$- \exp\left\{ - \lambda(t - t_o)[(1 + P_M)(k^2 + k'^2) + 2P_M kk' + \frac{2R}{\lambda} + 2\frac{\in_{mkl} \Omega_m}{\lambda} - \frac{fS}{\lambda}] \right\} \Bigg] dK'.$$

------------(2.16.9)

Multiplying both sides of equation (2.16.9) by k^2, we get

$$\frac{\partial H}{\partial t} + 2\lambda\left(k^2 + \frac{R}{\lambda}\right)H = G,$$ -------- (2.16.10)

where, H=$2\pi k^2 \langle \psi_i \psi_k^{'}(\hat{K}) \rangle$ is the magnetic energy spectrum function and G is the magnetic energy transfer term is given by

$$G = -\frac{\xi_0}{\nu(t - t_0)} \int_0^\infty (k^3 k'^5 - k^5 k'^3) \Bigg[\exp\left\{ - \lambda(t - t_0)[(1 + P_M)(k^2 + k'^2) - 2P_M kk' + \frac{2R}{\lambda} + \frac{2\in_{mkl}\Omega_m}{\lambda} - \frac{fS}{\lambda}] \right\}$$

$$- \exp\left\{ - \lambda(t - t_0)[(1 + P_M)(k^2 + k'^2) + 2P_M kk' + \frac{2R}{\lambda} + \frac{2\in_{mkl}\Omega_m}{\lambda} - \frac{fS}{\lambda}] \right\} \Bigg] dK'$$

--------- (2.16.11)

Integrating equation (2.16.11) with respect to K', we have

$$G = \frac{-\xi_0 P_M \sqrt{\pi}}{4\lambda^{3/2}(t - t_0)^{3/2}(1 + P_M)^{5/2}} \exp[-\{2R + \frac{2\in_{mkl}\Omega_m}{\lambda} - \frac{fS}{\lambda}\}(t - t_0)] \exp\left[- \lambda(t - t_0)(\frac{1 + 2P_M}{1 + P_M})k^2 \right]$$

$$\times \left[\frac{15P_M k^4}{4\nu^2(t - t_0)^2(1 + P_M)} + \frac{1}{(t - t_0)}\left\{ \frac{5P_M^2}{\nu(1 + P_M)^2} - \frac{3}{2\nu} \right\}k^6 + \frac{P_M}{(1 + P_M)}\left\{ \frac{P_M^2}{(1 + P_M)^2} - 1 \right\}k^8 \right]$$

-------------- (2.16.12)

The series of equation (2.16.7) contains only even powers of k and start with k^4 and the equation represents the transfer function arising owing to consideration of magnetic field at three points at a time.

It is interesting to note that if we integrate equation (2.16.12) over all wave numbers, we find that

$$\int_0^\infty G dk = 0$$ -------- (2.16.13)

which indicates that the expression for G satisfies the condition of continuity and homogeneity.

The linear equation (2.16.10) can be solved to give

$$H = \exp\left[-2\lambda(k^2 + \frac{R}{\lambda})(t - t_o)\right]\int G \exp\left[2\lambda(k^2 + \frac{R}{\lambda})(t - t_o)\right]dt$$

$$+ J(k)\exp\left[-2\lambda(k^2 + \frac{R}{\lambda})(t - t_o)\right] \qquad \text{-------- (2.16.14)}$$

where, J(K)= $\frac{N_0 k^2}{\pi}$ is a constant of integration and can be obtained as by Corrsin[32].

Substituting the values of G from equation (2.16.12) in to equation (2.16.14) and integrating with respect to t, we get

$$H = \frac{N_0 k^2}{\pi}\exp\left[-2\lambda(k^2 + \frac{R}{\lambda})(t - t_o)\right] + \frac{\xi_0 \sqrt{\pi}\, p_M}{4\lambda^{3/2}(1 + P_M)^{7/2}}$$

$$\times \exp[-\{2R + 2\in_{mkl}\Omega_m - fS\}(t - t_o)]\exp\left[-\lambda k^2\left\{\frac{1 + 2p_M}{(1 + p_M)}\right\}(t - t_o)\right]$$

$$\left[\frac{3p_M, k^4}{2\lambda^2 P_M (t - t_o)^{5/2}} + \frac{(7p_M - 6)k^6}{3\lambda(1 + p_M)(t - t_o)^{3/2}} - \frac{4(3p_M^2 - 2p_M + 3)k^8}{3(1 + p_M)^2 (t - t_o)^{1/2}}\right.$$

$$\left. + \frac{8\lambda^{1/2}(3p_M^2 - 2p_M + 3)k^9}{3(1 + p_M)^{5/2}} N(\omega)\right] \qquad \text{--------- (2.16.15)}$$

where, $N(\omega) = e^{-\omega^2}\int_o^{\omega} e^{x^2}dx, \qquad \omega = k\sqrt{\frac{\lambda(t - t_o)}{(1 + p_M)}}$.

By setting $\hat{r} = 0,\ j = i,\ d\hat{K} = -2\pi k^2 d(\cos\theta)dk$ and $H = 2\pi k^2 \langle \psi_i \psi_j^{'}(\hat{K})\rangle$ in equation

(2.14.7),

we get the expression for magnetic energy decay with the fluctuating concentration as

$$\frac{\langle h^2\rangle}{2} = \frac{\langle h_i h_i^{'}\rangle}{2} = \int_0^{\infty} H d\hat{k}. \qquad \text{---------- (2.16.16)}$$

Substituting equation (2.16.15) in to (2.16.16) and after integration with respect to k, we get

$$\frac{\langle h^2\rangle}{2} = \exp[-2R(t - t_0)]\left[\frac{N_o (t - t_o)^{-3/2}}{8\lambda^{3/2}\sqrt{2\pi}} + \exp[-\{2\in_{mkl}\Omega_m - fS\}] \times \frac{\xi_0 \pi (t - t_o)^{-5}}{4\lambda^6 (1 + p_M)(1 + 2p_M)}\right.$$

$$\times\left\{\frac{9}{16}+\frac{5p_M(7p_M-6)}{16(1+2p_M)}-\frac{35p_M(3p_M^2-2p_M+3)}{8(1+2p_M)^2}+\frac{8p_M(3p_M^2-2p_M+3)}{3.2^6.(1+2p_M)^3}\sum_{n=0}^{\infty}\frac{1.3.5....(2n+9)}{n!(2n+1)2^{2n}(1+p_M)^n}\right\}\Bigg]\Bigg]$$

or
$$\frac{\langle h^2\rangle}{2}=\exp[-2R(t-t_0)]\left[\frac{N_o(t-t_o)^{-3/2}}{8\lambda^{3/2}\sqrt{2\pi}}+\exp[-\{2\in_{mkl}\Omega_m-fS\}]\xi_0 Z(t-t_o)^{-5}\right]$$

----------(2.16.17)

where,

$$Z=\frac{\pi}{(1+p_M)(1+2p_M)^{5/2}}\Big[\frac{9}{16}+\frac{5p_M(7p_M-6)}{16(1+2p_M)}-\frac{35p_M(3p_M^2-2p_M+3)}{8(1+2p_M)^2}$$

$$+\frac{8p_M(3p_M^2-2p_M+3)}{3.2^6.(1+2p_M)^3}+----\Big]$$

Thus the decay law for magnetic energy fluctuation of dusty fluid MHD turbulence in a rotating system governing the concentration of a dilute contaminant undergoing a first order chemical reaction before the final period may be written as

$$\langle h^2\rangle=\exp[-2R(t-t_0)]\left[A(t-t_0)^{-3/2}+\exp[-\{2\in_{mkl}\Omega_m-fs\}]B(t-t_0)^{-5}\right]$$ ---------(2.16.18)

where, $A=\dfrac{N_0}{8\lambda^{3/2}\sqrt{2\pi}}$, $B=\xi_0 Z$

2.17. Results and Discussion:

In equation (2.16.18) we obtained the decay law for magnetic energy fluctuation of dusty fluid MHD turbulence governing the concentration of a dilute contaminant undergoing a first order chemical reaction before the final period in a rotating system considering three-point correlation after neglecting quadruple correlation terms.

For clean fluid, f=0 then the equation (2.16.18) reduces to the equation (2.5.18) of this chapter in part-A.

For non-rotating system, $\Omega_m=0$, then the equation (2.16.18) reduces to the equation (2.10.18) of this chapter in part-B.

If the system is non-rotating and the fluid is clean i.e. $\Omega_m = 0$, and f=0 then the equation (2.16.18) becomes

$$\langle h^2 \rangle = \exp[-2R(t-t_0)]\left[A(t-t_0)^{-3/2} + B(t-t_0)^{-5} \right] \quad \text{----------(2.17.1)}$$

which was obtained earlier by Sarker and Islam [128]

In absence of chemical reaction, i.e, R=0 then the equation (2.17.1) becomes

$$\langle h^2 \rangle = \left[A(t-t_0)^{-3/2} + B(t-t_0)^{-5} \right] \quad \text{-----------(2.17.2)}$$

which was obtained earlier by Sarker and Kishor [120].

This study shows that due to the effect of rotation of fluid in the magnetic field with chemical reaction of the first order in the concentrarion in presence of dust particle in a rotating system the magnetic field fluctuation i,e.the turbulent energy decays more rapidly than the energy for non-rotating clean fluid and the faster rate is governed by $\exp[-\{2 \in_{mkl} \Omega_m\} - fs]$. Here the chemical reaction (R≠0) in MHD turbulence causes the concentration to decay more they would for non-rotating clean fluid and it is governed by $\exp\left[-\{2RT_M + 2 \in_{mkl} \Omega_m - fs\}\right]$.

The first term of right hand side of equation (2.16.18) corresponds to the energy of magnetic field fluctuation of concentration for the two-point correlation and the second term represents magnetic energy for the three-point correlation. In equation (2.16.18), the term associated with the three-point correlation die out faster than the two-point correlation. For large times the last term in the equation (2.16.18) becomes negligible, leaving the -3/2 power decay law for the final period. If higher order correlations are considered in the analysis, it appears that more terms of higher power of time would be added to the equation (2.16.18).

CHAPTER-III

PART-A

STATISTICAL THEORY OF CERTAIN DISTRIBUTION FUNCTIONS IN MHD TURBULENT FLOW FOR VELOCITY AND CONCENTRATION UNDERGOING A FIRST ORDER REACTION IN A ROTATING SYSTEM

3.1. Introduction:

The kinetic theory of gases and the statistical theory of fluid mechanics are the two major and distinct areas of investigations in statistical mechanics. In the past, several authors discussed the distribution functions in the statistical theory of turbulence. Lundgren [83] derived a hierarchy of coupled equations for multi-point turbulence velocity distribution functions, which resemble with BBGKY hierarchy of equations of Ta-Yu-Wu [141] in the kinetic theory of gasses. Kishore [60] studied the distributions functions in the statistical theory of MHD turbulence of an incompressible fluid. Pope [109] derived the transport equation for the joint probability density function of velocity and scalars in turbulent flow. Kishore and Singh [62] derived the transport equation for the bivariate joint distribution function of velocity and temperature in turbulent flow. Dixit and Upadhyay [40] considered the distribution functions in the statistical theory of MHD turbulence of an incompressible fluid in the presence of the coriolis force. Kollman and Janicka [75] derived the transport equation for the probability density function of a scalar in turbulent shear flow and considered a closure model based on gradient –flux model.But at this stage, one is met with the difficulty that the N-point distribution function depends upon the N+1-point distribution function and thus result is an unclosed system. This so-called "closer problem" is encountered in turbulence, kinetic theory and other non-linear system. Sarker and Kishore [119] discussed the distribution functions in the statistical theory of convective MHD turbulence of an incompressible fluid. Also Sarker and Kishore [126] studied the distribution functions in the statistical theory of convective MHD turbulence of mixture of a miscible incompressible fluid. Sarker and Islam [138] studied the Distribution functions in the statistical theory of convective MHD turbulence of an incompressible fluid in a rotating system. Islam and Sarker [57] also studied Distribution functions in the statistical theory of MHD turbulence for velocity and concentration undergoing a first order reaction.

Using the above theories, we have studied the distribution function for simultaneous velocity, magnetic, temperature, concentration fields and reaction in MHD turbulent flow undergoing a first order reaction in a rotating system. Finally, the transport equations for evolution of distribution functions have been derived and various properties of the distribution function have been discussed. The resulting one-point equation is compared with the first equation of BBGKY hierarchy of equations and the closure difficulty is to be removed as in the case of ordinary turbulence.

3.2. Basic Equations:

The equations of motion and the equation of continuity for viscous incompressible dusty fluid MHD turbulent flow, the diffusion equations for the temperature and concentration undergoing a first order chemical reaction in a rotating system are given by

$$\frac{\partial u_\alpha}{\partial t}+\frac{\partial}{\partial x_\beta}\left(u_\alpha u_\beta - h_\alpha h_\beta\right)=-\frac{\partial w}{\partial x_\alpha}+\nu\nabla^2 u_\alpha - 2\in_{m\alpha\beta}\Omega_m u_\alpha \quad \text{---------} (3.2.1)$$

$$\frac{\partial h_\alpha}{\partial t}+\frac{\partial}{\partial x_\beta}\left(h_\alpha u_\beta - u_\alpha h_\beta\right)=\lambda\nabla^2 h_\alpha \quad \text{---------} (3.2.2)$$

$$\frac{\partial \theta}{\partial t}+u_\beta\frac{\partial \theta}{\partial x_\beta}=\gamma\nabla^2\theta \quad \text{---------} (3.2.3)$$

$$\frac{\partial C}{\partial t}+u_\beta\frac{\partial C}{\partial x_\beta}=D\nabla^2 C - RC \quad \text{---------} (3.2.4)$$

with $\frac{\partial u_\alpha}{\partial x_\alpha}=\frac{\partial v_\alpha}{\partial x_\alpha}=\frac{\partial h_\alpha}{\partial x_\alpha}=0$, ---------- (3.2.5)

where,

$u_\alpha(x,t)$, α – component of turbulent velocity

$h_\alpha(x,t)$, α – component of magnetic field

$\theta(x,t)$, temperature fluctuation

C, concentration of contaminants

v_α, dust particle velocity

R, constant reaction rate

$\in_{m\alpha\beta}$, alternating tensor

N, constant number of density of the dust particle

$w(\hat{x},t) = P/\rho + \frac{1}{2}|\vec{h}|^2 + \frac{1}{2}|\hat{\Omega}\times\hat{x}|^2$, total pressure

$P(\hat{x},t)$, hydrodynamic pressure

ρ, fluid density

Ω, angular velocity of a uniform rotation

ν, Kinetic viscosity

$\lambda = (4\pi\mu\sigma)^{-1}$, magnetic diffusivity

$\gamma = \frac{k_T}{\rho c_p}$, thermal diffusivity,

K= Stokes's resistance coefficient which for spherical particle of radius r is 6πμr.

c_p, specific heat at constant pressure,

k_T, thermal conductivity

σ, electrical conductivity

μ, magnetic permeability

D, diffusive co-efficient for contaminants.

The repeated suffices are assumed over the values 1, 2 and 3 and unrepeated suffices may take any of these values. Here u, h and x are vector quantities in the whole process.
The total pressure w which, occurs in equation (3.2.1) may be eliminated with the help of the equation obtained by taking the divergence of equation (3.2.1)

$$\nabla^2 w = -\frac{\partial^2}{\partial x_\alpha \partial x_\beta}\left(u_\alpha u_\beta - h_\alpha h_\beta\right) = -[\ \frac{\partial u_\alpha}{\partial x_\beta}\frac{\partial u_\beta}{\partial x_\alpha} - \frac{\partial h_\alpha}{\partial x_\beta}\frac{\partial h_\beta}{\partial x_\alpha}\ ]. \quad \text{---------} (3.2.6)$$

In a conducting infinite fluid only the particular solution of the Equation (3.2.6) is related, so that

$$w = \frac{1}{4\pi}\int [\ \frac{\partial u'_\alpha}{\partial x'_\beta}\frac{\partial u'_\beta}{\partial x'_\alpha} - \frac{\partial h'_\alpha}{\partial x'_\beta}\frac{\partial h'_\beta}{\partial x'_\alpha}\ ]\frac{\partial \bar{x}'}{|\bar{x}' - \bar{x}|}. \quad \text{----------} (3.2.7)$$

Hence equation (3.2.1) – (3.2.4) becomes

$$\frac{\partial u_\alpha}{\partial t}+\frac{\partial}{\partial x_\beta}\left(u_\alpha u_\beta-h_\alpha h_\beta\right)=-\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha}\int\left[\frac{\partial u'_\alpha}{\partial x'_\beta}\frac{\partial u'_\beta}{\partial x'_\alpha}-\frac{\partial h'_\alpha}{\partial x'_\beta}\frac{\partial h'_\beta}{\partial x'_\alpha}\right]\frac{d\bar{x}'}{\left|\bar{x}'-\bar{x}\right|}+\nu\nabla^2 u_\alpha$$
$$-2\in_{m\alpha\beta}\Omega_m u_\alpha \qquad \text{----------- (3.2.8)}$$

$$\frac{\partial h_\alpha}{\partial t}+\frac{\partial}{\partial x_\beta}\left(h_\alpha u_\beta-u_\alpha h_\beta\right)=\lambda\nabla^2 h_\alpha \qquad \text{------------- (3.2.9)}$$

$$\frac{\partial\theta}{\partial t}+u_\beta\frac{\partial\theta}{\partial x_\beta}=\gamma\nabla^2\theta \qquad \text{----------- (3.2.10)}$$

$$\frac{\partial C}{\partial t}+u_\beta\frac{\partial C}{\partial x_\beta}=D\nabla^2 C-RC \qquad \text{----------- (3.2.11)}$$

3.3. Formulation of the Problem:

We consider the turbulence and the concentration fields are homogeneous, the chemical reaction and the local mass transfer have no effect on the velocity field and the reaction rate and the diffusivity are constant. We also consider a large ensemble of identical fluids in which each member is an infinite incompressible reacting and heat conducting fluid in turbulent state. The fluid velocity u, Alfven velocity h, temperature θ and concentration C, are randomly distributed functions of position and time and satisfy their field. Different members of ensemble are subjected to different initial conditions and our aim is to find out a way by which we can determine the ensemble averages at the initial time. Certain microscopic properties of conducting fluids, such as total energy, total pressure, stress tensor which are nothing but ensemble averages at a particular time, can be determined with the help of the bivariate distribution functions (defined as the averaged distribution functions with the help of Dirac delta-functions). Our present aim is to construct the distribution functions, study its properties and derive an equation for its evolution of this distribution functions.

3.4. Distribution Function in MHD Turbulence and Their Properties:

In MHD turbulence, we may consider the fluid velocity u, Alfven velocity h, temperature θ, concentration C and constant reaction rate R at each point of the flow field. Lundgren [83] has studied the flow field on the basis of one variable character only (namely the fluid u), but we can study it for two or more variable characters as well. The corresponding to each point of the flow field, we have four measurable characteristics. We represent the four variables by v, g, ϕ and ψ and denote the pairs of these variables at the points $\bar{x}^{(1)},\bar{x}^{(2)},-----,\bar{x}^{(n)}$ as

$\left(\bar{v}^{(1)},\bar{g}^{(1)},\phi^{(1)},\psi^{(1)}\right), \left(\bar{v}^{(2)},\bar{g}^{(2)},\phi^{(2)},\psi^{(2)}\right), ------\left(\bar{v}^{(n)},\bar{g}^{(n)},\phi^{(n)},\psi^{(n)}\right)$ at a fixed instant of time. It is possible that the same pair may be occur more than once; therefore, we simplify the problem by an assumption that the distribution is discrete (in the sense that no pairs occur more than once). Symbolically we can express the distribution as

$$\left\{ \left(\bar{v}^{(1)},\bar{g}^{(1)},\phi^{(1)},\psi^{(1)}\right), \left(\bar{v}^{(2)},\bar{g}^{(2)},\phi^{(2)},\psi^{(2)}\right), ------\left(\bar{v}^{(n)},\bar{g}^{(n)},\phi^{(n)},\psi^{(n)}\right) \right\}.$$

Instead of considering discrete points in the flow field, if we consider the continuous distribution of the variables $\bar{v},\bar{g},\phi$ and ψ over the entire flow field, statistically behavior of the fluid may be described by the distribution function $F(\bar{v},\bar{g},\phi,\psi)$ which is normalized so that

$$\int F(\bar{v},\bar{g},\phi,\psi)d\bar{v},d\bar{g}d\phi d\psi = 1$$

where, the integration ranges over all the possible values of v, g, ϕ and ψ. We shall make use of the same normalization condition for the discrete distributions also. The distribution functions of the above quantities can be defined in terms of Dirac delta functions.

The one-point distribution function $F_1^{(1)}\left(v^{(1)},g^{(1)},\phi^{(1)},\psi^{(1)}\right)$, defined so that $F_1^{(1)}\left(v^{(1)},g^{(1)},\phi^{(1)},\psi^{(1)}\right)dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)}$ is the probability that the fluid velocity, Alfven velocity, temperature and concentration field at a time t are in the element $dv^{(1)}$ about $v^{(1)}$, $dg^{(1)}$ about $g^{(1)}$, $d\phi^{(1)}$ about $\phi^{(1)}$ and $d\psi^{(1)}$ about $\psi^{(1)}$ respectively and is given by

$$F_1^{(1)}\left(v^{(1)},g^{(1)},\phi^{(1)},\psi^{(1)}\right) = \left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right) \right\rangle \quad \text{--------} (3.4.1)$$

where δ is the Dirac delta-function defined as

$$\int \delta(\bar{u}-\bar{v})d\bar{v} = \begin{cases} 1 & \text{at the point } \bar{u}=\bar{v} \\ 0 & \text{elsewhere} \end{cases}$$

Two-point distribution function is given by

$$F_2^{(1,2)} = \left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\right.$$

$$\left.\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(C^{(2)}-\psi^{(2)}\right) \right\rangle \quad \text{------------} (3.4.2)$$

and three point distribution function is given by

$$F_3^{(1,2,3)} = \left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\right.$$

$$\times\,\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(C^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(C^{(3)}-\psi^{(3)}\right)\rangle.$$

---------- (3.4.3)

Similarly, we can define an infinite numbers of multi-point distribution functions $F_4^{(1,2,3,4)}$, $F_5^{(1,2,3,4,5)}$ and so on.

The distribution functions so constructed have the following properties:

(A) Reduction Properties:

Integration with respect to pair of variables at one-point, lowers the order of distribution function by one. For example,

$$\iiiint F_1^{(1)}dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)}=1\ ,$$

$$\iiiint F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}=F_1^{(1)}\ ,$$

$$\iiiint F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)}=F_2^{(1,2)}\qquad \text{etc.}$$

Also the integration with respect to any one of the variables, reduces the number of Delta-functions from the distribution function by one as

$$\int F_1^{(1)}dv^{(1)}=\left\langle\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\right\rangle,$$

$$\int F_1^{(1)}dg^{(1)}=\left\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\right\rangle,$$

$$\int F_1^{(1)}d\phi^{(1)}=\left\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\right\rangle$$

and $\int F_2^{(1,2)}dv^{(2)}=\left\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\right.$

$$\left.\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(C^{(2)}-\psi^{(2)}\right)\right\rangle.$$

(B) Separation Properties:

The pairs of variables at the two points are statistically independent of each other if these points are far apart from each other in the flow field i.e.,

$$Lim_{\left|\bar{x}^{(2)}\to\bar{x}^{(1)}\right|\to\infty}\ F_2^{(1,2)} = F_1^{(1)}F_1^{(2)}$$

and similarly,

$$Lim_{\left|\bar{x}^{(3)}\to\bar{x}^{(2)}\right|\to\infty} F_3^{(1,2,3)} = F_2^{(1,2)}F_1^{(3)} \qquad \text{etc.}$$

(C) Co-incidence Property:

When two points coincide in the flow field, the components at these points should be obviously the same that is $F_2^{(1,2)}$ must be zero. Thus $\bar{v}^{(2)} = \bar{v}^{(1)}$, $g^{(2)} = g^{(1)}$, $\phi^{(2)} = \phi^{(1)}$ and $\psi^{(2)} = \psi^{(1)}$, but $F_1^{(1,2)}$ must also have the property.

$$\iiiint F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} = F_1^{(1)}$$

and hence it follows that

$$Lim_{\left|\bar{x}^{(2)}\to\bar{x}^{(1)}\right|\to\infty}\ \int F_2^{(1,2)} = F_1^{(1)}\delta\left(v^{(2)} - v^{(1)}\right)\delta\left(g^{(2)} - g^{(1)}\right)\delta\left(\phi^{(2)} - \phi^{(1)}\right)\delta\left(\psi^{(2)} - \psi^{(1)}\right).$$

Similarly,

$$Lim_{\left|\bar{x}^{(3)}\to\bar{x}^{(2)}\right|\to\infty}\ \int F_3^{(1,2,3)} = F_2^{(1,2)}\delta\left(v^{(3)} - v^{(1)}\right)\delta\left(g^{(3)} - g^{(1)}\right)\delta\left(\phi^{(3)} - \phi^{(1)}\right)\delta\left(\psi^{(3)} - \psi^{(1)}\right) \qquad \text{etc.}$$

(D) Symmetric Conditions:

$$F_n^{(1,2,r,------s,-----n)} = F_n^{(1,2,-----s,---r,---n)}.$$

(E) Incompressibility Conditions:

(i) $$\iint \frac{\partial F_n^{(1,2,---n)}}{\partial x_\alpha^{(r)}} v_\alpha^{(r)} d\bar{v}^{(r)} d\bar{h}^{(r)} = 0,$$

(ii) $$\iint \frac{\partial F_n^{(1,2,---n)}}{\partial x_\alpha^{(r)}} h_\alpha^{(r)} d\bar{v}^{(r)} d\bar{h}^{(r)} = 0.$$

3.5. Continuity Equation in Terms of Distribution Functions:

An infinite number of continuity equations can be derived for the convective MHD turbulent flow and the continuity equations can be easily expressed in terms of distribution functions and are obtained directly by div $u = 0$. Taking ensemble average of equation (3.2.5)

$$0 = \left\langle \frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}} \right\rangle = \left\langle \frac{\partial}{\partial x_\alpha^{(1)}} u_\alpha^{(1)} \iiiint F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \right\rangle$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \left\langle u_\alpha^{(1)} \iiiint F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \right\rangle$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \iiiint \langle \; u_\alpha^{(1)} \; \rangle \langle \; F_1^{(1)} \; \rangle dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)}$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \iiiint v_\alpha^{(1)} F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)}$$

$$= \iiiint \frac{\partial F_1^{(1)}}{\partial x_\alpha^{(1)}} v_\alpha^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \qquad \text{------------ (3.5.1)}$$

and similarly,

$$0 = \iiiint \frac{\partial F_1^{(1)}}{\partial x_\alpha^{(1)}} g_\alpha^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \qquad \text{------------ (3.5.2)}$$

which are the first order continuity equations in which only one point distribution function is involved. For second-order continuity equations, if we multiply the continuity equation by

$$\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(C^{(2)} - \psi^{(2)}\right)$$

and if we take the ensemble average, we obtain

$$o = \left\langle \; \delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(C^{(2)} - \psi^{(2)}\right)\frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}} \; \right\rangle$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \left\langle \; \delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(C^{(2)} - \psi^{(2)}\right) u_\alpha^{(1)} \; \right\rangle$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \int \Big\langle \; u_\alpha^{(1)} \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)$$

$$\times \delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(C^{(2)} - \psi^{(2)}\right) \; \Big\rangle$$

$$o = \frac{\partial}{\partial x_{\alpha}^{(1)}} \iiiint v_{\alpha}^{(1)} F_2^{(1,2)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \qquad \text{---------- (3.5.3)}$$

and similarly,

$$o = \frac{\partial}{\partial x_{\alpha}^{(1)}} \iiiint g_{\alpha}^{(1)} F_2^{(1,2)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} . \qquad \text{----------- (3.5.4)}$$

The Nth – order continuity equations are

$$o = \frac{\partial}{\partial x_{\alpha}^{(1)}} \iiiint v_{\alpha}^{(1)} F_N^{(1,2,---,N)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \qquad \text{---------- (3.5.5)}$$

and $$o = \frac{\partial}{\partial x_{\alpha}^{(1)}} \iiiint g_{\alpha}^{(1)} F_N^{(1,2,\ldots\ldots\ldots,N)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} . \qquad \text{----------- (3.5.6)}$$

The continuity equations are symmetric in their arguments i.e.;

$$\frac{\partial}{\partial x_{\alpha}^{(r)}} \iiiint \left(v_{\alpha}^{(r)} F_N^{(1,2,\ldots\ldots\ldots;r.N)} dv^{(r)} dg^{(r)} d\phi^{(r)} d\psi^{(r)} \right)$$
$$= \frac{\partial}{\partial x_{\alpha}^{(s)}} \iiiint v_{\alpha}^{(s)} F_N^{(1,2,\ldots\ldots r,s,\ldots\ldots N)} dv^{(s)} dg^{(s)} d\phi^{(s)} d\psi^{(s)} \qquad \text{---------- (3.5.7)}$$

Since the divergence property is an important property and it is easily verified by the use of the property of distribution function as

$$\frac{\partial}{\partial x_{\alpha}^{(1)}} \iiiint v_{\alpha}^{(1)} F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} = \frac{\partial}{\partial x_{\alpha}^{(1)}} \left\langle u_{\alpha}^{(1)} \right\rangle = \left\langle \frac{\partial u_{\alpha}^{(1)}}{\partial x_{\alpha}(1)} \right\rangle = o \qquad \text{---------- (3.5.8)}$$

and all the properties of the distribution function obtained in section(3.4) can also be verified.

3.6. Equations for Evolution of Distribution Functions:

The equations (3.2.8)-(3.2.11) will be used to convert these into a set of equations for the variation of the distribution function with time. This, in fact, is done by making use of the definitions of the constructed distribution functions, differentiating them partially with respect to time, making some suitable operations on the right-hand side of the equation so obtained and lastly replacing the time derivative of v, h, θ and C from the equations (3.2.8)-(3.2.11).

Differentiating equation (3.4.1), and then using equations (3.2.8)-(3.2.11) we get,

$$\frac{\partial F_1^{(1)}}{\partial t}=\frac{\partial}{\partial t}\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\right\rangle$$

$$=\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial}{\partial t}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)$$

$$\times\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial}{\partial t}\delta\left(h^{(1)}-g^{(1)}\right)\ \rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\frac{\partial}{\partial t}\delta\left(C^{(1)}-\psi^{(1)}\right)\ \rangle$$

$$=\langle -\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial u^{(1)}}{\partial t}\frac{\partial}{\partial v^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial h^{(1)}}{\partial t}\frac{\partial}{\partial g^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\ \rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial \theta^{(1)}}{\partial t}\frac{\partial}{\partial \phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\ \rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\frac{\partial c^{(1)}}{\partial t}\frac{\partial}{\partial \psi^{(1)}}\delta\left(C^{(1)}-\psi^{(1)}\right)\ \rangle.$$

------ -- (3.6.1)

Using equations (3.2.8) – (3.2.11) in the equation (3.6.1), we get

$$\frac{\partial F_1^{(1)}}{\partial t}=\langle -\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\Big\{-\frac{\partial}{\partial x_\beta}\left(u_\alpha^{(1)}u_\beta^{(1)}-h_\alpha^{(1)}h_\beta^{(1)}\right)$$

$$-\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha}\int\left[\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}}-\frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}}\right]\frac{d\bar{x}'}{|\bar{x}'-\bar{x}|}+\nu\nabla^2 u_\alpha^{(1)}-2\in_{m\alpha\beta}\Omega_m u_\alpha^{(1)}\Big\}$$

$$\times\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\Big\{-\frac{\partial}{\partial x_\beta^{(1)}}\left(h_\alpha^{(1)}u_\beta^{(1)}-u_\alpha^{(1)}h_\beta^{(1)}\right)$$

$$+\lambda\nabla^2 h_\alpha^{(1)}\Big\}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\ \rangle+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)$$

$$\times\left\{-u_\beta^{(1)}\frac{\partial \theta^{(1)}}{\partial x_\beta}+\gamma\nabla^2\theta^{(1)}\right\}\times\frac{\partial}{\partial \phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\ \rangle+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)$$

$$\times\delta\left(\theta^{(1)}-\phi^{(1)}\right)\left\{-u_\beta^{(1)}\frac{\partial c^{(1)}}{\partial x_\beta^{(1)}}+D\nabla^2 C\right\}\frac{\partial}{\partial \psi^{(1)}}\delta\left(C^{(1)}-\psi^{(1)}\right)\ \rangle$$

$$=\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial u_\alpha^{(1)}u_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$+\langle -\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial h_\alpha^{(1)}h_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$+\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(1)}}\int\left[\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}}-\frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}}\right]$$

$$\times\frac{d\bar{x}'}{\left|\bar{x}'-\bar{x}\right|}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle+\langle -\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\nu\nabla^2u_\alpha^{(1)}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$+\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)2\in_{m\alpha\beta}\Omega_m u_\alpha^{(1)}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial h_\alpha^{(1)}u_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\ \rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial u_\alpha^{(1)}h_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial g_\alpha}\delta\left(h^{(1)}-g^{(1)}\right)\ \rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\lambda\nabla^2h_\alpha^{(1)}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\ \rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)u_\beta^{(1)}\frac{\partial\theta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\ \rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\gamma\nabla^2\theta^{(1)}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\ \rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)u_\beta^{(1)}\frac{\partial C^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial\psi^{(1)}}\delta\left(C^{(1)}-\psi^{(1)}\right)\ \rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)D\nabla^2C^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(C^{(1)}-\psi^{(1)}\right)\ \rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)RC^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(C^{(1)}-\psi^{(1)}\right)\ \rangle \quad \text{-----------} \ (3.6.2)$$

Various terms in the equation (3.6.2) can be simplified as that they may be expressed in terms of one point and two point distribution functions. For example,

The first term on the right hand side of the above equation is simplified as follows:

$$\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial u_\alpha^{(1)}u_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$=\left\langle u_\beta^{(1)}\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle$$

$$=\left\langle -u_\beta^{(1)}\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle$$

$$=\left\langle -u_\beta^{(1)}\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle,\quad (\text{since } \frac{\partial u_\alpha^{(1)}}{\partial v_\alpha^{(1)}}=1)$$

$$=\left\langle -\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)u_\beta^{(1)}\frac{\partial}{\partial x_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle. \qquad \text{-----------} (3.6.3)$$

Similarly, seventh, tenth and twelfth terms of right hand-side of equation (3.6.2) can be simplified as follows;

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial h_\alpha^{(1)}u_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\ \rangle$$

$$=\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)u_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\right\rangle \qquad \text{-----------} (3.6.4)$$

Tenth term,

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)u_\beta^{(1)}\frac{\partial \theta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial \phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\ \rangle$$

$$=\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)u_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\right\rangle \qquad \text{-----------} (3.6.5)$$

and twelfth term

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)u_\beta^{(1)}\frac{\partial C^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial \psi^{(1)}}\delta\left(C^{(1)}-\psi^{(1)}\right)\ \rangle$$

$$=\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)u_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(C^{(1)}-\psi^{(1)}\right)\right\rangle. \qquad \text{-----------} (3.6.6)$$

Adding (3.6.3) – (3.6.6), we get

$$\left\langle -\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)u_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle$$

$$+\langle -\delta(u^{(1)}-v^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(C^{(1)}-\psi^{(1)})u_{\beta}^{(1)}\frac{\partial}{\partial x_{\beta}^{(1)}}\delta(h^{(1)}-g^{(1)})\rangle$$

$$+\langle -\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(C^{(1)}-\psi^{(1)})u_{\beta}^{(1)}\frac{\partial}{\partial x_{\beta}^{(1)}}\delta(\theta^{(1)}-\phi^{(1)})\rangle$$

$$+\langle -\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})u_{\beta}^{(1)}\frac{\partial}{\partial x_{\beta}^{(1)}}\delta(C^{(1)}-\psi^{(1)})\rangle$$

$$=-\frac{\partial}{\partial x_{\beta}^{(1)}}\langle u_{\beta}^{(1)}\langle -\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(C^{(1)}-\psi^{(1)})\rangle$$

$$=-\frac{\partial}{\partial x_{\beta}^{(1)}}v_{\beta}^{(1)}F_1^{(1)}$$ [Using the properties of distribution functions]

$$=-v_{\beta}^{(1)}\frac{\partial F_1^{(1)}}{\partial x_{\beta}^{(1)}} \quad \text{----------- (3.6.7)}$$

Similarly second and eighth terms on the right hand-side of the equation (3.6.2) can be simplified as

$$\langle -\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(C^{(1)}-\psi^{(1)})\frac{\partial h_{\alpha}^{(1)}h_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta(u^{(1)}-v^{(1)})\rangle=-g_{\beta}^{(1)}\frac{\partial g_{\alpha}^{(1)}}{\partial v_{\alpha}^{(1)}}\frac{\partial}{\partial x_{\beta}^{(1)}}F_1^{(1)}$$

----------- (3.6.8)

and $$\langle -\delta(u^{(1)}-v^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(C^{(1)}-\psi^{(1)})\frac{\partial u_{\alpha}^{(1)}h_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial g_{\alpha}}\delta(h^{(1)}-g^{(1)})\rangle$$

$$=-g_{\beta}^{(1)}\frac{\partial v_{\alpha}^{(1)}}{\partial g_{\alpha}^{(1)}}\frac{\partial}{\partial x_{\beta}^{(1)}}F_1^{(1)} \quad \text{----------- (3.6.9)}$$

Fourth term can be reduced as

$$\langle -\nu\nabla^2 u_{\alpha}^{(1)}\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(C^{(1)}-\psi^{(1)})\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta(u^{(1)}-v^{(1)})$$

$$=-\nu\frac{\partial}{\partial v_{\alpha}^{(1)}}\langle\nabla^2 u_{\alpha}^{(1)}[\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(C^{(1)}-\psi^{(1)})]\rangle$$

$$=-\nu\frac{\partial}{\partial v_{\alpha}^{(1)}}\frac{\partial^2}{\partial x_{\beta}^{(1)}\partial x_{\beta}^{(1)}}\langle u_{\alpha}^{(1)}[\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(C^{(1)}-\psi^{(1)})]\rangle$$

$$=-\nu \frac{\partial}{\partial v_\alpha^{(1)}} Lim_{\bar{x}^{(2)} \to \bar{x}^{(1)}} \frac{\partial^2}{\partial x_\beta^{(2)} \partial x_\beta^{(2)}} \left\langle u_\alpha^{(2)} \left[\delta\left(u^{(1)} - v^{(1)}\right) \delta\left(h^{(1)} - g^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(C^{(1)} - \psi^{(1)}\right) \right] \right\rangle$$

$$=-\nu \frac{\partial}{\partial v_\alpha^{(1)}} Lim_{\bar{x}^{(2)} \to \bar{x}^{(1)}} \frac{\partial^2}{\partial x_\beta^{(2)} \partial x_\beta^{(2)}} \left\langle \iiiint u_\alpha^{(2)} \delta\left(u^{(2)} - v^{(2)}\right) \delta\left(h^{(2)} - g^{(2)}\right) \delta\left(\theta^{(2)} - \phi^{(2)}\right) \delta\left(C^{(2)} - \psi^{(2)}\right) \right)$$

$$\times \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(h^{(1)} - g^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(C^{(1)} - \psi^{(1)}\right) dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)}$$

$$=-\nu \frac{\partial}{\partial v_\alpha^{(1)}} Lim_{\bar{x}^{(2)} \to \bar{x}^{(1)}} \frac{\partial^2}{\partial x_\beta^{(2)} \partial x_\beta^{(2)}} \iiiint v_\alpha^{(2)} F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} \qquad \text{----------} (3.6.10)$$

Ninth, eleventh and thirteen terms of the right hand side of equation (3.6.2)

$$\left\langle -\delta\left(u^{(1)} - v^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(C^{(1)} - \psi^{(1)}\right) \lambda \nabla^2 h_\alpha^{(1)} \frac{\partial}{\partial g_\alpha^{(1)}} \delta\left(h^{(1)} - g^{(1)}\right) \right\rangle$$

$$= \left\langle -\lambda \nabla^2 h_\alpha^{(1)} \frac{\partial}{\partial g_\alpha^{(1)}} \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(C^{(1)} - \psi^{(1)}\right) \frac{\partial}{\partial g_\alpha^{(1)}} \delta\left(h^{(1)} - g^{(1)}\right) \right\rangle$$

$$=-\lambda \frac{\partial}{\partial g_\alpha^{(1)}} Lim_{\bar{x}^{(2)} \to \bar{x}^{(1)}} \frac{\partial^2}{\partial x_\beta^{(2)} \partial x_\beta^{(2)}} \iiiint g_\alpha^{(2)} F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} \qquad \text{-----------} (3.6.11)$$

$$\left\langle -\delta\left(u^{(1)} - v^{(1)}\right) \delta\left(h^{(1)} - g^{(1)}\right) \delta\left(C^{(1)} - \psi^{(1)}\right) \gamma \nabla^2 \theta^{(1)} \frac{\partial}{\partial \phi^{(1)}} \delta\left(\theta^{(1)} - \phi^{(1)}\right) \right\rangle$$

$$= \left\langle -\gamma \nabla^2 \theta^{(1)} \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(h^{(1)} - g^{(1)}\right) \delta\left(C^{(1)} - \psi^{(1)}\right) \frac{\partial}{\partial \phi^{(1)}} \delta\left(\theta^{(1)} - \phi^{(1)}\right) \right\rangle$$

$$=-\gamma \frac{\partial}{\partial \phi^{(1)}} Lim_{\bar{x}^{(2)-} \to \bar{x}^{(1)}} \frac{\partial^2}{\partial x_\beta^{(2)} \partial x_\beta^{(2)}} \int \phi^{(2)} F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} \qquad \text{------------} (3.6.12)$$

$$\left\langle -\delta\left(u^{(1)} - v^{(1)}\right) \delta\left(h^{(1)} - g^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) D \nabla^2 C^{(1)} \frac{\partial}{\partial \psi^{(1)}} \delta\left(C^{(1)} - \psi^{(1)}\right) \right\rangle$$

$$= \left\langle -D \nabla^2 C^{(1)} \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(h^{(1)} - g^{(1)}\right) \delta\left(C^{(1)} - \psi^{(1)}\right) \frac{\partial}{\partial \phi^{(1)}} \delta\left(\theta^{(1)} - \phi^{(1)}\right) \right\rangle$$

$$=-D \frac{\partial}{\partial \psi^{(1)}} Lim_{\bar{x}^{(2)} \to \bar{x}^{(1)}} \frac{\partial^2}{\partial x_\beta^{(2)} \partial x_\beta^{(2)}} \iiiint \psi^{(2)} F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} \qquad \text{---------} (3.6.13)$$

Now, we reduce the third term of right hand side of equation (3.6.2)

$$\left\langle \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(1)}}\int\left[\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}}-\frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}}\right]\frac{d\bar{x}'}{\left|\bar{x}'-\bar{x}\right|}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle$$

$$=\frac{\partial}{\partial v_\alpha^{(1)}}\frac{1}{4\pi}\iiiint\!\!\int\frac{\partial}{\partial x_\alpha^{(1)}}\left(\frac{1}{\left|\bar{x}^{(2)}-\bar{x}^{(1)}\right|}\right)\left(\frac{\partial v_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial v_\beta^{(2)}}{\partial x_\alpha^{(2)}}-\frac{\partial g_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial g_\beta^{(2)}}{\partial x_\alpha^{(2)}}\right)F_2^{(1,2)}dx^{(2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

------(3.6.14)

Fifth term of right hand side of equation (3.6.2)

$$\left\langle \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)2\in_{m\alpha\beta}\Omega_m u_\alpha^{(1)}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle$$

$$=\left\langle 2\in_{m\alpha\beta}\Omega_m u_\alpha^{(1)}\frac{\partial}{\partial v_\alpha^{(1)}}\left[\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\right]\right\rangle$$

$$=2\in_{m\alpha\beta}\Omega_m\frac{\partial}{\partial v_\alpha^{(1)}}\left\langle u_\alpha^{(1)}\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\right\rangle$$

$$=2\in_{m\alpha\beta}\Omega_m\frac{\partial u_\alpha^{(1)}}{\partial v_\alpha^{(1)}}\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\right\rangle$$

$$=2\in_{m\alpha\beta}\Omega_m F_1^{(1)}$$

----------- (3.6.15)

And, the last term of the equation (3.6.2) reduces to

$$\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)RC^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(C^{(1)}-\psi^{(1)}\right)\right\rangle$$

$$=-R\psi^{(1)}\frac{\partial}{\partial\psi^{(1)}}F_1^{(1)}$$

-----------(3.6.16)

Substituting the results (3.6.3) – (3.6.16) in equation (3.6.2) we get the transport equation for one point distribution function $F_1^{(1)}(v,g,\phi,\psi)$ in MHD turbulence for concentration undergoing a first order reaction in a rotating system in presence of dust particles as

$$\frac{\partial F_1^{(1)}}{\partial t}+v_\beta^{(1)}\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}+g_\beta^{(1)}\left(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}\right)\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}-\frac{\partial}{\partial v_\alpha^{(1)}}\left[\frac{1}{4\pi}\int\!\!\int\!\!\int\!\!\int\!\!\int\frac{\partial}{\partial x_\alpha^{(1)}}\left(\frac{1}{\left|\bar{x}^{(2)}-\bar{x}^{(1)}\right|}\right)\right.$$

$$\left.\times\left(\frac{\partial v_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial v_\beta^{(2)}}{\partial x_\alpha^{(2)}}-\frac{\partial g_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial g_\beta^{(2)}}{\partial x_\alpha^{(2)}}\right)F_2^{(1,2)}dx^{(2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}\right]$$

$$+\nu\frac{\partial}{\partial v_\alpha^{(1)}}Lim_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\int\!\!\int\!\!\int\!\!\int v_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+\lambda\frac{\partial}{\partial g_\alpha^{(1)}}Lim_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\int\!\!\int\!\!\int\!\!\int g_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+\gamma\frac{\partial}{\partial \phi^{(1)}}Lim_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\int\!\!\int\!\!\int\!\!\int \phi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+D\frac{\partial}{\partial \psi^{(1)}}Lim_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\int\!\!\int\!\!\int\!\!\int \psi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+2\in_{m\alpha\beta}\Omega_m F_1^{(1)}-R\psi^{(1)}\frac{\partial}{\partial\psi^{(1)}}F_1^{(1)}=0 \qquad \text{-----------} (3.6.17)$$

Similarly, an equation for two-point distribution function $F_2^{(1,2)}$ in MHD turbulence for concentration undergoing a first order reaction in a rotating system can be derived by differentiating equation (3.4.2) and using equations (3.2.2), (3.2.3),(3.2.4),(3.2.8) and simplifying in the same manner, which is

$$\frac{\partial F_2^{(1,2)}}{\partial t}+\left(v_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}+v_\beta^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}\right)F_2^{(1,2)}+g_\beta^{(1)}\left(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}\right)\frac{\partial}{\partial x_\beta^{(1)}}F_2^{(1,2)}$$

$$+g_\beta^{(2)}\left(\frac{\partial g_\alpha^{(2)}}{\partial v_\alpha^{(2)}}+\frac{\partial v_\alpha^{(2)}}{\partial g_\alpha^{(2)}}\right)\frac{\partial}{\partial x_\beta^{(2)}}F_2^{(1,3)}-\frac{\partial}{\partial v_\alpha^{(1)}}\left[\frac{1}{4\pi}\int\!\!\int\!\!\int\!\!\int\!\!\int\frac{\partial}{\partial x_\alpha^{(1)}}\left(\frac{1}{\left|\bar{x}^{(3)}-\bar{x}^{(1)}\right|}\right)\right.$$

$$\left.\times\left(\frac{\partial v_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial v_\beta^{(3)}}{\partial x_\alpha^{(3)}}-\frac{\partial g_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial g_\beta^{(3)}}{\partial x_\alpha^{(3)}}\right)F_3^{(1,2,3)}dx^{(3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{3)}\right]$$

$$-\frac{\partial}{\partial v_\alpha^{(2)}}\left[\frac{1}{4\pi}\int\!\!\int\!\!\int\!\!\int\!\!\int\frac{\partial}{\partial x_\alpha^{(2)}}\left(\frac{1}{\left|\bar{x}^{(3)}-\bar{x}^{(2)}\right|}\right)\left(\frac{\partial v_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial v_\beta^{(3)}}{\partial x_\alpha^{(3)}}-\frac{\partial g_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial g_\beta^{(3)}}{\partial x_\alpha^{(3)}}\right)\right.$$

$$\left.\times F_3^{(1,2,3)}dx^{(3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{3)}\right]$$

$$+\nu\left(\frac{\partial}{\partial v_\alpha^{(1)}}Lim_{\bar{x}^{(3)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial v_\alpha^{(2)}}Lim_{\bar{x}^{(3)}\to\bar{x}^{(2)}}\right)\frac{\partial^2}{\partial x_\beta^{(3)}\partial x_\beta^{(3)}}\iiiint v_\alpha^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)}$$

$$+\lambda\left(\frac{\partial}{\partial g_\alpha^{(1)}}Lim_{\bar{x}^{(3)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial g_\alpha^{(2)}}Lim_{\bar{x}^{(3)}\to\bar{x}^{(2)}}\right)\frac{\partial^2}{\partial x_\beta^{(3)}\partial x_\beta^{(3)}}\iiiint g_\alpha^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)}$$

$$+\gamma\left(\frac{\partial}{\partial \phi^{(1)}}Lim_{\bar{x}^{(3)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial \phi^{(2)}}Lim_{\bar{x}^{(3)}\to\bar{x}^{(2)}}\right)\frac{\partial^2}{\partial x_\beta^{(3)}\partial x_\beta^{(3)}}\iiiint \phi^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)}$$

$$+D\left(\frac{\partial}{\partial \psi^{(1)}}Lim_{\bar{x}^{(3)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial \psi^{(2)}}Lim_{\bar{x}^{(3)}\to\bar{x}^{(2)}}\right)\frac{\partial^2}{\partial x_\beta^{(3)}\partial x_\beta^{(3)}}\iiiint \psi^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)}$$

$$+2\in_{m\alpha\beta}\Omega_m F_2^{(1,2)}-R\psi^{(1)}\frac{\partial}{\partial\psi^{(1)}}F_1^{(1)}=0 \qquad \text{---------} (3.6.18)$$

Following this way, we can derive the equations for evolution of $F_3^{(1,2,3,)}$, $F_4^{(1,2,3,4)}$ and so on. Logically, it is possible to have an equation for every F_n (n is an integer) but the system of equations so obtained is not closed. It seems that certain approximations will be required thus obtained.

3.7. Results and Discussion:

If the system is non rotating then $\Omega m=0$, the transport equation for one point distribution function in MHD turbulent flow (3.6.17) becomes

$$\frac{\partial F_1^{(1)}}{\partial t}+v_\beta^{(1)}\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}+g_\beta^{(1)}\left(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}\right)\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}-\frac{\partial}{\partial v_\alpha^{(1)}}\left[\frac{1}{4\pi}\iiiint\int\left(\frac{\partial}{\partial x_\alpha^{(1)}}\frac{1}{\left|\bar{x}^{(2)}-\bar{x}^{(1)}\right|}\right)\right.$$

$$\left.\times\left(\frac{\partial v_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial v_\beta^{(2)}}{\partial x_\alpha^{(2)}}-\frac{\partial g_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial g_\beta^{(2)}}{\partial x_\alpha^{(2)}}\right)F_2^{(1,2)}dx^{(2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}\right]$$

$$+\nu\frac{\partial}{\partial v_\alpha^{(1)}}Lim_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiiint v_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+\lambda \frac{\partial}{\partial g_{\alpha}^{(1)}} Lim_{\bar{x}^{(2)} \to \bar{x}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(2)} \partial x_{\beta}^{(2)}} \iiiint g_{\alpha}^{(2)} F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)}$$

$$+\gamma \frac{\partial}{\partial \phi^{(1)}} Lim_{\bar{x}^{(2)} \to \bar{x}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(2)} \partial x_{\beta}^{(2)}} \iiiint \phi^{(2)} F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)}$$

$$+D \frac{\partial}{\partial \psi^{(1)}} Lim_{\bar{x}^{(2)} \to \bar{x}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(2)} \partial x_{\beta}^{(2)}} \iiiint \psi^{(2)} F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)}$$

$$- R\psi^{(1)} \frac{\partial}{\partial \psi^{(1)}} F_1^{(1)} = 0 \qquad \text{-------(3.7.1)}$$

which was obtained earlier by Islam and Sarker [57].

We can exhibit an analogy of this equation with the first equation in BBGKY hierarchy in the kinetic theory of gases. The first equation of BBGKY hierarchy is given as

$$\frac{\partial F_1^{(1)}}{\partial t} + \frac{1}{m} v_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} F_1^{(1)} = n \iint \frac{\partial \psi_{1,2}}{\partial x_{\alpha}^{(1)}} \frac{\partial F_2^{(1,2)}}{\partial v_{\alpha}^{(1)}} d\bar{x}^{(2)} d\bar{v}^{(2)} \qquad \text{------------ (3.7.2)}$$

where $\psi_{1,2} = \psi \left| v_{\alpha}^{(2)} - v_{\alpha}^{(1)} \right|$ is the inter molecular potential.

If we drop the viscous, magnetic and thermal diffusive, concentration terms and constant reaction terms from the one point evolution equation (3.7.2), we have

$$\frac{\partial F_1^{(1)}}{\partial t} + v_{\beta}^{(1)} \frac{\partial F_1^{(1)}}{\partial x_{\beta}^{(1)}} + g_{\beta}^{(1)} \left(\frac{\partial g_{\alpha}^{(1)}}{\partial v_{\alpha}^{(1)}} + \frac{\partial v_{\alpha}^{(1)}}{\partial g_{\alpha}^{(1)}} \right) \frac{\partial F_1^{(1)}}{\partial x_{\beta}^{(1)}} - \frac{\partial}{\partial v_{\alpha}^{(1)}} \left[\frac{1}{4\pi} \iiiint\int \frac{\partial}{\partial x_{\alpha}^{(1)}} \left(\frac{1}{\left| \bar{x}^{(2)} - \bar{x}^{(1)} \right|} \right) \right.$$

$$\left. \times \left(\frac{\partial v_{\alpha}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial v_{\beta}^{(2)}}{\partial x_{\alpha}^{(2)}} - \frac{\partial g_{\alpha}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial g_{\beta}^{(2)}}{\partial x_{\alpha}^{(2)}} \right) F_2^{(1,2)} dx^{(2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} \right] = 0 \ . \qquad \text{---------- (3.7.3)}$$

The existence of the term $\frac{\partial g_{\alpha}^{(1)}}{\partial v_{\alpha}^{(1)}} + \frac{\partial v_{\alpha}^{(1)}}{\partial g_{\alpha}^{(1)}}$

can be explained on the basis that two characteristics of the flow field are related to each other and describe the interaction between the two modes (velocity and magnetic) at a single point $x^{(1)}$.

The transport equation for distribution function of velocity, magnetic, temperature, concentration and reaction have been shown here to provide an advantageous basis for modeling the turbulent flows in a rotating system. Here we have made an attempt for the modeling of various terms such as fluctuating pressure, viscosity and diffusivity in order to close the equation for distribution function of velocity, magnetic, temperature, concentration and reaction. It is also possible to construct such type of distribution functions in variable density follows. The advantage of constructing such type hierarchy is to provide simultaneous information about velocity, magnetic temperature, concentration and reaction without knowledge of scale of turbulence.

CHAPTER-III

PART-B

STATISTICAL THEORY OF CERTAIN DISTRIBUTION FUNCTIONS IN MHD TURBULENT FLOW UNDERGOING A FIRST ORDER REACTION IN PRESENCE OF DUST PARTICLES

3.8. Introduction:

In this paper, we have studied the distribution function for simultaneous velocity, magnetic, temperature, concentration fields and reaction in MHD turbulence in presence of dust particles. Finally, the transport equations for evolution of distribution functions have been derived and various properties of the distribution function have been discussed. The obtained one-point equation is compared with the first equation of BBGKY hierarchy of equations in the kinetic theory of gases.

3.9. Basic Equations:

The equations of motion and continuity for viscous incompressible dusty fluid MHD turbulent flow, the diffusion equations for the temperature and concentration undergoing a first order chemical reaction are given by

$$\frac{\partial u_\alpha}{\partial t}+\frac{\partial}{\partial x_\beta}\left(u_\alpha u_\beta - h_\alpha h_\beta\right) = -\frac{\partial w}{\partial x_\alpha}+\nu\nabla^2 u_\alpha + f\left(u_\alpha - v_\alpha\right) \qquad \text{---------} (3.9.1)$$

$$\frac{\partial h_\alpha}{\partial t}+\frac{\partial}{\partial x_\beta}\left(h_\alpha u_\beta - u_\alpha h_\beta\right) = \lambda\nabla^2 h_\alpha \qquad \text{---------} (3.9.2)$$

$$\frac{\partial \theta}{\partial t}+u_\beta\frac{\partial \theta}{\partial x_\beta} = \gamma\nabla^2\theta \qquad \text{---------} (3.9.3)$$

$$\frac{\partial C}{\partial t}+u_\beta\frac{\partial C}{\partial x_\beta} = D\nabla^2 C - RC \qquad \text{---------} (3.9.4)$$

with $\dfrac{\partial u_\alpha}{\partial x_\alpha} = \dfrac{\partial v_\alpha}{\partial x_\alpha} = \dfrac{\partial h_\alpha}{\partial x_\alpha} = 0$, ---------- (3.9.5)

where

$w(\hat{x},t) = P/\rho + \frac{1}{2}\left|\vec{h}\right|^2$ total pressure and D is the diffusive co-efficient for contaminants.

The repeated suffices are assumed over the values 1, 2 and 3 and unrepeated suffices may take any of these values. Here u, h and x are vector quantities in the whole process.

The total pressure w which, occurs in equation (3.9.1) may be eliminated with the help of the equation obtained by taking the divergence of equation (3.9.1)

$$\nabla^2 w = -\frac{\partial^2}{\partial x_\alpha \partial x_\beta}\left(u_\alpha u_\beta - h_\alpha h_\beta\right) = -\left[\ \frac{\partial u_\alpha}{\partial x_\beta}\frac{\partial u_\beta}{\partial x_\alpha} - \frac{\partial h_\alpha}{\partial x_\beta}\frac{\partial h_\beta}{\partial x_\alpha}\ \right]. \quad \text{--------- (3.9.6)}$$

In a conducting infinite fluid only the particular solution of the Equation (3.9.6) is related, so that

$$w = \frac{1}{4\pi}\int\left[\ \frac{\partial u'_\alpha}{\partial x'_\beta}\frac{\partial u'_\beta}{\partial x'_\alpha} - \frac{\partial h'_\alpha}{\partial x'_\beta}\frac{\partial h'_\beta}{\partial x'_\alpha}\ \right]\frac{\partial \bar{x}'}{\left|\bar{x}' - \bar{x}\right|}. \quad \text{---------- (3.9.7)}$$

Hence equation (3.9.1) – (3.9.4) becomes

$$\frac{\partial u_\alpha}{\partial t} + \frac{\partial}{\partial x_\beta}\left(u_\alpha u_\beta - h_\alpha h_\beta\right) = -\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha}\int\left[\ \frac{\partial u'_\alpha}{\partial x'_\beta}\frac{\partial u'_\beta}{\partial x'_\alpha} - \frac{\partial h'_\alpha}{\partial x'_\beta}\frac{\partial h'_\beta}{\partial x'_\alpha}\ \right]\frac{d\bar{x}'}{\left|\bar{x}' - \bar{x}\right|} + \nu\nabla^2 u_\alpha + f\left(u_\alpha - v_\alpha\right)$$

------------- (3.9.8)

$$\frac{\partial h_\alpha}{\partial t} + \frac{\partial}{\partial x_\beta}\left(h_\alpha u_\beta - u_\alpha h_\beta\right) = \lambda\nabla^2 h_\alpha \quad \text{------------- (3.9.9)}$$

$$\frac{\partial \theta}{\partial t} + u_\beta\frac{\partial \theta}{\partial x_\beta} = \gamma\nabla^2\theta \quad \text{----------- (3.9.10)}$$

$$\frac{\partial C}{\partial t} + u_\beta\frac{\partial C}{\partial x_\beta} = D\nabla^2 C - RC \quad \text{----------- (3.9.11)}$$

3.10. Continuity Equation in Terms of Distribution Functions:

An infinite number of continuity equations can be derived for the convective MHD turbulent flow and the continuity equations can be easily expressed in terms of distribution functions and are obtained directly by div $u = 0$. Taking ensemble average of equation (3.9.5)

$$0 = \left\langle \frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}} \right\rangle = \left\langle \frac{\partial}{\partial x_\alpha^{(1)}} u_\alpha^{(1)} \iiiint F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \right\rangle$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \left\langle u_\alpha^{(1)} \iiiint F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \right\rangle$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \iiiint \langle\ u_\alpha^{(1)}\ \rangle \langle\ F_1^{(1)}\ \rangle dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)}$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \iiiint v_\alpha^{(1)} F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)}$$

$$= \iiiint \frac{\partial F_1^{(1)}}{\partial x_\alpha^{(1)}} v_\alpha^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \quad \text{------------ (3.10.1)}$$

and similarly,

$$0 = \iiiint \frac{\partial F_1^{(1)}}{\partial x_\alpha^{(1)}} g_\alpha^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \quad \text{------------ (3.10.2)}$$

which are the first order continuity equations in which only one point distribution function is involved. For second-order continuity equations, if we multiply the continuity equation by

$$\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(C^{(2)} - \psi^{(2)}\right)$$

and if we take the ensemble average, we obtain

$$0 = \left\langle\ \delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(C^{(2)} - \psi^{(2)}\right)\frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}}\ \right\rangle$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \left\langle\ \delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(C^{(2)} - \psi^{(2)}\right)u_\alpha^{(1)}\ \right\rangle$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \int \langle\ u_\alpha^{(1)} \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)$$

$$\times \delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(C^{(2)}-\psi^{(2)}\right)\rangle$$

$$0=\frac{\partial}{\partial x_{\alpha}^{(1)}}\iiiint v_{\alpha}^{(1)}F_{2}^{(1,2)}dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)} \qquad \text{----------} (3.10.3)$$

and similarly, $$0=\frac{\partial}{\partial x_{\alpha}^{(1)}}\iiiint g_{\alpha}^{(1)}F_{2}^{(1,2)}dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)}. \qquad \text{-----------} (3.10.4)$$

The Nth – order continuity equations are

$$0=\frac{\partial}{\partial x_{\alpha}^{(1)}}\iiiint v_{\alpha}^{(1)}F_{N}^{(1,2,---,N)}dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)} \qquad \text{----------} (3.10.5)$$

and $$0=\frac{\partial}{\partial x_{\alpha}^{(1)}}\iiiint g_{\alpha}^{(1)}F_{N}^{(1,2,.........,N)}dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)}. \qquad \text{-----------} (3.10.6)$$

The continuity equations are symmetric in their arguments i.e.;

$$\frac{\partial}{\partial x_{\alpha}^{(r)}}\iiiint\left(v_{\alpha}^{(r)}F_{N}^{(1,2,.........;r.N)}dv^{(r)}dg^{(r)}d\phi^{(r)}d\psi^{(r)}\right)$$
$$=\frac{\partial}{\partial x_{\alpha}^{(s)}}\iiiint v_{\alpha}^{(s)}F_{N}^{(1,2,....r,s,....N)}dv^{(s)}dg^{(s)}d\phi^{(s)}d\psi^{(s)} \qquad \text{------------} (3.10.7)$$

Since the divergence property is an important property and it is easily verified by the use of the property of distribution function as

$$\frac{\partial}{\partial x_{\alpha}^{(1)}}\iiiint v_{\alpha}^{(1)}F_{1}^{(1)}dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)}=\frac{\partial}{\partial x_{\alpha}^{(1)}}\langle u_{\alpha}^{(1)}\rangle=\langle\frac{\partial u_{\alpha}^{(1)}}{\partial x_{\alpha}(1)}\rangle=0 \qquad \text{----------}(3.10.8)$$

3.11. Equations for Evolution of Distribution Functions:

The equations (3.9.8)-(3.9.11) will be used to convert these into a set of equations for the variation of the distribution function with time. This, in fact, is done by making use of the definitions of the constructed distribution functions, differentiating them partially with respect to time, making some suitable operations on the right-hand side of the equation so obtained and lastly replacing the time derivative of v,h,θ and C from the equations (3.9.8)-(3.9.11).

Using the equations (3.9.8)-(3.9.11) we get,

$$\frac{\partial F_1^{(1)}}{\partial t}=\frac{\partial}{\partial t}\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\right\rangle$$

$$=\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial}{\partial t}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)$$

$$\times\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial}{\partial t}\delta\left(h^{(1)}-g^{(1)}\right)\ \rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\frac{\partial}{\partial t}\delta\left(C^{(1)}-\psi^{(1)}\right)\ \rangle$$

$$=\langle-\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial u^{(1)}}{\partial t}\frac{\partial}{\partial v^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial h^{(1)}}{\partial t}\frac{\partial}{\partial g^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\ \rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial \theta^{(1)}}{\partial t}\frac{\partial}{\partial \phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\ \rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\frac{\partial C}{\partial t}\frac{\partial}{\partial \psi^{(1)}}\delta\left(C^{(1)}-\psi^{(1)}\right)\ \rangle. \text{------ -- (3.11.1)}$$

Using equations (3.9.8) – (3.9.11) in the equation (3.11.1), we get

$$\frac{\partial F_1^{(1)}}{\partial t}=\langle-\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\left\{-\frac{\partial}{\partial x_\beta}\left(u_\alpha^{(1)}u_\beta^{(1)}-h_\alpha^{(1)}h_\beta^{(1)}\right)\right.$$

$$\left.-\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha}\iint\left[\ \frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}}-\frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}}\ \right]\frac{d\bar{x}'}{\left|\bar{x}'-\bar{x}\right|}+\nu\nabla^2u_\alpha^{(1)}+f\left(u_\alpha^{(1)}-v_\alpha^{(1)}\right)\right\}$$

$$\times\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\left\{-\frac{\partial}{\partial x_\beta^{(1)}}\left(h_\alpha^{(1)}u_\beta^{(1)}-u_\alpha^{(1)}h_\beta^{(1)}\right)\right.$$

$$\left.+\lambda\nabla^2h_\alpha^{(1)}\right\}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\ \rangle+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)$$

$$\times\left\{-u_\beta^{(1)}\frac{\partial\theta^{(1)}}{\partial x_\beta}+\gamma\nabla^2\theta^{(1)}\right\}\times\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\ \rangle+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)$$

$$\times\delta\left(\theta^{(1)}-\phi^{(1)}\right)\left\{-u_{\beta}^{(1)}\frac{\partial c^{(1)}}{\partial x_{\beta}^{(1)}}+D\nabla^{2}C\right\}\frac{\partial}{\partial\psi^{(1)}}\delta\left(C^{(1)}-\psi^{(1)}\right)\rangle$$

$$=\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial u_{\alpha}^{(1)}u_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle-\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial h_{\alpha}^{(1)}h_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial v_{\alpha}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{1}{4\pi}\frac{\partial}{\partial x_{\alpha}^{(1)}}\int\left[\frac{\partial u_{\alpha}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial u_{\beta}^{(1)}}{\partial x_{\alpha}^{(1)}}-\frac{\partial h_{\alpha}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial h_{\beta}^{(1)}}{\partial x_{\alpha}^{(1)}}\right]$$

$$\times\frac{d\bar{x}'}{\left|\bar{x}'-\bar{x}\right|}\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle+\langle-\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\nu\nabla^{2}u_{\alpha}^{(1)}\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle-\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)f\left(u_{\alpha}{}^{(1)}-v_{\alpha}{}^{(1)}\right)\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial h_{\alpha}^{(1)}u_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial g_{\alpha}^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial u_{\alpha}^{(1)}h_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial g_{\alpha}}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\lambda\nabla^{2}h_{\alpha}^{(1)}\frac{\partial}{\partial g_{\alpha}^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)u_{\beta}^{(1)}\frac{\partial\theta^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\gamma\nabla^{2}\theta^{(1)}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)u_{\beta}^{(1)}\frac{\partial C^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial\psi^{(1)}}\delta\left(C^{(1)}-\psi^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)D\nabla^{2}C^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(C^{(1)}-\psi^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)RC^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(C^{(1)}-\psi^{(1)}\right)\rangle \qquad \text{-----------} (3.11.2)$$

Various terms in the equation (3.11.2) can be simplified as the equations (3.6.3) – (3.6.14) and the Sixth term of right hand side of equation (3.11.2)

$$\langle -\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)f\left(\ u_{\alpha}^{(1)}-v_{\alpha}^{(1)}\ \right)\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$=-\langle\ f\left(u_{\alpha}^{(1)}-v_{\alpha}^{(1)}\right)\frac{\partial}{\partial v_{\alpha}^{(1)}}\left[\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\right]\ \rangle$$

$$=-f\left(\ u_{\alpha}^{(1)}-v_{\alpha}^{(1)}\ \right)\frac{\partial}{\partial v_{\alpha}^{(1)}}\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\ \rangle$$

$$=-f\left(\ u_{\alpha}^{(1)}-v_{\alpha}^{(1)}\ \right)\frac{\partial}{\partial v_{\alpha}^{(1)}}F_{1}^{(1)} \qquad \text{------------ (3.11.3)}$$

And, the last term of the equation (3.11.2) reduces to

$$\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)RC^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(C^{(1)}-\psi^{(1)}\right)\ \rangle$$

$$=-R\psi^{(1)}\frac{\partial}{\partial\psi^{(1)}}F_{1}^{(1)} \qquad \text{-----------(3.11.4)}$$

Substituting the results (3.6.3) – (3.6.14) and (3.11.3) – (3.11.4) in equation (3.11.2) we get the transport equation for one point distribution function $F_1^{(1)}(v,g,\phi,\psi)$ in MHD turbulence for concentration undergoing a first order reaction in a rotating system in presence of dust particles as

$$\frac{\partial F_{1}^{(1)}}{\partial t}+v_{\beta}^{(1)}\frac{\partial F_{1}^{(1)}}{\partial x_{\beta}^{(1)}}+g_{\beta}^{(1)}\left(\frac{\partial g_{\alpha}^{(1)}}{\partial v_{\alpha}^{(1)}}+\frac{\partial v_{\alpha}^{(1)}}{\partial g_{\alpha}^{(1)}}\right)\frac{\partial F_{1}^{(1)}}{\partial x_{\beta}^{(1)}}-\frac{\partial}{\partial v_{\alpha}^{(1)}}\left[\frac{1}{4\pi}\int\iiiint\frac{\partial}{\partial x_{\alpha}^{(1)}}\left(\frac{1}{\left|\overline{x}^{(2)}-\overline{x}^{(1)}\right|}\right)\right.$$

$$\left.\times\left(\frac{\partial v_{\alpha}^{(2)}}{\partial x_{\beta}^{(2)}}\frac{\partial v_{\beta}^{(2)}}{\partial x_{\alpha}^{(2)}}-\frac{\partial g_{\alpha}^{(2)}}{\partial x_{\beta}^{(2)}}\frac{\partial g_{\beta}^{(2)}}{\partial x_{\alpha}^{(2)}}\right)F_{2}^{(1,2)}dx^{(2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}\right]$$

$$+v\frac{\partial}{\partial v_{\alpha}^{(1)}}\underset{\overline{x}^{(2)}\to\overline{x}^{(1)}}{Lim}\frac{\partial^{2}}{\partial x_{\beta}^{(2)}\partial x_{\beta}^{(2)}}\iiiint v_{\alpha}^{(2)}F_{2}^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+\lambda\frac{\partial}{\partial g_{\alpha}^{(1)}}\underset{\overline{x}^{(2)}\to\overline{x}^{(1)}}{Lim}\frac{\partial^{2}}{\partial x_{\beta}^{(2)}\partial x_{\beta}^{(2)}}\iiiint g_{\alpha}^{(2)}F_{2}^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+\gamma\frac{\partial}{\partial\phi^{(1)}}Lim_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiiint\phi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+D\frac{\partial}{\partial\psi^{(1)}}Lim_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiiint\psi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+f\left(u_\alpha^{(1)}-v_\alpha^{(1)}\right)\frac{\partial}{\partial v_\alpha^{(1)}}F_1^{(1)}-R\psi^{(1)}\frac{\partial}{\partial\psi^{(1)}}F_1^{(1)}=0 \qquad \text{-----------} (3.11.5)$$

Similarly, an equation for two-point distribution function $F_2^{(1,2)}$ in MHD dusty fluid turbulence for concentration undergoing a first order reaction can be derived by using equations (3.9.2), (3.9.3),(3.9.4),(3.9.8) and simplifying in the same manner, which is

$$\frac{\partial F_2^{(1,2)}}{\partial t}+\left(v_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}+v_\beta^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}\right)F_2^{(1,2)}+g_\beta^{(1)}\left(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}\right)\frac{\partial}{\partial x_\beta^{(1)}}F_2^{(1,2)}$$

$$+g_\beta^{(2)}\left(\frac{\partial g_\alpha^{(2)}}{\partial v_\alpha^{(2)}}+\frac{\partial v_\alpha^{(2)}}{\partial g_\alpha^{(2)}}\right)\frac{\partial}{\partial x_\beta^{(2)}}F_2^{(1,3)}-\frac{\partial}{\partial v_\alpha^{(1)}}\left[\frac{1}{4\pi}\iiiiint\frac{\partial}{\partial x_\alpha^{(1)}}\left(\frac{1}{\left|\bar{x}^{(3)}-\bar{x}^{(1)}\right|}\right)\right.$$

$$\left.\times\left(\frac{\partial v_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial v_\beta^{(3)}}{\partial x_\alpha^{(3)}}-\frac{\partial g_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial g_\beta^{(3)}}{\partial x_\alpha^{(3)}}\right)F_3^{(1,2,3)}dx^{(3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{3)}\right]$$

$$-\frac{\partial}{\partial v_\alpha^{(2)}}\left[\frac{1}{4\pi}\iiiiint\frac{\partial}{\partial x_\alpha^{(2)}}\left(\frac{1}{\left|\bar{x}^{(3)}-\bar{x}^{(2)}\right|}\right)\left(\frac{\partial v_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial v_\beta^{(3)}}{\partial x_\alpha^{(3)}}-\frac{\partial g_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial g_\beta^{(3)}}{\partial x_\alpha^{(3)}}\right)\right.$$

$$\left.\times F_3^{(1,2,3)}dx^{(3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{3)}\right]$$

$$+\nu\left(\frac{\partial}{\partial v_\alpha^{(1)}}Lim_{\bar{x}^{(3)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial v_\alpha^{(2)}}Lim_{\bar{x}^{(3)}\to\bar{x}^{(2)}}\right)\frac{\partial^2}{\partial x_\beta^{(3)}\partial x_\beta^{(3)}}\iiiint v_\alpha^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)}$$

$$+\lambda\left(\frac{\partial}{\partial g_\alpha^{(1)}}Lim_{\bar{x}^{(3)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial g_\alpha^{(2)}}Lim_{\bar{x}^{(3)}\to\bar{x}^{(2)}}\right)\frac{\partial^2}{\partial x_\beta^{(3)}\partial x_\beta^{(3)}}\iiiint g_\alpha^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)}$$

$$+\gamma\left(\frac{\partial}{\partial\phi^{(1)}}Lim_{\bar{x}^{(3)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial\phi^{(2)}}Lim_{\bar{x}^{(3)}\to\bar{x}^{(2)}}\right)\frac{\partial^2}{\partial x_\beta^{(3)}\partial x_\beta^{(3)}}\iiiint\phi^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)}$$

$$+D\left(\frac{\partial}{\partial\psi^{(1)}}Lim_{\bar{x}^{(3)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial\psi^{(2)}}Lim_{\bar{x}^{(3)}\to\bar{x}^{(2)}}\right)\frac{\partial^2}{\partial x_\beta^{(3)}\partial x_\beta^{(3)}}\iiiint\psi^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)}$$

$$+f\left(u_\alpha^{(1)}-v_\alpha^{(1)}\right)\frac{\partial}{\partial v_\alpha^{(2)}}F_2^{(1,2)}-R\psi^{(1)}\frac{\partial}{\partial\psi^{(1)}}F_1^{(1)}=0 \qquad \text{---------} (3.11.6)$$

Continuing this way, we can derive the equations for evolution of $F_3^{(1,2,3,)}$, $F_4^{(1,2,3,4)}$ and so on. Logically, it is possible to have an equation for every F_n (n is an integer) but the system of equations so obtained is not closed. It seems that certain approximations will be required thus obtained.

3.12. Results and Discussion:

If the fluid is clean then f=0 , the transport equation for one point distribution function in MHD turbulent flow (3.11.5) becomes

$$\frac{\partial F_1^{(1)}}{\partial t}+v_\beta^{(1)}\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}+g_\beta^{(1)}\left(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}\right)\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}-\frac{\partial}{\partial v_\alpha^{(1)}}\left[\frac{1}{4\pi}\iiiint\!\!\int\left(\frac{\partial}{\partial x_\alpha^{(1)}}\frac{1}{\left|\bar{x}^{(2)}-\bar{x}^{(1)}\right|}\right)\right.$$

$$\left.\times\left(\frac{\partial v_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial v_\beta^{(2)}}{\partial x_\alpha^{(2)}}-\frac{\partial g_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial g_\beta^{(2)}}{\partial x_\alpha^{(2)}}\right)F_2^{(1,2)}dx^{(2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}\right]$$

$$+\nu\frac{\partial}{\partial v_\alpha^{(1)}}Lim_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiiint v_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+\lambda\frac{\partial}{\partial g_\alpha^{(1)}}Lim_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiiint g_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+\gamma\frac{\partial}{\partial\phi^{(1)}}Lim_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiiint\phi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+D\frac{\partial}{\partial\psi^{(1)}}Lim_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiiint\psi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}-R\psi^{(1)}\frac{\partial}{\partial\psi^{(1)}}F_1^{(1)}=0$$

------------ (3.12.1)

which was obtained earlier by Islam and Sarker [57].

We can exhibit an analogy of this equation with the first equation in BBGKY hierarchy in the kinetic theory of gases. The first equation of BBGKY hierarchy is given as

$$\frac{\partial F_1^{(1)}}{\partial t}+\frac{1}{m}v_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}F_1^{(1)}=n\iint\frac{\partial\psi_{1,2}}{\partial x_\alpha^{(1)}}\frac{\partial F_2^{(1,2)}}{\partial v_\alpha^{(1)}}d\overline{x}^{(2)}d\overline{v}^{(2)} \qquad \text{------------ (3.12.2)}$$

where $\psi_{1,2}=\psi\left|v_\alpha^{(2)}-v_\alpha^{(1)}\right|$ is the inter molecular potential.

If we drop the viscous, magnetic and thermal diffusive, concentration terms and constant reaction terms from the one point evolution equation (3.12.1), we have

$$\frac{\partial F_1^{(1)}}{\partial t}+v_\beta^{(1)}\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}+g_\beta^{(1)}\left(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}\right)\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}-\frac{\partial}{\partial v_\alpha^{(1)}}\left[\frac{1}{4\pi}\iiiint\!\!\int\frac{\partial}{\partial x_\alpha^{(1)}}\left(\frac{1}{\left|\overline{x}^{(2)}-\overline{x}^{(1)}\right|}\right)\right.$$

$$\left.\times\left(\frac{\partial v_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial v_\beta^{(2)}}{\partial x_\alpha^{(2)}}-\frac{\partial g_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial g_\beta^{(2)}}{\partial x_\alpha^{(2)}}\right)F_2^{(1,2)}dx^{(2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}\right]=0 \qquad \text{---------- (3.12.3)}$$

The existence of the term $\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}$

can be explained on the basis that two characteristics of the flow field are related to each other and describe the interaction between the two modes (velocity and magnetic) at a single point $x^{(1)}$.

In order to close the system of equations for the distribution functions, some approximations are required. If we consider the collection of ionized particles, i.e. in plasma turbulence case, it can be provided closure form easily by decomposing $F_2^{(1,2)}$ as $F_1^{(1)}\,F_1^{(2)}$. But such type of approximations can be possible if there is no interaction or correlation between two particles. If we decompose $F_2^{(1,2)}$ as

$$F_2^{(1,2)}=(1+\epsilon)\,F_1^{(1)}\,F_1^{(2)} \quad \text{And} \quad F_3^{(1,2,3)}=(1+\epsilon)^2\,F_1^{(1)}\,F_1^{(2)}\,F_1^{(3)}$$

where, ϵ is the correlation coefficient between the particles. If there is no correlation between the particles, ϵ will be zero and distribution function can be decomposed in usual way. Here we are considering such type of approximation only to provide closed from of the equation i.e., to approximate two-point equation as one point equation.

The transport equation for distribution function of velocity, magnetic, temperature, concentration and reaction have been shown here to provide an advantageous basis for modeling the turbulent flows in presence of dust particles. Here we have made an attempt for the modeling of various terms such as fluctuating pressure, viscosity and diffusivity in order to close the equation for distribution function of velocity, magnetic, temperature, concentration and reaction. It is also possible to construct such type of distribution functions in variable density follows. The advantage of constructing such type hierarchy is to provide simultaneous information about velocity, magnetic temperature, concentration and reaction without knowledge of scale of turbulence.

CHAPTER-III

PART-C

STATISTICAL THEORY OF CERTAIN DISTRIBUTION FUNCTIONS IN MHD TURBULENCE IN A ROTATING SYSTEM UNDERGOING A FIRST ORDER REACTION IN PRESENCE OF DUST PARTICLES

3.13. Introduction:

In this work, we have studied the distribution function in the statistical theory for simultaneous velocity, magnetic, temperature, concentration fields and reaction in MHD turbulence in a rotating system in presence of dust particles. Finally, the transport equations for evolution of distribution functions have been derived and various properties of the distribution function have been discussed. The resulting one-point equation is compared with the first equation of BBGKY hierarchy of equations and the closure difficulty is to be removed as in the case of ordinary turbulence.

3.14. Basic Equations:

The equations of motion and the equation of continuity for viscous incompressible dusty fluid MHD turbulent flow, the diffusion equations for the temperature and concentration undergoing a first order chemical reaction in a rotating system are given by

$$\frac{\partial u_\alpha}{\partial t}+\frac{\partial}{\partial x_\beta}\left(u_\alpha u_\beta - h_\alpha h_\beta\right)=-\frac{\partial w}{\partial x_\alpha}+\nu\nabla^2 u_\alpha - 2\in_{m\alpha\beta}\Omega_m u_\alpha + f\left(u_\alpha - v_\alpha\right) \quad \text{---------} (3.14.1)$$

$$\frac{\partial h_\alpha}{\partial t}+\frac{\partial}{\partial x_\beta}\left(h_\alpha u_\beta - u_\alpha h_\beta\right)=\lambda\nabla^2 h_\alpha \quad \text{---------} (3.14.2)$$

$$\frac{\partial \theta}{\partial t}+u_\beta\frac{\partial \theta}{\partial x_\beta}=\gamma\nabla^2\theta \quad \text{---------} (3.14.3)$$

$$\frac{\partial C}{\partial t}+u_\beta\frac{\partial C}{\partial x_\beta}=D\nabla^2 C - RC \quad \text{---------} (3.14.4)$$

with $\dfrac{\partial u_\alpha}{\partial x_\alpha}=\dfrac{\partial v_\alpha}{\partial x_\alpha}=\dfrac{\partial h_\alpha}{\partial x_\alpha}=0$, ---------- (3.14.5)

where, Ω, angular velocity of a uniform rotation.

The total pressure w which, occurs in equation (3.14.1) may be eliminated with the help of the equation obtained by taking the divergence of equation (3.14.1)

$$\nabla^2 w=-\frac{\partial^2}{\partial x_\alpha \partial x_\beta}\left(u_\alpha u_\beta-h_\alpha h_\beta\right)=-\left[\ \frac{\partial u_\alpha}{\partial x_\beta}\frac{\partial u_\beta}{\partial x_\alpha}-\frac{\partial h_\alpha}{\partial x_\beta}\frac{\partial h_\beta}{\partial x_\alpha}\ \right]. \qquad \text{---------} (3.14.6)$$

In a conducting infinite fluid only the particular solution of the Equation (3.14.6) is related, so that

$$w=\frac{1}{4\pi}\int\left[\ \frac{\partial u'_\alpha}{\partial x'_\beta}\frac{\partial u'_\beta}{\partial x'_\alpha}-\frac{\partial h'_\alpha}{\partial x'_\beta}\frac{\partial h'_\beta}{\partial x'_\alpha}\ \right]\frac{\partial \bar{x}'}{|\bar{x}'-\bar{x}|}. \qquad \text{----------} (3.14.7)$$

Hence equation (3.14.1) – (3.14.4) becomes

$$\frac{\partial u_\alpha}{\partial t}+\frac{\partial}{\partial x_\beta}\left(u_\alpha u_\beta-h_\alpha h_\beta\right)=-\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha}\int\left[\ \frac{\partial u'_\alpha}{\partial x'_\beta}\frac{\partial u'_\beta}{\partial x'_\alpha}-\frac{\partial h'_\alpha}{\partial x'_\beta}\frac{\partial h'_\beta}{\partial x'_\alpha}\ \right]\frac{d\bar{x}'}{|\bar{x}'-\bar{x}|}+\nu\nabla^2 u_\alpha$$

$$-2\in_{m\alpha\beta}\Omega_m u_\alpha+f\left(u_\alpha-v_\alpha\right) \qquad \text{-------------} (3.14.8)$$

$$\frac{\partial h_\alpha}{\partial t}+\frac{\partial}{\partial x_\beta}\left(h_\alpha u_\beta-u_\alpha h_\beta\right)=\lambda\nabla^2 h_\alpha \qquad \text{-------------} (3.14.9)$$

$$\frac{\partial \theta}{\partial t}+u_\beta\frac{\partial \theta}{\partial x_\beta}=\gamma\nabla^2\theta \qquad \text{-----------} (3.14.10)$$

$$\frac{\partial C}{\partial t}+u_\beta\frac{\partial C}{\partial x_\beta}=D\nabla^2 C-RC \qquad \text{-----------} (3.14.11)$$

3.15. Continuity Equation in Terms of Distribution Functions:

An infinite number of continuity equations can be derived for the convective MHD turbulent flow and the continuity equations can be easily expressed in terms of distribution functions and are obtained directly by div $u=0$. Taking ensemble average of equation (3.14.5)

$$0=\left\langle \frac{\partial u_{\alpha}^{(1)}}{\partial x_{\alpha}^{(1)}} \right\rangle = \left\langle \frac{\partial}{\partial x_{\alpha}^{(1)}} u_{\alpha}^{(1)} \iiiint F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \right\rangle$$

$$= \frac{\partial}{\partial x_{\alpha}^{(1)}} \left\langle u_{\alpha}^{(1)} \iiiint F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \right\rangle$$

$$= \frac{\partial}{\partial x_{\alpha}^{(1)}} \iiiint \langle\ u_{\alpha}^{(1)}\ \rangle \langle\ F_1^{(1)}\ \rangle dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)}$$

$$= \frac{\partial}{\partial x_{\alpha}^{(1)}} \iiiint v_{\alpha}^{(1)} F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)}$$

$$= \iiiint \frac{\partial F_1^{(1)}}{\partial x_{\alpha}^{(1)}} v_{\alpha}^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \qquad \text{------------ (3.15.1)}$$

and similarly, $$0 = \iiiint \frac{\partial F_1^{(1)}}{\partial x_{\alpha}^{(1)}} g_{\alpha}^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \qquad \text{------------ (3.15.2)}$$

which are the first order continuity equations in which only one point distribution function is involved. For second-order continuity equations, if we multiply the continuity equation by

$$\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(C^{(2)} - \psi^{(2)}\right)$$

and if we take the ensemble average, we obtain

$$0 = \left\langle\ \delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(C^{(2)} - \psi^{(2)}\right)\frac{\partial u_{\alpha}^{(1)}}{\partial x_{\alpha}^{(1)}}\ \right\rangle$$

$$= \frac{\partial}{\partial x_{\alpha}^{(1)}} \left\langle\ \delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(C^{(2)} - \psi^{(2)}\right) u_{\alpha}^{(1)}\ \right\rangle$$

$$= \frac{\partial}{\partial x_{\alpha}^{(1)}} \int \Big\langle\ u_{\alpha}^{(1)} \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)$$

$$\times \delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(C^{(2)} - \psi^{(2)}\right)\ \Big\rangle$$

$$0 = \frac{\partial}{\partial x_{\alpha}^{(1)}} \iiiint v_{\alpha}^{(1)} F_2^{(1,2)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \qquad \text{---------- (3.15.3)}$$

and similarly,

$$0 = \frac{\partial}{\partial x_{\alpha}^{(1)}} \iiiint g_{\alpha}^{(1)} F_2^{(1,2)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)}\ . \qquad \text{----------- (3.15.4)}$$

The Nth – order continuity equations are

$$0=\frac{\partial}{\partial x_{\alpha}^{(1)}}\iiiint v_{\alpha}^{(1)}F_{N}^{(1,2,---,N)}dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)} \qquad \text{---------- (3.15.5)}$$

and

$$0=\frac{\partial}{\partial x_{\alpha}^{(1)}}\iiiint g_{\alpha}^{(1)}F_{N}^{(1,2,.........,N)}dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)}. \qquad \text{----------- (3.15.6)}$$

The continuity equations are symmetric in their arguments i.e.;

$$\frac{\partial}{\partial x_{\alpha}^{(r)}}\iiiint\left(v_{\alpha}^{(r)}F_{N}^{(1,2,..........;r.N)}dv^{(r)}dg^{(r)}d\phi^{(r)}d\psi^{(r)}\right)=\frac{\partial}{\partial x_{\alpha}^{(s)}}\iiiint v_{\alpha}^{(s)}F_{N}^{(1,2,.....r,s,.....N)}dv^{(s)}dg^{(s)}d\phi^{(s)}d\psi^{(s)}$$

----------- (3.15.7)

Since the divergence property is an important property and it is easily verified by the use of the property of distribution function as

$$\frac{\partial}{\partial x_{\alpha}^{(1)}}\iiiint v_{\alpha}^{(1)}F_{1}^{(1)}dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)}=\frac{\partial}{\partial x_{\alpha}^{(1)}}\left\langle u_{\alpha}^{(1)}\right\rangle=\left\langle\frac{\partial u_{\alpha}^{(1)}}{\partial x_{\alpha}(1)}\right\rangle=0 \qquad \text{----------(3.15.8)}$$

3.16. Equations for Evolution of Distribution Functions:

The equations (3.14.8)-(3.14.11) will be used to convert these into a set of equations for the variation of the distribution function with time. This, in fact, is done by making use of the definitions of the constructed distribution functions, differentiating them partially with respect to time, making some suitable operations on the right-hand side of the equation so obtained and lastly replacing the time derivative of v, h, θ and C from the equations (3.14.8)-(3.14.11).

Using equations (3.14.8) - (3.14.11) we get,

$$\frac{\partial F_{1}^{(1)}}{\partial t}=\frac{\partial}{\partial t}\left\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\right\rangle$$

$$=\left\langle\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial}{\partial t}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle+\left\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\right.$$

$$\left.\times\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial}{\partial t}\delta\left(h^{(1)}-g^{(1)}\right)\right\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\frac{\partial}{\partial t}\delta\left(C^{(1)}-\psi^{(1)}\right)\ \rangle$$

$$=\langle-\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial u^{(1)}}{\partial t}\frac{\partial}{\partial v^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial h^{(1)}}{\partial t}\frac{\partial}{\partial g^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\ \rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial \theta^{(1)}}{\partial t}\frac{\partial}{\partial \phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\ \rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\frac{\partial C}{\partial t}\frac{\partial}{\partial \psi^{(1)}}\delta\left(C^{(1)}-\psi^{(1)}\right)\ \rangle. \quad \text{------- -- (3.16.1)}$$

Using equations (3.14.8) – (3.14.11) in the equation (3.16.1), we get

$$\frac{\partial F_1^{(1)}}{\partial t}=\langle-\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\Big\{-\frac{\partial}{\partial x_\beta}\left(u_\alpha^{(1)}u_\beta^{(1)}-h_\alpha^{(1)}h_\beta^{(1)}\right)$$

$$-\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha}\iint\left[\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}}-\frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}}\right]\frac{d\bar{x}'}{|\bar{x}'-\bar{x}|}+\nu\nabla^2u_\alpha^{(1)}-2\in_{m\alpha\beta}\Omega_m u_\alpha^{(1)}+f\left(u_\alpha^{(1)}-v_\alpha^{(1)}\right)\Big\}$$

$$\times\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\Big\{-\frac{\partial}{\partial x_\beta^{(1)}}\left(h_\alpha^{(1)}u_\beta^{(1)}-u_\alpha^{(1)}h_\beta^{(1)}\right)$$

$$+\lambda\nabla^2h_\alpha^{(1)}\Big\}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\ \rangle+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)$$

$$\times\left\{-u_\beta^{(1)}\frac{\partial\theta^{(1)}}{\partial x_\beta}+\gamma\nabla^2\theta^{(1)}\right\}\times\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\ \rangle+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)$$

$$\times\delta\left(\theta^{(1)}-\phi^{(1)}\right)\left\{-u_\beta^{(1)}\frac{\partial c^{(1)}}{\partial x_\beta^{(1)}}+D\nabla^2C\right\}\frac{\partial}{\partial\psi^{(1)}}\delta\left(C^{(1)}-\psi^{(1)}\right)\ \rangle$$

$$=\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial u_\alpha^{(1)}u_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$+\langle-\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial h_\alpha^{(1)}h_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$+\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(1)}}\int\left[\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}}-\frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}}\right]$$

$$\times \frac{d\bar{x}'}{|\bar{x}'-\bar{x}|}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle+\langle-\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\nu\nabla^2 u_\alpha^{(1)}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)2\in_{m\alpha\beta}\Omega_m u_\alpha^{(1)}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle-\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)f\left(u_\alpha^{(1)}-v_\alpha^{(1)}\right)\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial h_\alpha^{(1)}u_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial u_\alpha^{(1)}h_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial g_\alpha}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\lambda\nabla^2 h_\alpha^{(1)}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)u_\beta^{(1)}\frac{\partial\theta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\gamma\nabla^2\theta^{(1)}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)u_\beta^{(1)}\frac{\partial C^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial\psi^{(1)}}\delta\left(C^{(1)}-\psi^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)D\nabla^2 C^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(C^{(1)}-\psi^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)RC^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(C^{(1)}-\psi^{(1)}\right)\rangle \quad \text{-----------} (3.16.2)$$

Various terms in the equation (3.16.2) can be simplified as the equations (3.6.3) – (3.6.14) and the Fifth and sixth terms of right hand side of equation (3.16.2)

$$\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)2\in_{m\alpha\beta}\Omega_m u_\alpha^{(1)}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$=\langle\ 2\in_{m\alpha\beta}\Omega_m u_\alpha^{(1)}\frac{\partial}{\partial v_\alpha^{(1)}}\left[\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\right]\rangle$$

$$= 2 \in_{m\alpha\beta} \Omega_m \frac{\partial}{\partial v_\alpha^{(1)}} \langle\ u_\alpha^{(1)} \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)\ \rangle$$

$$= 2 \in_{m\alpha\beta} \Omega_m \frac{\partial u_\alpha^{(1)}}{\partial v_\alpha^{(1)}} \langle\ \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)\ \rangle$$

$$= 2 \in_{m\alpha\beta} \Omega_m F_1^{(1)} \qquad \text{-----------} (3.16.3)$$

and $\langle -\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right) f(\ u_\alpha^{(1)} - v_\alpha^{(1)}\ ) \frac{\partial}{\partial v_\alpha^{(1)}} \delta\left(u^{(1)} - v^{(1)}\right)\ \rangle$

$$= -\langle\ f\left(u_\alpha^{(1)} - v_\alpha^{(1)}\right) \frac{\partial}{\partial v_\alpha^{(1)}} \left[\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)\right]\ \rangle$$

$$= -f(\ u_\alpha^{(1)} - v_\alpha^{(1)}\ ) \frac{\partial}{\partial v_\alpha^{(1)}} \langle\ \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)\ \rangle$$

$$= -f(\ u_\alpha^{(1)} - v_\alpha^{(1)}\ ) \frac{\partial}{\partial v_\alpha^{(1)}} F_1^{(1)}\ . \qquad \text{------------} (3.16.4)$$

And, the last term of the equation (3.16.2) reduces to

$$\langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right) R C^{(1)} \frac{\partial}{\partial \psi^{(1)}} \delta\left(C^{(1)} - \psi^{(1)}\right)\ \rangle$$

$$= -R\psi^{(1)} \frac{\partial}{\partial \psi^{(1)}} F_1^{(1)} \qquad \text{-----------}(3.16.5)$$

Substituting the results (3.6.3) – (3.6.14) and (3.16.3) - (3.16.5) in equation (3.16.2) we get the transport equation for one point distribution function $F_1^{(1)}(v, g, \phi, \psi)$ in MHD turbulence for concentration undergoing a first order reaction in a rotating system in presence of dust particles as

$$\frac{\partial F_1^{(1)}}{\partial t} + v_\beta^{(1)} \frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}} + g_\beta^{(1)} \left(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}} + \frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}} \right) \frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}} - \frac{\partial}{\partial v_\alpha^{(1)}} \left[\frac{1}{4\pi} \int\!\!\int\!\!\int\!\!\int\!\!\int \frac{\partial}{\partial x_\alpha^{(1)}} \left(\frac{1}{\left|\bar{x}^{(2)} - \bar{x}^{(1)}\right|} \right) \right.$$

$$\left. \times \left(\frac{\partial v_\alpha^{(2)}}{\partial x_\beta^{(2)}} \frac{\partial v_\beta^{(2)}}{\partial x_\alpha^{(2)}} - \frac{\partial g_\alpha^{(2)}}{\partial x_\beta^{(2)}} \frac{\partial g_\beta^{(2)}}{\partial x_\alpha^{(2)}} \right) F_2^{(1,2)} dx^{(2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} \right]$$

$$+ \nu \frac{\partial}{\partial v_\alpha^{(1)}} Lim_{\overline{\bar{x}^{(2)} \to \bar{x}^{(1)}}} \frac{\partial^2}{\partial x_\beta^{(2)} \partial x_\beta^{(2)}} \int\!\!\int\!\!\int\!\!\int v_\alpha^{(2)} F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)}$$

$$+\lambda\frac{\partial}{\partial g_\alpha^{(1)}}Lim_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiiint g_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+\gamma\frac{\partial}{\partial \phi^{(1)}}Lim_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiiint \phi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+D\frac{\partial}{\partial \psi^{(1)}}Lim_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiiint \psi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+2\in_{m\alpha\beta}\Omega_m F_1^{(1)}+f\left(u_\alpha^{(1)}-v_\alpha^{(1)}\right)\frac{\partial}{\partial v_\alpha^{(1)}}F_1^{(1)}-R\psi^{(1)}\frac{\partial}{\partial\psi^{(1)}}F_1^{(1)}=0\ . \qquad \text{---------}\ (3.16.6)$$

Similarly, an equation for two-point distribution function $F_2^{(1,2)}$ in MHD dusty fluid turbulence for concentration undergoing a first order reaction in a rotating system can be derived by using the equations (3.14.2), (3.14.3),(3.14.4),(3.14.8) and simplifying in the same manner, which is

$$\frac{\partial F_2^{(1,2)}}{\partial t}+\left(v_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}+v_\beta^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}\right)F_2^{(1,2)}+g_\beta^{(1)}\left(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}\right)\frac{\partial}{\partial x_\beta^{(1)}}F_2^{(1,2)}$$

$$+g_\beta^{(2)}\left(\frac{\partial g_\alpha^{(2)}}{\partial v_\alpha^{(2)}}+\frac{\partial v_\alpha^{(2)}}{\partial g_\alpha^{(2)}}\right)\frac{\partial}{\partial x_\beta^{(2)}}F_2^{(1,3)}-\frac{\partial}{\partial v_\alpha^{(1)}}\left[\frac{1}{4\pi}\iiiint\!\!\int\frac{\partial}{\partial x_\alpha^{(1)}}\left(\frac{1}{\left|\bar{x}^{(3)}-\bar{x}^{(1)}\right|}\right)\right.$$

$$\left.\times\left(\frac{\partial v_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial v_\beta^{(3)}}{\partial x_\alpha^{(3)}}-\frac{\partial g_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial g_\beta^{(3)}}{\partial x_\alpha^{(3)}}\right)F_3^{(1,2,3)}dx^{(3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{3)}\right]$$

$$-\frac{\partial}{\partial v_\alpha^{(2)}}\left[\frac{1}{4\pi}\iiiint\!\!\int\frac{\partial}{\partial x_\alpha^{(2)}}\left(\frac{1}{\left|\bar{x}^{(3)}-\bar{x}^{(2)}\right|}\right)\left(\frac{\partial v_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial v_\beta^{(3)}}{\partial x_\alpha^{(3)}}-\frac{\partial g_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial g_\beta^{(3)}}{\partial x_\alpha^{(3)}}\right)\right.$$

$$\left.\times F_3^{(1,2,3)}dx^{(3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{3)}\right]$$

$$+\nu\left(\frac{\partial}{\partial v_\alpha^{(1)}}Lim_{\bar{x}^{(3)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial v_\alpha^{(2)}}Lim_{\bar{x}^{(3)}\to\bar{x}^{(2)}}\right)\frac{\partial^2}{\partial x_\beta^{(3)}\partial x_\beta^{(3)}}\iiiint v_\alpha^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)}$$

$$+\lambda\left(\frac{\partial}{\partial g_\alpha^{(1)}}Lim_{\bar{x}^{(3)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial g_\alpha^{(2)}}Lim_{\bar{x}^{(3)}\to\bar{x}^{(2)}}\right)\frac{\partial^2}{\partial x_\beta^{(3)}\partial x_\beta^{(3)}}\iiiint g_\alpha^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{3)}d\psi^{(3)}$$

$$+\gamma\left(\frac{\partial}{\partial\phi^{(1)}}Lim_{\bar{x}^{(3)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial\phi^{(2)}}Lim_{\bar{x}^{(3)}\to\bar{x}^{(2)}}\right)\frac{\partial^2}{\partial x_\beta^{(3)}\partial x_\beta^{(3)}}\iiiint\phi^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)}$$

$$+D\left(\frac{\partial}{\partial\psi^{(1)}}Lim_{\bar{x}^{(3)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial\psi^{(2)}}Lim_{\bar{x}^{(3)}\to\bar{x}^{(2)}}\right)\frac{\partial^2}{\partial x_\beta^{(3)}\partial x_\beta^{(3)}}\iiiint\psi^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)}$$

$$+2\in_{m\alpha\beta}\Omega_m F_2^{(1,2)}+f\left(u_\alpha^{(1)}-v_\alpha^{(1)}\right)\frac{\partial}{\partial v_\alpha^{(2)}}F_2^{(1,2)}-R\psi^{(1)}\frac{\partial}{\partial\psi^{(1)}}F_1^{(1)}=0\ . \text{ --------- } (3.16.7)$$

Following this way, we can derive the equations for evolution of $F_3^{(1,2,3,)}$, $F_4^{(1,2,3,4)}$ and so on.

Logically, it is possible to have an equation for every F_n (n is an integer) but the system of equations so obtained is not closed. It seems that certain approximations will be required thus obtained.

3.17. Results and Discussion:

For clean fluid, f=0, the equation (3.16.6) reduces to the equation (3.6.17) of this chapter in part-A.

For non-rotating system, $\Omega_m=0$, the equation (3.16.6) reduces to the equation (3.11.5) of this chapter in part-B.

If the reaction constant R=0, the transport equation for one point distribution function in MHD turbulent flow (3.16.6) becomes

$$\frac{\partial F_1^{(1)}}{\partial t}+v_\beta^{(1)}\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}+g_\beta^{(1)}\left(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}\right)\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}-\frac{\partial}{\partial v_\alpha^{(1)}}\left[\frac{1}{4\pi}\iiiiint\frac{\partial}{\partial x_\alpha^{(1)}}\left(\frac{1}{\left|\bar{x}^{(2)}-\bar{x}^{(1)}\right|}\right)\right.$$

$$\left.\times\left(\frac{\partial v_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial v_\beta^{(2)}}{\partial x_\alpha^{(2)}}-\frac{\partial g_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial g_\beta^{(2)}}{\partial x_\alpha^{(2)}}\right)F_2^{(1,2)}dx^{(2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}\right]$$

$$+\nu\frac{\partial}{\partial v_\alpha^{(1)}}Lim_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiiint v_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+\lambda\frac{\partial}{\partial g_\alpha^{(1)}}Lim_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiiint g_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+\gamma\frac{\partial}{\partial\phi^{(1)}}\underset{\bar{x}^{(2)}\to\bar{x}^{(1)}}{Lim}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiiint\phi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+D\frac{\partial}{\partial\psi^{(1)}}\underset{\bar{x}^{(2)}\to\bar{x}^{(1)}}{Lim}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiiint\psi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+2\in_{m\alpha\beta}\Omega_m F_1^{(1)}+f\left(u_\alpha^{(1)}-v_\alpha^{(1)}\right)\frac{\partial}{\partial v_\alpha^{(1)}}F_1^{(1)}=0\ . \qquad \text{---------} (3.17.1)$$

which was obtained earlier by Azad and sarker [7].

If the fluid is clean and the system is non rotating then f=0 and Ωm=0, the transport equation for one point distribution function in MHD turbulent flow (3.16.6) becomes

$$\frac{\partial F_1^{(1)}}{\partial t}+v_\beta^{(1)}\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}+g_\beta^{(1)}\left(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}\right)\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}-\frac{\partial}{\partial v_\alpha^{(1)}}\left[\frac{1}{4\pi}\iiiint\int\left(\frac{\partial}{\partial x_\alpha^{(1)}}\frac{1}{\left|\bar{x}^{(2)}-\bar{x}^{(1)}\right|}\right)\right.$$

$$\left.\times\left(\frac{\partial v_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial v_\beta^{(2)}}{\partial x_\alpha^{(2)}}-\frac{\partial g_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial g_\beta^{(2)}}{\partial x_\alpha^{(2)}}\right)F_2^{(1,2)}dx^{(2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}\right]$$

$$+\nu\frac{\partial}{\partial v_\alpha^{(1)}}\underset{\bar{x}^{(2)}\to\bar{x}^{(1)}}{Lim}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiiint v_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+\lambda\frac{\partial}{\partial g_\alpha^{(1)}}\underset{\bar{x}^{(2)}\to\bar{x}^{(1)}}{Lim}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiiint g_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+\gamma\frac{\partial}{\partial\phi^{(1)}}\underset{\bar{x}^{(2)}\to\bar{x}^{(1)}}{Lim}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiiint\phi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+D\frac{\partial}{\partial\psi^{(1)}}\underset{\bar{x}^{(2)}\to\bar{x}^{(1)}}{Lim}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiiint\psi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}-R\psi^{(1)}\frac{\partial}{\partial\psi^{(1)}}F_1^{(1)}=0$$

------------(3.17.2)

which was obtained earlier by Islam and Sarker [57].

CHAPTER-IV

PART-A

FIRST ORDER REACTANT IN MAGNETO-HYDRODYNAMIC TURBULENCE BEFORE THE FINAL PERIOD OF DECAY FOR THE CASE OF MULTI-POINT AND MULTI-TIME IN A ROTATING SYSTEM

4.1. Introduction:

The essential characteristic of turbulent flows is that turbulent fluctuations are random in nature. In geophysical flows, the system is usually rotating with a constant angular velocity. Such large-scale flows are generally turbulent. When the motion is referred to axes, which rotate steadily with the bulk of the fluid, the coriolis and centrifugal force must be supposed to act on the fluid. The coriolis force due to rotation plays an important role in a rotating system of turbulent flow, while the centrifugal force with the potential is incorporated into the pressure. Kishore and Dixit [61], Kishore and Singh [63], Dixit and Upadhyay [39], Kishore and Golsefied [66] discussed the effect of coriolis force on acceleration covariance in ordinary and MHD turbulent flow. Funada, Tuitiya and Ohji [47] considered the effect of coriolis force on turbulent motion in presence of strong magnetic field. Kishore and Sarker [71] studied the rate of change of vorticity covariance in MHD turbulence in a rotating system. Sarker [123] studied the thermal decay process of MHD turbulence in a rotating system.

Deissler [36, 37] developed a theory "decay of homogeneous turbulence for times before the final period". Using Deissler's theory, Loeffler and Deissler [81] studied the decay of temperature fluctuations in homogeneous turbulence before the final period. In their approach they considered the two and three-point correlation equations and solved these equations after neglecting fourth and higher order correlation terms. Using Deissler theory, Kumar and Patel [73] studied the first-order reactant in homogeneous turbulence before the final period of decay for the case of multi-point and single-time correlation. Kumar and Patel [74] extended their problem [73] for the case of multi-point and multi-time concentration correlation. Patel [106] also studied in detail the same problem to carry out the numerical results. Sarker and Kishore [120] studied the decay of MHD turbulence at time before the final period using chandrasekher's relation [27]. Sarker and Islam [127] studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time. Sarker and.Azad [137] studied the Decay of MHD turbulence before the final period for the case of multi-point and multi-time in a rotating system. Islam and Sarker [56] also studied the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time.

In this chapter, using the above theories we have studied the magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction in MHD turbulence before the final period of decay for the case of multi-point and multi-time in a rotating system. Here, we have considered the two-point, two-time and three-point, three-time correlation equations and solved these equations after neglecting the fourth-order correlation terms. Finally we obtained the decay law for magnetic field energy fluctuation of concentration of dilute contaminant undergoing a first order chemical reaction in MHD turbulence for the case of multi-point and multi-time in a rotating system is obtained. If the fluid is non-rotating, the equation reduces to one obtained earlier by Islam and Sarker[56].

4.2. Basic Equations:

The equations of motion and continuity for viscous, incompressible MHD turbulent flow in a rotating system are given by

$$\frac{\partial u_i}{\partial t}+\frac{\partial}{\partial x_k}(u_i u_k - h_i h_k) = -\frac{\partial w}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_k \partial x_k} - 2\in_{mkl}\Omega_m u_i , \qquad \text{-------- (4.2.1)}$$

$$\frac{\partial h_i}{\partial t}+\frac{\partial}{\partial x_k}(h_i u_k - u_i h_k) = \lambda\frac{\partial^2 h_i}{\partial x_k \partial x_k} , \qquad \text{-------- (4.2.2)}$$

$$\frac{\partial v_i}{\partial t}+v_k\frac{\partial v_i}{\partial x_K} = -\frac{K}{m_s}(v_i - u_i) , \qquad \text{---------- (4.2.3)}$$

with

$$\frac{\partial u_i}{\partial x_i} = \frac{\partial v_i}{\partial x_i} = \frac{\partial h_i}{\partial x_i} = 0 . \qquad \text{----------- (4.2.4)}$$

Here, u_i, turbulence velocity component; h_i, magnetic field fluctuation component; $w(\hat{x},t)=\frac{P}{\rho}+\frac{1}{2}\langle h^2 \rangle+\frac{1}{2}|\hat{\Omega}\times\hat{x}|^2$, total MHD pressure $p(\hat{x},t)$, hydrodynamic pressure; ρ, fluid density; ν, Kinematic viscosity; $\lambda = {}^{\nu}/_{P_M}$, magnetic diffusivity; P_M, magnetic prandtl number; x_k, space co-ordinate; $m_s = \frac{4}{3}\pi R_s{}^3 \rho_s$, mass of single spherical dust particle of radius R_s; Ω_m,,constant angular velocity component; $\in_{mkl}$, alternating tensor, $\gamma = \frac{K}{\rho c_p}$, thermal diffusivity, K= Stokes's resistance coefficient which for spherical particle of radius r is 6πμr. The subscripts can take on the values 1, 2 or 3 and the repeated subscripts in a term indicate a summation;

4.3. Two-Point, Two-Time Correlation and Spectral Equations:

Under the condition that (i) the turbulence and the concentration magnetic field are homogeneous (ii) the chemical reaction has no effect on the velocity field and (iii) the reaction rate and the magnetic diffusivity are constant, the induction equation of a magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction at the points p and p' separated by the vector $\hat{r}$ could be written as

$$\frac{\partial h_i}{\partial t} + u_k \frac{\partial h_i}{\partial x_k} - h_k \frac{\partial u_i}{\partial x_k} = \lambda \frac{\partial^2 h_i}{\partial x_k \partial x_k} - Rh_i \qquad \text{----------- (4.3.1)}$$

and $$\frac{\partial h'_j}{\partial t'} + u'_k \frac{\partial h'_j}{\partial x'_k} - h'_k \frac{\partial u'_j}{\partial x'_k} = \lambda \frac{\partial^2 h'_j}{\partial x'_k \partial x'_k} - Rh'_j, \qquad \text{----------- (4.3.2)}$$

where R is the constant reaction rate.

Multiplying equation (4.3.1) by h'_j and equation (4.3.2) by h_i and taking ensemble average, we get

$$\frac{\partial \langle h_i h'_j \rangle}{\partial t} + \frac{\partial}{\partial x_k}\left[\langle u_k h_i h'_j \rangle - \langle u_i h_k h'_j \rangle\right] = \lambda \frac{\partial^2 \langle h_i h'_j \rangle}{\partial x_k \partial x_k} - R\langle h_i h'_j \rangle \qquad \text{----------- (4.3.3)}$$

And $$\frac{\partial \langle h_i h'_j \rangle}{\partial t'} + \frac{\partial}{\partial x'_k}\left[\langle u'_k h_i h'_j \rangle - \langle u'_j h_i h'_k \rangle\right] = \lambda \frac{\partial^2 \langle h_i h'_j \rangle}{\partial x'_k \partial x'_k} - R\langle h_i h'_j \rangle \qquad \text{----------- (4.3.4)}$$

Angular bracket $\langle - - - - - \rangle$ is used to denote an ensemble average.

Using the transformations

$$\frac{\partial}{\partial x_k}=-\frac{\partial}{\partial r_k},\quad \frac{\partial}{\partial x'_k}=\frac{\partial}{\partial r_k},\quad \left(\frac{\partial}{\partial t}\right)t'=\left(\frac{\partial}{\partial t}\right)\Delta t-\frac{\partial}{\partial \Delta t},\quad \frac{\partial}{\partial t'}=\frac{\partial}{\partial \Delta t} \qquad \text{--------- (4.3.5)}$$

into equations (4.3.3) and (4.3.4), we obtain

$$\frac{\partial\langle h_i h'_j\rangle}{\partial t}+\frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle-\langle u'_j h_i h'_k\rangle\right](\hat{r},\Delta t,t)-\frac{\partial}{\partial r_k}\left[\langle u_k h_i h'_j\rangle-\langle u_i h_k h'_j\rangle\right](\hat{r},\Delta t,t)$$
$$=2\lambda\frac{\partial^2\langle h_i h'_j\rangle}{\partial r_k\partial r_k}-2R\langle h_i h'_j\rangle \qquad \text{--------- (4.3.6)}$$

and $\frac{\partial\langle h_i h'_j\rangle}{\partial \Delta t}+\frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle-\langle u'_j h_i h'_k\rangle\right](\hat{r},\Delta t,t)=\lambda\frac{\partial^2\langle h_i h'_j\rangle}{\partial r_k\partial r_k}-R\langle h_i h'_j\rangle$ -------- (4.3.7)

Using the relations of Chandrasekhar [27]

$$\langle u_k h_i h'_j\rangle=-\langle u'_k h_i h'_j\rangle,\quad \langle u'_j h_i h'_k\rangle=\langle u_i h_k h'_j\rangle,$$

equations (4.3.6) and (4.3.7) become

$$\frac{\partial\langle h_i h'_j\rangle}{\partial t}+2\frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle-\langle u_i h_k h'_j\rangle\right]=2\lambda\frac{\partial^2\langle h_i h'_j\rangle}{\partial r_k\partial r_k}-2R\langle h_i h'_j\rangle \qquad \text{--------- (4.3.8)}$$

and $\frac{\partial\langle h_i h'_j\rangle}{\partial t}+\frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle-\langle u_i h_k h'_j\rangle\right]=\lambda\frac{\partial^2\langle h_i h'_j\rangle}{\partial r_k\partial r_k}-R\langle h_i h'_j\rangle$ ---------- (4.3.9)

Now we write equations (4.3.8) and (4.3.9) in spectral form in order to reduce it to an ordinary differential equation by use of the following three-dimensional Fourier transforms:

$$\langle h_i h'_j\rangle(\hat{r},\Delta t,t)=\int_{-\infty}^{\infty}\langle \psi_i\psi'_j\rangle(\hat{K},\Delta t,t)\exp\left[\hat{i}(\hat{K}.\hat{r})\right]d\hat{K} \qquad \text{---------- (4.3.10)}$$

and $\langle u_i h_k h'_j\rangle(\hat{r},\Delta t,t)=\int_{-\infty}^{\infty}\langle \alpha_i\psi_k\psi'_j\rangle(\hat{K},\Delta t,t)\exp\left[\hat{i}(\hat{K}.\hat{r})\right]d\hat{K}$ --------- (4.3.11)

Interchanging the subscripts i and j then interchanging the points p and p′ gives

$$\langle u'_k h_i h'_j\rangle(\hat{r},\Delta t,t)=\langle u_k h_i h'_j\rangle(-\hat{r},-\Delta t,t+\Delta t)$$
$$=\int_{-\infty}^{\infty}\langle \alpha_i\psi_i\psi'_j\rangle(-\hat{K},-\Delta t,t+\Delta t)\exp\left[\hat{i}(\hat{K}.\hat{r})\right]d\hat{K} \qquad \text{--------- (4.3.12)}$$

where, $\hat{K}$ is known as a wave number vector and $d\hat{K}$ =dK$_1$ dK$_2$ dK$_3$. The magnitude of $\hat{K}$ has the dimension 1/length and can be considered to be the reciprocal of an eddy size.

Substituting of equation (4.3.10) to (4.3.12) in to equations (4.3.8) and (4.3.9) leads to the spectral equations

$$\frac{\partial\langle\psi_i\psi'_j\rangle}{\partial t}+2\left[\lambda k^2+R\right]\langle\psi_i\psi'_j\rangle=2ik_k\left[\langle\alpha_i\psi_k\psi'_j\rangle(\hat{K},\Delta t,t)-\langle\alpha_k\psi_i\psi'_j\rangle(-\hat{K},-\Delta t,t+\Delta t)\right]$$

------------- (4.3.13)

and $$\frac{\partial\langle\psi_i\psi'_j\rangle}{\partial\Delta t}+\left[\lambda K^2+R\right]\langle\psi_i\psi'_j\rangle=ik_k\left[\langle\alpha_i\psi_k\psi'_j\rangle(\hat{K},\Delta t,t)-\langle\alpha_k\psi_i\psi'_j\rangle(-\hat{K},-\Delta t,t+\Delta t)\right]$$

-----------(4.3.14)

The tensor equations (4.3.13) and (4.3.14) becomes a scalar equation by contraction of the indices i and j

$$\frac{\partial\langle\psi_i\psi'_i\rangle}{\partial t}+2\left[\lambda K^2+R\right]\langle\psi_i\psi'_i\rangle=2ik_k\left[\langle\alpha_i\psi_k\psi'_i\rangle(\hat{K},\Delta t,t)-\langle\alpha_k\psi_i\psi'_i\rangle(-\hat{K},-\Delta t,t+\Delta t)\right]$$

------------ (4.3.15)

and $$\frac{\partial\langle\psi_i\psi'_i\rangle}{\partial\Delta t}+\left[\lambda K^2+R\right]\langle\psi_i\psi'_i\rangle=ik_k\left[\langle\alpha_i\psi_k\psi'_i\rangle(\hat{K},\Delta t,t)-\langle\alpha_k\psi_i\psi'_i\rangle(-\hat{K},-\Delta t,t+\Delta t)\right]$$

--------- (4.3.16)

The terms on the right side of equations (4.3.15) and (4.3.16) are collectively proportional to what is known as the magnetic energy transfer terms.

4.4. Three-Point, Three-Time Correlation and Spectral Equations:

Similar procedure can be used to find the three-point correlation equations. For this purpose we take the momentum equation of MHD turbulence in a rotating system at the point P and the induction equations of magnetic field fluctuations, governing the concentration of a dilute contaminant undergoing a first order chemical reaction at p′ and p″ separated by the vector $\hat{r}$ and $\hat{r}'$ as

$$\frac{\partial u_l}{\partial t}+u_k\frac{\partial u_l}{\partial x_k}-h_k\frac{\partial h_l}{\partial x_k}=-\frac{\partial w}{\partial x_l}+\nu\frac{\partial^2 u_l}{\partial x_k\partial x_k}-2\in_{mkl}\Omega_m u_l,$$

------- (4.4.1)

$$\frac{\partial h'_i}{\partial t'}+u'_k\frac{\partial h'_i}{\partial x'_k}-h'_k\frac{\partial u'_i}{\partial x'_k}=\lambda\frac{\partial^2 h'_i}{\partial x'_k\partial x'_k}-Rh'_i,$$

-------- (4.4.2)

$$\frac{\partial h_j''}{\partial t''}+u_k''\frac{\partial h_j''}{\partial x_k''}-h_k''\frac{\partial u_j''}{\partial x_k''}=\lambda\frac{\partial^2 h_j''}{\partial x_k''\partial x_k''}-Rh_j'' . \quad \text{-------- (4.4.3)}$$

Multiplying equation (4.4.1) by $h'_i\, h_j''$, equation (4.4.2) by $u_l h_j''$ and equation (4.4.3) by $u_l h_i'$, taking ensemble average, one obtains

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t}+\frac{\partial}{\partial x_k}\left[\langle u_k u_l h_i' h_j''\rangle-\langle h_k h_l h_i' h_j''\rangle\right]=\frac{\partial\langle w h_i' h_j''\rangle}{\partial x_l}+\nu\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k\partial x_k}-2\in_{mkl}\Omega_m\langle u_l h_i' h_j''\rangle$$

-------- (4.4.4)

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t'}+\frac{\partial}{\partial x_k'}\left[\langle u_l u_k' h_i' h_j''\rangle-\langle u_l u_i' h_k' h_j''\rangle\right]=\lambda\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k'\partial x_k'}-R\langle u_l h_i' h_j''\rangle \quad \text{-------- (4.4.5)}$$

and

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t''}+\frac{\partial}{\partial x_k''}\left[\langle u_l u_k'' h_i' h_j''\rangle-\langle u_l u_j'' h_i' h_k''\rangle\right]=\lambda\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k''\partial x_k''}-R\langle u_l h_i' h_j''\rangle \quad \text{-------- (4.4.6)}$$

Using the transformations

$$\frac{\partial}{\partial x_k}=-\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'}\right),\ \frac{\partial}{\partial x_k'}=\frac{\partial}{\partial r_k},\quad \frac{\partial}{\partial x_k''}=\frac{\partial}{\partial r_k'}\ ,$$

$$\left(\frac{\partial}{\partial t}\right)t',t''=\left(\frac{\partial}{\partial t}\right)\Delta t,\Delta t'-\frac{\partial}{\partial\Delta t}-\frac{\partial}{\partial\Delta t'}\ ,$$

$$\frac{\partial}{\partial t'}=\frac{\partial}{\partial\Delta t},\qquad \frac{\partial}{\partial t''}=\frac{\partial}{\partial\Delta t'}$$

into equations (4.4.4) to (4.4.6), we have

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t}-\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'}\right)\left[\langle u_k u_l h_i' h_j''\rangle-\langle h_k h_l h_i' h_j''\rangle\right]+\frac{\partial}{\partial r_k}\left[\langle u_l u_k' h_i' h_j''\rangle-\langle u_l u_i' h_k' h_j''\rangle\right]$$

$$+\frac{\partial}{\partial r_k'}\left[\langle u_l u_k'' h_i' h_j''\rangle-\langle u_l u_j'' h_i' h_k''\rangle\right]=-\left(\frac{\partial}{\partial r_l}+\frac{\partial}{\partial r_l'}\right)\langle w h_i' h_j''\rangle+\nu\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'}\right)^2\langle u_l h_i' h_j''\rangle$$

$$+\lambda\left[\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k\partial r_k}+\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k'\partial r_k'}\right]-2\in_{mkl}\Omega_m\langle u_l h_i' h_j''\rangle \quad \text{---------(4.4.7)}$$

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial \Delta t} + \frac{\partial}{\partial r_k}\left[\langle u_l u_k' h_i' h_j''\rangle - \langle u_l u_i' h_k' h_j''\rangle\right] = \lambda \frac{\partial^2 \langle u_l h_i' h_j''\rangle}{\partial r_k \partial r_k} - R\langle u_l h_i h_j\rangle \qquad \text{----------} (4.4.8)$$

and

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial \Delta t'} + \frac{\partial}{\partial r_k'}\left[\langle u_l u_k'' h_i' h_j''\rangle - \langle u_l u_j'' h_i' h_k''\rangle\right] = \lambda \frac{\partial^2 \langle u_l h_i' h_j''\rangle}{\partial r_k' \partial r_k'} - R\langle u_l h_i h_j\rangle \,.$$

---------(4.4.9)

In order to convert equations (4.4.7)–(4.4.9) to spectral form, we can define the following six dimensional Fourier transforms:

$$\langle u_l h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_l \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}'$$

-------------(4.4.10)

$$\langle u_l u_k' h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_l \phi_K' \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}'$$

----------(4.4.11)

$$\langle w h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \gamma \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}'$$

----------(4.4.12)

$$\langle u_k u_l h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_K \phi_l \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}'$$

-----------(4.4.13)

$$\langle h_k h_l h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \beta_K \beta_l \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}'$$

------------ (4.4.14)

$$\langle u_l u_i' h_K' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_l \phi_i' \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}'$$

--------- (4.4.15)

Interchanging the points P′ and P″ along with the indices i and j result in the relations

$$\langle u_l u_k'' h_i' h_j''\rangle = \langle u_l u_k' h_i' h_j''\rangle \qquad \text{------------}(4.4.16)$$

By use of these facts and the equations (4.4.10)-(4.4.16), we can write equations (4.4.7)-(4.4.9) in the form

$$\frac{\partial}{\partial t}\langle \phi_l \beta_i' \beta_j'' \rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) + \lambda\left[(1+P_M)(k^2+k'^2) + 2P_M kk' + \frac{2R}{\lambda} + \frac{1}{\lambda}(2\in_{mkl}\Omega_m)\right]$$

$$\times \langle \phi_l \beta_i' \beta_j'' \rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) = \left[i(k_k + k_k')\langle \phi_k \phi_l \beta_i' \beta'' \rangle - i(k_k + k_k')\langle \beta_k \beta_l \beta_i' \beta_j'' \rangle\right.$$

$$\left. - i(k_k + k_k')\langle \phi_l \phi_k' \beta_i' \beta_j'' \rangle + i(k_k + k_k')\langle \phi_l \phi_i' \beta_k' \beta_j'' \rangle - i(k_l + k_l')\langle \gamma \beta_i' \beta_j'' \rangle\right](\hat{K},\hat{K}',\Delta t,\Delta t',t)$$

---------- (4.4.17)

$$\frac{\partial}{\partial \Delta t}\langle \phi_l \beta_i' \beta_j'' \rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) + \lambda\left[K^2 + \frac{R}{\lambda}\right]\langle \phi_l \beta_i' \beta_j'' \rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)$$

$$= -ik_k\langle \phi_l\, \phi_k' \beta_i' \beta_j'' \rangle(\hat{K},\hat{K}'\Delta t,\Delta t',t) + ik_k\langle \phi_l \phi_i' \beta_k' \beta_j'' \rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) \quad \text{--------- (4.4.18)}$$

and $$\frac{\partial}{\partial \Delta t'}\langle \phi_l \beta_i' \beta_j'' \rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) + \lambda\left[K^2 + \frac{R}{\lambda}\right]\langle \phi_l \beta_i' \beta_j'' \rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)$$

$$= -ik_k'\langle \phi_l\, \phi_k' \beta_i' \beta_j'' \rangle(\hat{K},\hat{K}'\Delta t,\Delta t',t) + ik_k'\langle \phi_l \phi_i' \beta_k' \beta_j'' \rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) \text{--------- (4.4.19)}$$

If the derivative with respect to x_l is taken of the momentum equation (4.4.1) for the point P, the equation multiplied by $h_i'h_j''$ and time average taken, the resulting equation

$$-\frac{\partial^2\langle wh_i'h_j''\rangle}{\partial x_l \partial x_l} = \frac{\partial^2}{\partial x_l \partial x_k}\left(\langle u_l u_k h_i' h_j''\rangle - \langle h_l h_k h_i' h_j''\rangle\right) \quad \text{---------- (4.4.20)}$$

Writing this equation in terms of the independent variables $\hat{r}$ and $\hat{r}'$

$$-\left[\frac{\partial^2}{\partial r_l \partial r_l} + 2\frac{\partial^2}{\partial r_l \partial r_l'} + \frac{\partial^2}{\partial r_l' \partial r_l'}\right]\langle wh_i'h_j''\rangle = \left[\frac{\partial^2}{\partial r_l \partial r_k} + \frac{\partial^2}{\partial r_l' \partial r_k} + \frac{\partial^2}{\partial r_l \partial r_k'} + \frac{\partial^2}{\partial r_l' \partial r_k'}\right]\times$$

$$\left(\langle u_l u_k h_i' h_j''\rangle - \langle h_l h_k h_i' h_j''\rangle\right) \quad \text{---------(4.4.21)}$$

Taking the Fourier transforms of equation (4.4.8)

$$-\langle \gamma \beta_i' \beta_j'' \rangle = \frac{(k_l k_k + k_l' k_k + k_l k_k' + k_l' k_k')\left(\langle \phi_l \phi_k \beta_i' \beta_j'' \rangle - \langle \beta_l \beta_k \beta_i' \beta_j'' \rangle\right)}{k_l k_l + 2k_l k_l' + k_l' k_l'} \quad \text{---------(4.4.22)}$$

Equation (4.4.22) can be used to eliminate $\langle \gamma \beta_i' \beta_j'' \rangle$ from equation (4.4.17)

The tensor equations (4.4.17) to (4.4.19) can be converted to scalar equation by contraction of the indices i and j and inner multiplication by k_l

$$\frac{\partial}{\partial t}k_l\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)+\lambda\Big[(1+P_M)(k^2+k'^2)+2P_M kk'+\frac{2R}{\lambda}+$$

$$\frac{1}{\lambda}(2\in_{mkl}\Omega_m)\Big]\langle\phi_l\beta_i''\beta_i''\rangle(\hat{K},\hat{K},\Delta t,\Delta t',t)=i(k_k+k_k')\langle\phi_k\phi_l\beta_i'\beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)$$

$$-i(k_k+k_k')\langle\beta_K\beta_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)-i(k_k+k_k')\langle\phi_l\phi_k'\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)$$

$$+i(k_k+k_k')\langle\phi_l\phi_i'\beta_k'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)-i(k_l+k_l')\langle\gamma\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)$$

--------- (4.4.23)

$$\frac{\partial}{\partial\Delta t}k_L\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)+\lambda\left[K^2+\frac{R}{\lambda}\right]\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)$$

$$=-ik_k\langle\phi_l\phi_k'\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)+ik_k\langle\phi_l\phi_i'\beta_k'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) \quad \text{--------- (4.4.24)}$$

and $$\frac{\partial}{\partial\Delta t'}k_l\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)+\lambda\left[K'^2+\frac{R}{\lambda}\right]\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)$$

$$=-ik_k'\langle\phi_l\phi_k'\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)+ik_k'\langle\phi_l\phi_i'\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) \quad \text{---------- (4.4.25)}$$

4.5. Solution for Times Before the Final Period:

It is known that the equation for final period of decay is obtained by considering the two-point correlations after neglecting third-order correlation terms. To study the decay for times before the final period, the three-point correlations are considered and the quadruple correlation terms are neglected because the quadruple correlation terms decays faster than the lower-order correlation terms. The term $\langle\gamma\beta_i'\beta_j''\rangle$ associated with the pressure fluctuations should also be neglected. Thus neglecting all the terms on the right hand side of equations (4.4.23) to (4.4.25), we get

$$\frac{\partial}{\partial t}K_l\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)+\lambda\Big[(1+P_M)(k^2+k'^2)+2P_M kk'+\frac{2R}{\lambda}$$

$$+\frac{1}{\lambda}(2\in_{mkl}\Omega_m)\Big]\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)=0 \quad \text{---------- (4.5.1)}$$

$$\frac{\partial}{\partial \Delta t} K_l \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda \left[k^2 + \frac{R}{\lambda} \right] \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0 \quad \text{---------} (4.5.2)$$

and $$\frac{\partial}{\partial \Delta t'} K_l \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda \left[k'^2 + \frac{R}{\lambda} \right] \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0 \quad \text{------} (4.5.3)$$

Integrating equations (4.5.1) to (4.5.3) between t_o and t, we obtain

$$k_l \langle \phi_l \beta_i' \beta_i'' \rangle = f_l \exp\left\{ -\lambda \left[(1 + P_M)(k^2 + k'^2) + 2P_M kk' \cos\theta + \frac{2R}{\lambda} + \frac{1}{\lambda}(2 \in_{mkl} \Omega_m) \right] (t - t_o) \right\},$$

$$k_l \langle \phi_l \beta_i' \beta_i'' \rangle = g_l \exp\left[-\lambda \left(K^2 + \frac{R}{\lambda} \right) \Delta t \right]$$

and $$k_l \langle \phi_l \beta_i' \beta_i'' \rangle = q_l \exp\left[-\lambda \left(k'^2 + \frac{R}{\lambda} \right) \Delta t' \right].$$

For these relations to be consistent, we have

$$k_l \langle \phi_l \beta_i' \beta_i'' \rangle = k_l \langle \phi_l \beta_i' \beta_i'' \rangle_o \exp\left\{ -\lambda \left[(1 + P_M)(k^2 + k'^2)(t - t_o) + k^2 \Delta t + k'^2 \Delta t' + 2P_M kk' \cos\theta (t - t_o) + \frac{2R}{\lambda} \left(t - t_0 + \frac{\Delta t + \Delta t'}{2} \right) + \left(\frac{2 \in_{mkl} \Omega_m}{\lambda} \right) (t - t_o) \right] \right\} \quad \text{--------} (4.5.4)$$

where θ is the angle between $\hat{K}$ and $\hat{K}'$ and $\langle \phi_l \beta_i' \beta_i'' \rangle_o$ is the value of $\langle \phi_l \beta_i' \beta_i'' \rangle$ at $t = t_o$, $\Delta t = \Delta t' = 0$, $\lambda = \frac{\nu}{P_M}$

By letting $\hat{r}' = 0$, $\Delta t' = 0$ in the equation (4.4.10) and comparing with equations (4.3.11) and (4.3.12) we get

$$\langle \alpha_i \psi_k \psi_i' \rangle (\hat{K}, \Delta t, t) = \int_{-\infty}^{\infty} \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, o, t) d\hat{K}' \quad \text{-----------} (4.5.5)$$

and $$\langle \alpha_i \psi_k \psi_i' \rangle (-\hat{K}, -\Delta t, t + \Delta t) = \int_{-\infty}^{\infty} \langle \phi_l \beta_i' \beta_i'' \rangle (-\hat{K}, \hat{K}', \Delta t, o, t) d\hat{K}' \quad \text{----------} (4.5.6)$$

Substituting equation (4.5.4) - (4.5.6) into equation (4.3.15), one obtains

$$\frac{\partial}{\partial t} \langle \psi_i \psi_i' \rangle (\hat{K}, \Delta t, t) + 2\lambda \left[k^2 + \frac{R}{\lambda} \right] \langle \psi_i \psi_i' \rangle (\hat{K}, \Delta t, t) = \int_{-\infty}^{\infty} 2ik_l \left[\langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, 0, t) \right.$$

$$-\langle\phi_l\beta_i'\beta_i''\rangle\left(-\hat{K},-\hat{K}',\Delta t,0,t\right)\,\big]_o \exp\Big[-\lambda\big\{(1+P_M)(k^2+k'^2)(t-t_o)$$

$$+k^2\Delta t+2P_M(t-t_o)kk'\cos\theta+\frac{2R}{\lambda}(t-t_0+\Delta t)+\left(\frac{2\in_{mkl}\Omega_m}{\lambda}\right)(t-t_o)\big\}\Big]d\hat{k}\ .$$

----------(4.5.7)

Now, $d\hat{K}'$ can be expressed in terms of k′ and θ as $-2\pi k' d(\cos\theta)dk'$ (cf. Deissler [37])

i.e. $d\hat{K}'=-2\pi k' d(\cos\theta)dk'$ --------- (4.5.8)

Substituting of equation (4.5.8) in equation (4.5.7) yields

$$\frac{\partial}{\partial t}\langle\psi_i\psi_i'\rangle\left(\hat{K},\Delta t,t\right)+2\lambda\left[k^2+\frac{R}{\lambda}\right]\langle\psi_i\psi_i'\rangle\left(\hat{K},\Delta t,t\right)=2\int_0^\infty 2\pi i k_l\Big[\langle\phi_l\beta_i'\beta_i''\rangle\left(\hat{K},\hat{K}'\right)$$

$$-\langle\phi_l\beta_i'\beta_i''\rangle\left(-\hat{K},-\hat{K}'\right)\Big]_o k'^2\left[\int_{-1}^{1}\exp\Big\{-\lambda\big[(1+P_M)(k^2+k'^2)(t-t_o)\right.$$

$$\left.+k^2\Delta t+2P_M(t-t_o)kk'\cos\theta+\frac{2R}{\lambda}(t-t_0+\Delta t/2)+\left(\frac{2\in_{mkl}\Omega_m}{\lambda}\right)(t-t_o)\big]\Big\}d(\cos\theta)\right]dk'$$

----------(4.5.9)

In order to find the solution completely and following Loeffler and Deissler [81] we assume that

$$ik_l\Big[\langle\phi_l\beta_i'\beta_i''\rangle\left(\hat{K},\hat{K}'\right)-\langle\phi_l\beta_i'\beta_i''\rangle\left(-\hat{K},-\hat{K}'\right)\Big]_o=\frac{-\delta_0}{(2\pi)^2}\left(k^2k'^4-k^4k'^2\right)$$ ---------- (4.5.10)

where δ_0 is a constant determined by the initial conditions. The negative sign is placed in front of δ_0 in order to make the transfer of energy from small to large wave numbers for positive value of δ_0.

Substituting equation (4.5.10) into equation (4.5.9), we get

$$\frac{\partial}{\partial t}2\pi\langle\psi_i\psi_i'\rangle\left(\hat{K},\Delta t,t\right)+2\lambda\left[k^2+R/\lambda\right]2\pi\langle\psi_i\psi_i'\rangle\left(\hat{K},\Delta t,t\right)=-2\delta_0\int_0^\infty\left(k^2k'^4-k^4k'^2\right)k'^2$$

$$\left[\int_{-1}^{1}\exp\Big\{-\lambda\big[(1+P_M)(k^2+k'^2)(t-t_o)+k^2\Delta t+2P_M(t-t_o)kk'\cos\theta\right.$$

$$+\frac{2R}{\lambda}(t-t_0+\Delta t/2)+\left(\frac{2\in_{mkl}\Omega_m}{\lambda}\right)(t-t_o)\Big]\Big\}d(\cos\theta)\Big]d\hat{k}' \quad \text{--------- (4.5.11)}$$

Multiplying both sides of equation (4.5.11) by k^2, we get

$$\frac{\partial E}{\partial t}+2\lambda k^2E=F \quad \text{---------- (4.5.12)}$$

where, $E=2\pi k^2\langle\psi_i\psi_i'\rangle$, E is the magnetic energy spectrum function and F is the magnetic energy transfer term and is given by

$$F=-2\delta_0\int_0^\infty\left(k^2k'^4-k^4k'^2\right)k^2k'^2\times\left[\int_{-1}^{1}\exp\{-\lambda[(1+P_M)(k^2+k'^2)(t-t_o)\right.$$
$$\left.+k^2\Delta t+2P_M(t-t_o)kk'\cos\theta+\frac{2R}{\lambda}(t-t_0+\Delta t/2)+\left(\frac{2\in_{mkl}\Omega_m}{\lambda}\right)(t-t_o)\Big]\Big\}d(\cos\theta)\right]dk'$$

-------- (4.5.13)

Integrating equation (4.5.13) with respect to cosθ and k', we have

$$F=-\frac{\delta_oP_M\sqrt{\pi}}{4\lambda^{3/2}(t-t_o)^{3/2}(1+P_M)^{5/2}}\exp\left\{-\left(\frac{2\in_{mkl}\Omega_m}{\lambda}\right)(t-t_o)\right\}\times$$

$$\exp\left[\frac{-k^2\lambda(1+2P_M)}{1+P_M}\left(t-t_o+\frac{1+P_M}{1+2P_M}\Delta t\right)-2R(t-t_0+\Delta t/2)\right]\times\left[\frac{15P_Mk^4}{4P_M^2\lambda^2(t-t_o)^2(1+P_M)}\right.$$

$$+\left\{\frac{5P_M^2}{(1+P_M)^2}-\frac{3}{2}\right\}\frac{k^6}{P_M\lambda(t-t_o)}+\left\{\frac{P_M^3}{(1+P_M)^3}-\frac{P_M}{1+P_M}\right\}k^8$$

$$-\frac{\delta_oP_M\sqrt{\pi}}{4\lambda^{3/2}(t-t_o+\Delta t)^{3/2}(1+P_M)^{5/2}}\exp\left\{-\left(\frac{2\in_{mkl}\Omega_m}{\lambda}\right)(t-t_o)\right\}\times$$

$$\exp\left[\frac{-k^2\lambda(1+2P_M)}{1+P_M}\left(t-t_o+\frac{P_M}{1+P_M}\Delta t\right)-2R(t-t_0+\Delta t/2)\right]\times\left[\frac{15P_Mk^4}{4\nu^2(t-t_o+\Delta t)^2(1+P_M)}\right.$$

$$\left.+\left\{\frac{5P_M^2}{(1+P_M)^2}-\frac{3}{2}\right\}\frac{k^6}{P_M\lambda(t-t_o+\Delta t)}+\left\{\frac{P_M^3}{(1+P_M)^3}-\frac{P_M}{1+P_M}\right\}k^8\right] \quad \text{------ (4.5.14)}$$

The series of equation (4.5.14) contains only even power of k and start with k^4 and the equation represents the transfer function arising owing to consideration of magnetic field at three-point and three-times.

If we integrate equation (4.5.14) for Δt=0 over all wave numbers, we find that

$$\int_0^\infty F dk = 0 \qquad \text{---------- (4.5.15)}$$

which indicates that the expression for F satisfies the condition of continuity and homogeneity. Physically it was to be expected as F is a measure of the energy transfer and the total energy transferred to all wave numbers must be zero.

The linear equation (4.5.12) can be solved to give

$$E = \exp\left[-2\lambda k^2 \left(t - t_o + \Delta t/2\right) \int F \exp\left[2\lambda (k^2 + R/\lambda)(t - t_o + \Delta t/2)\right] dt\right.$$

$$\left. + J(k)\exp\left[-2\lambda (k^2 + R/\lambda\right)(t - t_o + \Delta t/2)\right] \qquad \text{---------- (4.5.16)}$$

where $J(k) = \frac{N_o k^2}{\pi}$ is a constant of integration and can be obtained as by Corrsin[32].

Substituting the values of F from equation (4.5.14) into equation (4.5.16) gives the equation

$$E = \frac{N_o k^2}{\pi} \exp\left[-2\lambda (k^2 + R/\lambda)(t - t_o + \Delta t/2)\right] + \frac{\delta_o P_M \sqrt{\pi}}{4\lambda^{3/2}(1 + P_M)^{7/2}} \times$$

$$\exp\left[-(2 \in_{mkl} \Omega_m)(t - t_o)\right]$$

$$\exp\left[\frac{-k^2 \lambda (1 + 2P_M)}{1 + P_M}\left(t - t_o + \frac{1 + P_M}{1 + 2P_M}\Delta t\right) - 2R(t - t_0 + \Delta t/2)\right]$$

$$\left[\frac{3k^4}{2P_M \lambda^2 (t - t_0)^{5/2}} + \frac{(7P_M - 6)k^6}{3\lambda (1 + P_M)(t - t_0)^{3/2}} - \frac{4(3P_M^2 - 2P_M + 3)k^8}{3(1 + P_M)^2 (t - t_0)^{1/2}}\right.$$

$$\left. + \frac{8\sqrt{\lambda}(3P_M^2 - 2P_M + 3)k^9}{3(1 + P_M)^{5/2}} F(\omega)\right] + \frac{\delta_o P_M \sqrt{\pi}}{4\lambda^{3/2}(1 + P_M)^{7/2}} \exp\left[-(2 \in_{mkl} \Omega_m)(t - t_o)\right]$$

$$\exp\left[\frac{-k^2\lambda(1+2P_M)}{1+P_M}\left(t-t_o+\frac{P_M}{1+P_M}\Delta t\right)-2R(t-t_0+\Delta t/2)\right]$$

$$\left[\frac{3k^4}{2P_M\lambda^2(t-t_0+\Delta t)^{5/2}}+\frac{(7P_M-6)k^6}{3\lambda(1+P_M)(t-t_0+\Delta t)^{3/2}}\right.$$

$$\left.-\frac{4(3P_M^2-2P_M+3)k^8}{3(1+P_M)^2(t-t_0+\Delta t)^{1/2}}+\frac{8\sqrt{\lambda}(3P_M^2-2P_M+3)k^9F(\omega)}{(1+P_M)^{5/2}P_M^{1/2}}\right] \quad \text{---------} (4.5.17)$$

where $F(\omega)=e^{-\omega^2}\int_0^{\omega}e^{x^2}dx$,

$$\omega=k\sqrt{\frac{\lambda(t-t_0)}{1+P_M}} \quad \text{or} \quad k\sqrt{\frac{\lambda(t-t_0+\Delta t)}{1+P_M}}.$$

By setting $\hat{r}=0$, j=i, $d\hat{k}=-2\pi k^2 d(\cos\theta)dk$ and $E=2\pi k^2\langle\psi_i\psi'_j\rangle$ in equation (4.3.10), we get the expression for magnetic energy decay law as

$$\frac{\langle h_i h'_i\rangle}{2}=\int_0^{\infty}Edk \quad \text{---------} (4.5.18)$$

Substituting equation (4.5.17) into equation (4.5.18) and after integration, we get

$$\frac{\langle h_i h'_i\rangle}{2}=\frac{N_0}{8\sqrt{2\pi}\lambda^{3/2}(T+\Delta T/2)^{3/2}}\exp[-2R(T+\Delta T/2)]$$

$$+\frac{\pi\delta_0}{4\lambda^6(1+P_M)(1+2P_M)^{5/2}}\exp[-2R(T+\Delta T/2)]\exp[-(2\in_{mkl}\Omega_m)]$$

$$\times\left[\frac{9}{16T^{5/2}\left(T+\frac{1+P_M}{1+2P_M}\Delta T\right)^{3/2}}+\frac{9}{16(T+\Delta T)^{5/2}\left(T+\frac{P_M}{1+2P_M}\Delta T\right)^{5/2}}\right.$$

$$+\frac{5P_M(7P_M-6)}{16(1+2P_M)T^{3/2}\left(T+\frac{1+P_M}{1+2P_M}\Delta T\right)^{7/2}}+\frac{5P_M(7P_M-6)}{16(1+2P_M)(T+\Delta T)^{3/2}\left(T+\frac{P_M}{1+2P_M}\Delta T\right)^{7/2}}$$

$$+\frac{35P_M\left(3P_M^2-2P_M+3\right)}{8\left(1+2P_M\right)T^{1/2}\left(T+\frac{1+P_M}{1+2P_M}\Delta T\right)^{9/2}}+\frac{35P_M\left(3P_M^2-2P_M+3\right)}{8\left(1+2P_M\right)\left(T+\Delta T\right)^{1/2}\left(T+\frac{P_M}{1+2P_M}\Delta T\right)^{9/2}}$$

$$+\frac{8P_M\left(3P_M^2-2P_M+3\right)\left(1+2P_M\right)^{5/2}}{3.2^{23/2}\left(1+P_M\right)^{11/2}}\sum_{n=0}^{\infty}\frac{1.3.5............(2n+9)}{n!(2n+1)2^{2n}\left(1+P_M\right)^n}\times$$

$$\left\{\frac{T^{(2n+1)/2}}{\left(T+\Delta T/2\right)^{(2n+1)/2}}+\frac{\left(T+\Delta T\right)^{(2n+1)/2}}{\left(T+\Delta T/2\right)^{(2n+1)/2}}\right\}\Bigg] \qquad \text{---------} (4.5.19)$$

where, $T=t-t_0$.

For $T_m=T+\Delta T/2$, equation (4.5.19) takes the form

$$\frac{\langle h^2\rangle}{2}=\frac{\langle h_i h_i'\rangle}{2}=\exp[-2RT_m]\left[\frac{N_0}{8\sqrt{2\pi}\lambda^{3/2}T_m^{3/2}}+\frac{\pi\delta_0}{4\lambda^6\left(1+P_M\right)\left(1+2P_M\right)^{5/2}}\exp[-(2\in_{mkl}\Omega_m)]\right.$$

$$\times\left[\frac{9}{16\left(T_m-\Delta T/2\right)^{5/2}\left(T_m+\frac{\Delta T}{1+2P_M}\right)^{5/2}}+\frac{9}{16\left(T_m+\Delta T/2\right)^{5/2}\left(T_m-\frac{\Delta T}{2\left(1+2P_M\right)}\right)^{5/2}}\right.$$

$$+\frac{5P_M\left(7P_M-6\right)}{16\left(1+2P_M\right)\left(T_m-\frac{\Delta T}{2}\right)^{3/2}\left(T_m+\frac{\Delta T}{2\left(1+2P_M\right)}\right)^{7/2}}$$

$$\left.\left.+\frac{5P_M\left(7P_M-6\right)}{16\left(1+2P_M\right)\left(T_m+\frac{\Delta T}{2}\right)^{3/2}\left(T_m-\frac{\Delta T}{2\left(1+2P_M\right)}\right)^{7/2}}+---\right]\right] \qquad \text{-----------}(4.5.20)$$

This is the decay law of magnetic energy fluctuations of concentration of a dilute contaminant undergoing a first order chemical reaction before the final period for the case of multi-point and multi-time in MHD turbulence in a rotating system.

4.6. Results and Discussion:

In equation (4.5.20) we obtained the decay law of magnetic energy fluctuations of a dilute contaminant undergoing a first order chemical reaction before the final period considering three-point correlation terms for the case of multi-point and multi-time in MHD turbulence in a rotating system.

If the system is non-rotating, $\Omega_m = 0$ then the equation (4.5.20) becomes

$$\frac{\langle h^2\rangle}{2} = \exp[-2RT_m]\left[\frac{N_0}{8\sqrt{2\pi}\lambda^{3/2}T_m^{3/2}} + \frac{\pi\delta_0}{4\lambda^6(1+P_M)(1+2P_M)^{5/2}}\right.$$

$$\times\left[\frac{9}{16\left(T_m - \Delta T/2\right)^{5/2}\left(T_m + \frac{\Delta T}{1+2P_M}\right)^{5/2}} + \frac{9}{16\left(T_m + \Delta T/2\right)^{5/2}\left(T_m - \frac{\Delta T}{2(1+2P_M)}\right)^{5/2}}\right.$$

$$+\frac{5P_M(7P_M-6)}{16(1+2P_M)\left(T_m - \frac{\Delta T}{2}\right)^{3/2}\left(T_m + \frac{\Delta T}{2(1+2P_M)}\right)^{7/2}}$$

$$\left.\left.+\frac{5P_M(7P_M-6)}{16(1+2P_M)\left(T_m + \frac{\Delta T}{2}\right)^{3/2}\left(T_m - \frac{\Delta T}{2(1+2P_M)}\right)^{7/2}} + ---\right]\right] \quad ----------(4.6.1)$$

which was obtained earlier by Islam and Sarker [56].

If we put ΔT=0, R=0, in equation (4.6.1), we can easily find out

$$\frac{\langle h^2\rangle}{2} = \frac{\langle h_i h_i'\rangle}{2} = \frac{N_0 T^{-3/2}}{8\sqrt{2\pi}\lambda^{3/2}} + \frac{\pi\delta_0}{4\lambda^6(1+P_M)(1+2P_M)^{5/2}}T^{-5}\left\{\frac{9}{16} + \frac{5}{16}\frac{P_M(7P_M-6)}{1+2P_M} + -----\right\} \quad --------(4.6.2)$$

which is same as obtained earlier by Sarker and Kishor [120].

This study shows that due to the effect of rotation of fluid in the flow field with chemical reaction of the first order in the concentrarion the magnetic field fluctuation in MHD turbulence in a rotating system for the case of multi-point and multi-time i,e.the turbulent energy decays more rapidly than the energy for non-rotating fluid and the faster rate is governed by $\exp[-\{2\in_{mkl}\Omega_m\}]$. Here the chemical reaction (R≠0) in MHD turbulence for the case of multi-point and multi-time causes the concentration to decay more they would for non-rotating system and it is governed by $\exp\left[-\left\{2RT_M+\in_{mkl}\Omega_m\right\}\right]$

The first term of right hand side of equation (4.5.20) corresponds to the energy of magnetic field fluctuation of concentration for the two-point correlation and the second term represents magnetic energy for the three-point correlation. In equation (4.5.20), the term associated with the three-point correlation die out faster than the two-point correlation. If higher order correlations are considered in the analysis, it appears that more terms of higher power of time would be added to the equation (4.5.20). For large times the last term in the equation (4.5.20) becomes negligible, leaving the -3/2 power decay law for the final period.

CHAPTER- IV

PART-B

FIRST ORDER REACTANT IN MAGNETO-HYDRODYNAMIC TURBULENCE BEFORE THE FINAL PERIOD OF DECAY FOR THE CASE OF MULTI-POINT AND MULTI-TIME IN PRESENCE OF DUST PARTICLES

4.7. Introduction:

The relative motion of dust particle and the air will dissipate energy because of the drag between dust and air, and extract energy from turbulent intensity is reduced than the Reynolds stresses will be decreased and the force required to maintain a given flow rate will likewise be reduced.The behavior of dust particles in a turbulent flow depends on the concentration of the particles and the size of the particles with respect to the scale of turbulent fluid.

In the past, many researchers worked taking dust particles. Saffman [118] derived and equation that describe the motion of a fluid containing small dust particle, which is applicable to laminar flows as well as turbulent flow. Using the Saffman's equations Michael and Miller [92] discussed the motion of dusty gas occupying the semi-infinite space above a rigid plane boundary. Sarker and Rahman [124] considered dust particles on their won works. Sinha [134] studied the effect of dust particles on the acceleration covariance of ordinary turbulence. Kishore and Sinha [88] also studied the rate of change of vorticity covariance in dusty fluid turbulence. Sarker [121] discussed the vorticity covariance of dusty fluid turbulence in a rotating frame.

Deissler [36, 37] developed a theory "decay of homogeneous turbulence for times before the final period". Using Deissler's theory, Loeffler and Deissler [81] studied the decay of temperature fluctuations in homogeneous turbulence before the final period. In their approach they considered the two and three-point correlation equations and solved these equations after neglecting fourth and higher order correlation terms. Using Deissler theory, Kumar and Patel [73] studied the first-order reactant in homogeneous turbulence before the final period of decay for the case of multi-point and single-time correlation. Kumar and Patel [74] extended their problem [73] for the case of multi-point and multi-time concentration correlation. Patel

[106] also studied in detail the same problem to carry out the numerical results. Sarker and Kishore [120] studied the decay of MHD turbulence at time before the final period using chandrasekher's relation [27]. Sarker and Islam [127] studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time. Sarker and Azad [137] studied the Decay of MHD turbulence before the final period for the case of multi-point and multi-time in presence of dust particle. Islam and Sarker [56] also studied the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time.

Here, we have studied the magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction in MHD turbulence before the final period of decay for the case of multi-point and multi-time in presence of dust particle. Here, we have considered the two-point, two-time and three-point, three-time correlation equations and solved these equations after neglecting the fourth-order correlation terms. Finally we obtained the decay law for magnetic field energy fluctuation of concentration of dilute contaminant undergoing a first order chemical reaction in MHD turbulence for the case of multi-point and multi-time in presence of dust particle is obtained. If the fluid is clean , the equation reduces to one obtained earlier by [56].

4.8. Basic Equations:

The equations of motion and the equation of continuity for viscous, incompressible dusty fluid MHD turbulent flow are given by

$$\frac{\partial u_i}{\partial t}+\frac{\partial}{\partial x_k}(u_i u_k - h_i h_k) = -\frac{\partial w}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_k \partial x_k}+f(u_i - v_i)\,, \qquad \text{--------- (4.8.1)}$$

$$\frac{\partial h_i}{\partial t}+\frac{\partial}{\partial x_k}(h_i u_k - u_i h_k) = \lambda\frac{\partial^2 h_i}{\partial x_k \partial x_k}\,, \qquad \text{--------- (4.8.2)}$$

$$\frac{\partial v_i}{\partial t}+v_k\frac{\partial v_i}{\partial x_K} = -\frac{K}{m_s}(v_i - u_i)\,, \qquad \text{---------- (4.8.3)}$$

$$\text{with}\ \frac{\partial u_i}{\partial x_i}=\frac{\partial v_i}{\partial x_i}=\frac{\partial h_i}{\partial x_i}=0\ . \qquad \text{---------- (4.8.4)}$$

Here, u_i, turbulence velocity component; h_i, magnetic field fluctuation component; v_i, dust particle velocity component; $w(\hat{x},t)=\frac{P}{\rho}+\frac{1}{2}\langle h^2 \rangle$ total MHD pressure $p(\hat{x},t)$, hydrodynamic pressure; ρ, fluid density; ν, Kinematic viscosity; $\lambda = {}^{\nu}\!/_{P_M}$, magnetic diffusivity; P_M, magnetic prandtl number; x_k, space co-ordinate; the subscripts can take on the values 1, 2 or 3 and the repeated subscripts in a term indicate a summation; $\in_{mkl}$, alternating tensor; $f=\frac{KN}{\rho}$, dimension of frequency ; N, constant number density of dust particle, K is Stokes's resistance coefficient which for spherical particle of radius r is 6πμr. $m_s=\frac{4}{3}\pi R_s^3 \rho_s$, mass of single spherical dust particle of radius R_s; ρ_s, constant density of the material in dust particle.

4.9. Two-Point, Two-Time Correlation and Spectral Equations:

It is assumed that (i) the turbulence and the concentration magnetic field are homogeneous (ii) the chemical reaction has no effect on the velocity field and (iii) the reaction rate and the magnetic diffusivity are constant, the induction equation of a magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction at the points p and p' separated by the vector $\hat{r}$ could be written as

$$\frac{\partial h_i}{\partial t}+u_k\frac{\partial h_i}{\partial x_k}-h_k\frac{\partial u_i}{\partial x_k}=\lambda\frac{\partial^2 h_i}{\partial x_k \partial x_k}-Rh_i \qquad \text{---------} (4.9.1)$$

and $$\frac{\partial h'_j}{\partial t'}+u'_k\frac{\partial h'_j}{\partial x'_k}-h'_k\frac{\partial u'_j}{\partial x'_k}=\lambda\frac{\partial^2 h'_j}{\partial x'_k \partial x'_k}-Rh'_j\ . \qquad \text{-----------} (4.9.2)$$

where R is the constant reaction rate.

Multiplying equation (4.9.1) by h'_j and equation (4.9.2) by h_i and taking ensemble average, we get

$$\frac{\partial\langle h_i h'_j\rangle}{\partial t}+\frac{\partial}{\partial x_k}\left[\langle u_k h_i h'_j\rangle-\langle u_i h_k h'_j\rangle\right]=\lambda\frac{\partial^2\langle h_i h'_j\rangle}{\partial x_k \partial x_k}-R\langle h_i h'_j\rangle \qquad \text{-----------} (4.9.3)$$

and $$\frac{\partial\langle h_i h'_j\rangle}{\partial t'}+\frac{\partial}{\partial x'_k}\left[\langle u'_k h_i h'_j\rangle-\langle\ u'_j h_i h'_k\rangle\right]=\lambda\frac{\partial^2\langle h_i h'_j\rangle}{\partial x'_k \partial x'_k}-R\langle h_i h'_j\rangle\ . \qquad \text{-----------} (4.9.4)$$

Angular bracket $\langle - - - - - \rangle$ is used to denote an ensemble average.

Using the transformations

$$\frac{\partial}{\partial x_k} = -\frac{\partial}{\partial r_k}, \; \frac{\partial}{\partial x'_k} = \frac{\partial}{\partial r_k}, \; \left(\frac{\partial}{\partial t}\right)t' = \left(\frac{\partial}{\partial t}\right)\Delta t - \frac{\partial}{\partial \Delta t}, \; \frac{\partial}{\partial t'} = \frac{\partial}{\partial \Delta t} \qquad \text{--------- (4.9.5)}$$

into equations (4.9.3) and (4.9.4), we obtain

$$\frac{\partial\langle h_i h'_j\rangle}{\partial t} + \frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle - \langle u'_j h_i h'_k\rangle\right](\hat{r},\Delta t,t) - \frac{\partial}{\partial r_k}\left[\langle u_k h_i h'_j\rangle - \langle u_i h_k h'_j\rangle\right](\hat{r},\Delta t,t)$$

$$= 2\lambda\frac{\partial^2\langle h_i h'_j\rangle}{\partial r_k \partial r_k} - 2R\langle h_i h'_j\rangle \qquad \text{---------- (4.9.6)}$$

and $$\frac{\partial\langle h_i h'_j\rangle}{\partial \Delta t} + \frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle - \langle u'_j h_i h'_k\rangle\right](\hat{r},\Delta t,t) = \lambda\frac{\partial^2\langle h_i h'_j\rangle}{\partial r_k \partial r_k} - R\langle h_i h'_j\rangle \qquad \text{-------- (4.9.7)}$$

Using the relations of Chandrasekhar [27]

$$\langle u_k h_i h'_j\rangle = -\langle u'_k h_i h'_j\rangle, \quad \langle u'_j h_i h'_k\rangle = \langle u_i h_k h'_j\rangle.$$

Equations (4.9.6) and (4.9.7) become

$$\frac{\partial\langle h_i h'_j\rangle}{\partial t} + 2\frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle - \langle u_i h_k h'_j\rangle\right] = 2\lambda\frac{\partial^2\langle h_i h'_j\rangle}{\partial r_k \partial r_k} - 2R\langle h_i h'_j\rangle \qquad \text{--------- (4.9.8)}$$

and $$\frac{\partial\langle h_i h'_j\rangle}{\partial t} + \frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle - \langle u_i h_k h'_j\rangle\right] = \lambda\frac{\partial^2\langle h_i h'_j\rangle}{\partial r_k \partial r_k} - R\langle h_i h'_j\rangle\ . \qquad \text{---------- (4.9.9)}$$

Now we write equations (4.9.8) and (4.9.9) in spectral form in order to reduce it to an ordinary differential equation by use of the following three-dimensional Fourier transforms:

$$\langle h_i h'_j\rangle(\hat{r},\Delta t,t) = \int_{-\infty}^{\infty}\langle\psi_i\psi'_j\rangle(\hat{K},\Delta t,t)\exp\left[\hat{i}(\hat{K}.\hat{r})\right]d\hat{K}\ , \qquad \text{---------- (4.9.10)}$$

and $$\langle u_i h_k h'_j\rangle(\hat{r},\Delta t,t) = \int_{-\infty}^{\infty}\langle\alpha_i\psi_k\psi'_j\rangle(\hat{K},\Delta t,t)\exp\left[\hat{i}(\hat{K}.\hat{r})\right]d\hat{K} \qquad \text{--------- (4.9.11)}$$

Interchanging the subscripts i and j then interchanging the points p and p′ gives

$$\langle u'_k h_i h'_j\rangle(\hat{r},\Delta t,t) = \langle u_k h_i h'_j\rangle(-\hat{r},-\Delta t,t+\Delta t)$$

$$= \int_{-\infty}^{\infty}\langle\alpha_i\psi_i\psi'_j\rangle(-\hat{K},-\Delta t,t+\Delta t)\exp\left[\hat{i}(\hat{K}.\hat{r})\right]d\hat{K} \qquad \text{--------- (4.9.12)}$$

where, $\hat{K}$ is known as a wave number vector and $d\hat{K}$ =dK$_1$ dK$_2$ dK$_3$. The magnitude of $\hat{K}$ has the dimension 1/length and can be considered to be the reciprocal of an eddy size. Substituting of equation (4.9.10) to (4.9.12) in to equations (4.9.8) and (4.9.9) leads to the spectral equations

$$\frac{\partial\langle\psi_i\psi'_j\rangle}{\partial t}+2\left[\lambda k^2+R\right]\langle\psi_i\psi'_j\rangle=2ik_k\left[\langle\alpha_i\psi_k\psi'_j\rangle\left(\hat{K},\Delta t,t\right)-\langle\alpha_k\psi_i\psi'_j\rangle\left(-\hat{K},-\Delta t,t+\Delta t\right)\right] \quad \text{------------ (4.9.13)}$$

and $$\frac{\partial\langle\psi_i\psi'_j\rangle}{\partial\Delta t}+\left[\lambda K^2+R\right]\langle\psi_i\psi'_j\rangle=ik_k\left[\langle\alpha_i\psi_k\psi'_j\rangle\left(\hat{K},\Delta t,t\right)-\langle\alpha_k\psi_i\psi'_j\rangle\left(-\hat{K},-\Delta t,t+\Delta t\right)\right] \quad \text{----------(4.9.14)}$$

The tensor equations (4.9.13) and (4.9.14) becomes a scalar equation by contraction of the indices i and j

$$\frac{\partial\langle\psi_i\psi'_i\rangle}{\partial t}+2\left[\lambda K^2+R\right]\langle\psi_i\psi'_i\rangle=2ik_k\left[\langle\alpha_i\psi_k\psi'_i\rangle\left(\hat{K},\Delta t,t\right)-\langle\alpha_k\psi_i\psi'_i\rangle\left(-\hat{K},-\Delta t,t+\Delta t\right)\right] \quad \text{--------------(4.9.15)}$$

and $$\frac{\partial\langle\psi_i\psi'_i\rangle}{\partial\Delta t}+\left[\lambda K^2+R\right]\langle\psi_i\psi'_i\rangle=ik_k\left[\langle\alpha_i\psi_k\psi'_i\rangle\left(\hat{K},\Delta t,t\right)-\langle\alpha_k\psi_i\psi'_i\rangle\left(-\hat{K},-\Delta t,t+\Delta t\right)\right] \quad \text{--------(4.9.16)}$$

The terms on the right side of equations (4.9.15) and (4.9.16) are collectively proportional to what is known as the magnetic energy transfer terms.

4.10. Three-Point, Three-Time Correlation and Spectral Equations:

To find the three-point correlation equations, the procedure can be used as before. For this purpose we take the momentum equation of dusty fluid MHD turbulence at the point P and the induction equations of magnetic field fluctuations, governing the concentration of a dilute contaminant undergoing a first order chemical reaction at p′ and p″ separated by the vector $\hat{r}$ and $\hat{r}'$ as

$$\frac{\partial u_l}{\partial t}+u_k\frac{\partial u_l}{\partial x_k}-h_k\frac{\partial h_l}{\partial x_k}=-\frac{\partial w}{\partial x_l}+\nu\frac{\partial^2 u_l}{\partial x_k\partial x_k}+f\left(u_l-v_l\right), \quad \text{------- (4.10.1)}$$

$$\frac{\partial h'_i}{\partial t'}+u'_k\frac{\partial h'_i}{\partial x'_k}-h'_k\frac{\partial u'_i}{\partial x'_k}=\lambda\frac{\partial^2 h'_i}{\partial x'_k\partial x'_k}-Rh'_i, \quad \text{-------- (4.10.2)}$$

$$\frac{\partial h_j''}{\partial t''}+u_k''\frac{\partial h_j''}{\partial x_k''}-h_k''\frac{\partial u_j''}{\partial x_k''}=\lambda\frac{\partial^2 h_j''}{\partial x_k''\partial x_k''}-Rh_j'' . \qquad \text{------- (4.10.3)}$$

Multiplying equation (4.10.1) by $h'_i h_j''$, equation (4.10.2) by $u_l h_j''$ and equation (4.10.3) by $u_l h_i'$, taking ensemble average, one obtains

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t}+\frac{\partial}{\partial x_k}\left[\langle u_k u_l h_i' h_j''\rangle-\langle h_k h_l h_i' h_j''\rangle\right]=\frac{\partial\langle w h_i' h_j''\rangle}{\partial x_l}+\nu\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k\partial x_k}$$
$$+f(\langle u_l h_i' h_j''\rangle-\langle v_l h_i' h_j''\rangle) \qquad \text{---------- (4.10.4)}$$

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t'}+\frac{\partial}{\partial x_k'}\left[\langle u_l u_k' h_i' h_j''\rangle-\langle u_l u_i' h_k' h_j''\rangle\right]=\lambda\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k'\partial x_k'}-R\langle u_l h_i h_j''\rangle \qquad \text{-------- (4.10.5)}$$

and
$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t''}+\frac{\partial}{\partial x_k''}\left[\langle u_l u_k'' h_i' h_j''\rangle-\langle u_l u_j'' h_i' h_k''\rangle\right]=\lambda\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k''\partial x_k''}-R\langle u_l h_i h_j''\rangle . \qquad \text{-------- (4.10.6)}$$

Using the transformations

$$\frac{\partial}{\partial x_k}=-\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'}\right),\quad \frac{\partial}{\partial x_k'}=\frac{\partial}{\partial r_k},\quad \frac{\partial}{\partial x_k''}=\frac{\partial}{\partial r_k'} ,$$

$$\left(\frac{\partial}{\partial t}\right)t',t''=\left(\frac{\partial}{\partial t}\right)\Delta t,\Delta t'-\frac{\partial}{\partial\Delta t}-\frac{\partial}{\partial\Delta t'} ,$$

$$\frac{\partial}{\partial t'}=\frac{\partial}{\partial\Delta t},\quad \frac{\partial}{\partial t''}=\frac{\partial}{\partial\Delta t'}$$

into equations (4.10.4) to (4.10.6), we have

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t}-\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'}\right)\left[\langle u_k u_l h_i' h_j''\rangle-\langle h_k h_l h_i' h_j''\rangle\right]+\frac{\partial}{\partial r_k}\left[\langle u_l u_k' h_i' h_j''\rangle-\langle u_l u_i' h_k' h_j''\rangle\right]$$

$$+\frac{\partial}{\partial r_k'}\left[\langle u_l u_k'' h_i' h_j''\rangle-\langle u_l u_j'' h_i' h_k''\rangle\right]=-\left(\frac{\partial}{\partial r_l}+\frac{\partial}{\partial r_l'}\right)\langle w h_i' h_j''\rangle+\nu\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'}\right)^2\langle u_l h_i' h_j''\rangle$$
$$+\lambda\left[\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k\partial r_k}+\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k'\partial r_k'}\right]+f\left(\langle u_l h_i' h_j''\rangle-\langle v_l h_i' h_j''\rangle\right) \qquad \text{--------- (4.10.7)}$$

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial \Delta t}+\frac{\partial}{\partial r_k}\left[\langle u_l u_k' h_i' h_j''\rangle-\langle u_l u_i' h_k' h_j''\rangle\right]=\lambda\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k \partial r_k}-R\langle u_l h_i' h_j''\rangle \quad \text{------ (4.10.8)}$$

and

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial \Delta t'}+\frac{\partial}{\partial r_k'}\left[\langle u_l u_k'' h_i' h_j''\rangle-\langle u_l u_j'' h_i' h_k''\rangle\right]=\lambda\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial r_k' \partial r_k'}-R\langle u_l h_i' h_j''\rangle \quad \text{---------(4.10.9)}$$

In order to convert equations (4.10.7)–(4.10.9) to spectral form, we can define the following six dimensional Fourier transforms:

$$\langle u_l h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_l \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp[\hat{i}(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')]d\hat{K}d\hat{K}' \quad \text{-------------(4.10.10)}$$

$$\langle u_l u_k' h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_l \phi_K' \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp[\hat{i}(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')]d\hat{K}d\hat{K}' \quad \text{----------(4.10.11)}$$

$$\langle w h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \gamma \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp[\hat{i}(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')]d\hat{K}d\hat{K}' \quad \text{----------(4.10.12)}$$

$$\langle u_k u_l h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_K \phi_l \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp[\hat{i}(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')]d\hat{K}d\hat{K}' \quad \text{-----------(4.10.13)}$$

$$\langle h_k h_l h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \beta_K \beta_l \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp[\hat{i}(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')]d\hat{K}d\hat{K}' \quad \text{------------ (4.10.14)}$$

$$\langle u_l u_i' h_K' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_l \phi_i' \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp[\hat{i}(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')]d\hat{K}d\hat{K}' \quad \text{-------------(4.10.15)}$$

$$\langle v_l h_i' h_j''\rangle(\hat{r},\hat{r},\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \mu_l \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp[\hat{i}(\hat{K}.\hat{r}+\hat{K}'.\hat{r}')]d\hat{K}d\hat{K}' \quad \text{------------- (4.10.16)}$$

Interchanging the points P′ and P″ along with the indices i and j result in the relations

$$\langle u_l u_k'' h_i' h_j''\rangle=\langle u_l u_k' h_i' h_j''\rangle .$$

By use the equations (4.10.10)-(4.10.16) and the facts, we can write equations (4.10.7)-(4.10.9) in the form

$$\frac{\partial}{\partial t}\left\langle \phi_l \beta_i' \beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)+\lambda\left[\left(1+P_M\right)\left(k^2+k'^2\right)+2P_M kk'+\frac{2R}{\lambda}-\frac{1}{\lambda}f\right]$$

$$\times\left\langle \phi_l \beta_i' \beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)=\left[i\left(k_k+k_k'\right)\left\langle \phi_k\phi_l \beta_i' \beta''\right\rangle-i\left(k_k+k_k'\right)\left\langle \beta_k\beta_l \beta_i' \beta_j''\right\rangle\right.$$

$$-i\left(k_k+k_k'\right)\left\langle \phi_l\phi_k' \beta_i' \beta_j''\right\rangle+i\left(k_k+k_k'\right)\left\langle \phi_l\phi_i' \beta_k' \beta_j''\right\rangle-i\left(k_l+k_l'\right)\left\langle \gamma \beta_i' \beta_j''\right\rangle$$

$$\left.-f\left\langle \mu_l \beta_i' \beta_j''\right\rangle\right]\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right) \qquad \text{---------- (4.10.17)}$$

$$\frac{\partial}{\partial \Delta t}\left\langle \phi_l \beta_i' \beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)+\lambda\left[K^2+\frac{R}{\lambda}\right]\left\langle \phi_l \beta_i' \beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)$$

$$=-ik_k\left\langle \phi_l\,\phi_k' \beta_i' \beta_j''\right\rangle\left(\hat{K},\hat{K}'\Delta t,\Delta t',t\right)+ik_k\left\langle \phi_l\phi_i' \beta_k' \beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right) \qquad \text{--------- (4.10.18)}$$

and $\frac{\partial}{\partial \Delta t'}\left\langle \phi_l \beta_i' \beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)+\lambda\left[K^2+\frac{R}{\lambda}\right]\left\langle \phi_l \beta_i' \beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)$

$$=-ik_k'\left\langle \phi_l\,\phi_k' \beta_i' \beta_j''\right\rangle\left(\hat{K},\hat{K}'\Delta t,\Delta t',t\right)+ik_k'\left\langle \phi_l\phi_i' \beta_k' \beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right) \qquad \text{--------- (4.10.19)}$$

If the derivative with respect to x_l is taken of the momentum equation (4.10.1) for the point P, the equation multiplied by $h_i'h_j''$ and time average taken, the resulting equation

$$-\frac{\partial^2\left\langle wh_i'h_j''\right\rangle}{\partial x_l\partial x_l}=\frac{\partial^2}{\partial x_l\partial x_k}\left(\left\langle u_l u_k h_i'h_j''\right\rangle-\left\langle h_l h_k h_i'h_j''\right\rangle\right) \qquad \text{---------- (4.10.20)}$$

Writing this equation in terms of the independent variables $\hat{r}$ and $\hat{r}'$

$$-\left[\frac{\partial^2}{\partial r_l\partial r_l}+2\frac{\partial^2}{\partial r_l\partial r_l'}+\frac{\partial^2}{\partial r_l'\partial r_l'}\right]\left\langle wh_i'h_j''\right\rangle=\left[\frac{\partial^2}{\partial r_l\partial r_k}+\frac{\partial^2}{\partial r_l'\partial r_k}+\frac{\partial^2}{\partial r_l\partial r_k'}+\frac{\partial^2}{\partial r_l'\partial r_k'}\right]\times$$

$$\left(\left\langle u_l u_k h_i'h_j''\right\rangle-\left\langle h_l h_k h_i'h_j''\right\rangle\right). \qquad \text{---------(4.10.21)}$$

Taking the Fourier transforms of equation (4.10.8)

$$-\left\langle \gamma \beta_i' \beta_j''\right\rangle=\frac{\left(k_l k_k+k_l' k_k+k_l k_k'+k_l' k_k'\right)\left(\left\langle \phi_l\phi_k \beta_i' \beta_j''\right\rangle-\left\langle \beta_l\beta_k \beta_i' \beta_j''\right\rangle\right)}{k_l k_l+2k_l k_l'+k_l' k_l'}. \qquad \text{----------(4.10.22)}$$

Equation (4.10.22) can be used to eliminate $\langle \gamma\beta_i'\beta_j'' \rangle$ from equation (4.10.17)

The tensor equations (4.10.17) to (4.10.19) can be converted to scalar equation by contraction of the indices i and j and inner multiplication by k_l

$$\frac{\partial}{\partial t} k_l \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda \left[(1 + P_M)(k^2 + k'^2) + 2P_M kk' + \frac{2R}{\lambda} - \frac{1}{\lambda} f \right] \times$$

$$\langle \phi_l \beta_i'' \beta_i'' \rangle (\hat{K}, \hat{K}, \Delta t, \Delta t', t) = i(k_k + k_k')\langle \phi_k \phi_l \beta_i' \beta_j'' \rangle$$

$$(\hat{K}, \hat{K}', \Delta t, \Delta t', t) - i(k_k + k_k')\langle \beta_K \beta_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) - i(k_k + k_k')$$

$$\langle \phi_l \phi_k' \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + i(k_k + k_k')\langle \phi_l \phi_i' \beta_k' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) - i(k_l + k_l')$$

$$\langle \gamma \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) - f \langle \mu_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) \quad \text{---------} (4.10.23)$$

$$\frac{\partial}{\partial \Delta t} k_L \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda \left[K^2 + \frac{R}{\lambda} \right] \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t)$$

$$= -ik_k \langle \phi_l \phi_k' \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + ik_k \langle \phi_l \phi_i' \beta_k' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) \quad \text{---------} (4.10.24)$$

and $$\frac{\partial}{\partial \Delta t'} k_l \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda \left[K'^2 + \frac{R}{\lambda} \right] \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t)$$

$$= -ik_k' \langle \phi_l \phi_k' \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + ik_k' \langle \phi_l \phi_i' \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) \quad \text{.-------} (4.10.25)$$

4.11. Solution for Times Before the Final Period:

We know that the equation for final period of decay is obtained by considering the two-point correlations after neglecting third-order correlation terms. To study the decay for times before the final period, the three-point correlations are considered and the quadruple correlation terms are neglected because the quadruple correlation terms decays faster than the lower-order correlation terms. The term $\langle \gamma\beta_i'\beta_j'' \rangle$ associated with the pressure fluctuations should also be neglected. Thus neglecting all the terms on the right hand side of equations (4.10.23) to (4.10.25)

$$\frac{\partial}{\partial t}K_l\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)+\lambda\left[(1+P_M)(k^2+k'^2)+2P_M kk'+\frac{2R}{\lambda}-\frac{1}{\lambda}fs\right]\times$$

$$\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)=0 \qquad \text{---------- (4.11.1)}$$

$$\frac{\partial}{\partial \Delta t}K_l\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)+\lambda\left[k^2+\frac{R}{\lambda}\right]\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)=0 \qquad \text{----- (4.11.2)}$$

and $\frac{\partial}{\partial \Delta t'}K_l\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)+\lambda\left[k'^2+\frac{R}{\lambda}\right]\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)=0$ --------- (4.11.3)

where $\langle\mu_l\beta_i'\beta_i''\rangle=C\langle\phi_l\beta_i'\beta_i''\rangle$ and 1-C=S, here C and S are arbitrary constant.

Integrating equations (4.11.1) to (4.11.3) between t_o and t, we obtain

$$k_l\langle\phi_l\beta_i'\beta_i''\rangle=f_l\exp\left\{-\lambda\left[(1+P_M)(k^2+k'^2)+2P_M kk'\cos\theta+\frac{2R}{\lambda}-\frac{1}{\lambda}fs(t-t_o)\right\}\right.,$$

$$k_l\langle\phi_l\beta_i'\beta_i''\rangle=g_l\exp\left[-\lambda\left(K^2+\frac{R}{\lambda}\right)\Delta t\right]$$

and $k_l\langle\phi_l\beta_i'\beta_i''\rangle=q_l\exp\left[-\lambda\left(k'^2+\frac{R}{\lambda}\right)\Delta t'\right]$.

For these relations to be consistent, we have

$$k_l\langle\phi_l\beta_i'\beta_i''\rangle=k_l\langle\phi_l\beta_i'\beta_i''\rangle_o\exp\left\{-\lambda\left[(1+P_M)(k^2+k'^2)(t-t_o)+k^2\Delta t+k'^2\Delta t'\right.\right.$$

$$\left.\left.+2P_M kk'\cos\theta(t-t_o)+\frac{2R}{\lambda}\left(t-t_0+\frac{\Delta t+\Delta t'}{2}\right)-\frac{fs}{\lambda}(t-t_o)\right]\right\} \qquad \text{-------- (4.11.4)}$$

where, θ is the angle between $\hat{K}$ and $\hat{K}'$ and $\langle\phi_l\beta_i'\beta_i''\rangle_o$ is the value of $\langle\phi_l\beta_i'\beta_i''\rangle$ at $t=t_o$, $\Delta t=\Delta t'=0$, $\lambda=\frac{\nu}{P_M}$

By letting $\hat{r}'=0$, $\Delta t'=0$ in the equation (4.10.10) and comparing with equations (4.9.11) and (4.9.12) we get

$$\langle\alpha_i\psi_k\psi_i'\rangle(\hat{K},\Delta t,t)=\int_{-\infty}^{\infty}\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,o,t)d\hat{K}' \qquad \text{----------- (4.11.5)}$$

and $\langle \alpha_i \psi_k \psi_i' \rangle (-\hat{K}, -\Delta t, t+\Delta t) = \int_{-\infty}^{\infty} \langle \phi_l \beta_i' \beta_i'' \rangle (-\hat{K}, \hat{K}', \Delta t, o, t) d\hat{K}'$. ---------- (4.11.6)

Substituting equation (4.11.4) to (4.11.6) into equation (4.9.15), one obtains

$$\frac{\partial}{\partial t}\langle \psi_i \psi_i' \rangle (\hat{K}, \Delta t, t) + 2\lambda \left[k^2 + \frac{R}{\lambda}\right] \langle \psi_i \psi_i' \rangle (\hat{K}, \Delta t, t) = \int_{-\infty}^{\infty} 2ik_l \left[\langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, 0, t) \right.$$

$$\left. - \langle \phi_l \beta_i' \beta_i'' \rangle (-\hat{K}, -\hat{K}', \Delta t, 0, t) \right]_o \exp[-\lambda\{(1+P_M)(k^2 + k'^2)(t-t_o)$$

$$+ k^2 \Delta t + 2P_M (t-t_o) kk' \cos\theta + \frac{2R}{\lambda}(t - t_0 + \Delta t) - \frac{fs}{\lambda}(t-t_o)\}] \, d\hat{k} \quad \text{-----------(4.11.7)}$$

Now, $d\hat{K}'$ can be expressed in terms of k′ and θ as $-2\pi k' d(\cos\theta) dk'$ (cf. Deissler [37])

i.e. $d\hat{K}' = -2\pi k' d(\cos\theta) dk'$ --------- (4.11.8)

Substituting of equation (4.11.8) in equation (4.11.7) yields

$$\frac{\partial}{\partial t}\langle \psi_i \psi_i' \rangle (\hat{K}, \Delta t, t) + 2\lambda \left[k^2 + \frac{R}{\lambda}\right] \langle \psi_i \psi_i' \rangle (\hat{K}, \Delta t, t) = 2\int_0^{\infty} 2\pi i k_l \left[\langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}') \right.$$

$$\left. - \langle \phi_l \beta_i' \beta_i'' \rangle (-\hat{K}, -\hat{K}') \right]_o k'^2 \left[\int_{-1}^{1} \exp\{-\lambda[(1+P_M)(k^2 + k'^2)(t-t_o) \right.$$

$$\left. + k^2 \Delta t + 2P_M (t-t_o) kk' \cos\theta + \frac{2R}{\lambda}(t - t_0 + \Delta t/2) - \frac{fs}{\lambda}(t-t_o)]\} d(\cos\theta) \right] dk'$$

----------(4.11.9)

The quantity $[\langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}') - \langle \phi_l \beta_i' \beta_i'' \rangle (-\hat{K}, -\hat{K}')]_0$ depends on the initial conditions of the turbulence.

In order to find the solution completely and following Loeffler and Deissler [81] we assume that

$$ik_l \left[\langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}') - \langle \phi_l \beta_i' \beta_i'' \rangle (-\hat{K}, -\hat{K}') \right]_o = \frac{-\delta_0}{(2\pi)^2} (k^2 k'^4 - k^4 k'^2) \quad \text{---------- (4.11.10)}$$

where, δ_0 is a constant determined by the initial conditions. The negative sign is placed in front of δ_0 in order to make the transfer of energy from small to large wave numbers for positive value of δ_0.

Substituting equation (4.11.10) into equation (4.11.9), we get

$$\frac{\partial}{\partial t}2\pi\langle\psi_i\psi_i'\rangle(\hat{K},\Delta t,t)+2\lambda\left[k^2+R/\lambda\right]2\pi\langle\psi_i\psi_i'\rangle(\hat{K},\Delta t,t)=-2\delta_0\int_0^\infty(k^2k'^4-k^4k'^2)k'^2$$

$$\left[\int_{-1}^{1}\exp\{-\lambda[(1+P_M)(k^2+k'^2)(t-t_o)+k^2\Delta t+2P_M(t-t_o)kk'\cos\theta\right.$$

$$\left.+\frac{2R}{\lambda}(t-t_0+\Delta t/2)\ -\frac{fs}{\lambda}(t-t_o)]\}d(\cos\theta)\ \right]d\hat{k}' \qquad \text{---------} (4.11.11)$$

Multiplying both sides of equation (4.11.11) by k^2, we get

$$\frac{\partial E}{\partial t}+2\lambda k^2E=F \qquad \text{----------} (4.11.12)$$

where, $E=2\pi k^2\langle\psi_i\psi_i'\rangle$, E is the magnetic energy spectrum function and F is the magnetic energy transfer term and is given by

$$F=-2\delta_0\int_0^\infty(k^2k'^4-k^4k'^2)k^2k'^2\times\left[\int_{-1}^{1}\exp\{-\lambda[(1+P_M)(k^2+k'^2)(t-t_o)\right.$$

$$\left.+k^2\Delta t+2P_M(t-t_o)kk'\cos\theta\ +\frac{2R}{\lambda}(t-t_0+\Delta t/2)-\frac{fs}{\lambda}(t-t_o)]\}d(\cos\theta)\ \right]dk'$$

$$\text{--------}(4.11.13)$$

Integrating equation (4.11.13) with respect to cosθ and k' we have

$$F=-\frac{\delta_oP_M\sqrt{\pi}}{4\lambda^{3/2}(t-t_o)^{3/2}(1+P_M)^{5/2}}\exp\left\{\left(\frac{fs}{\lambda}\right)(t-t_o)\right\}\times$$

$$\exp\left[\frac{-k^2\lambda(1+2P_M)}{1+P_M}\left(t-t_o+\frac{1+P_M}{1+2P_M}\Delta t\right)-2R(t-t_0+\Delta t/2)\right]\times\left[\frac{15P_Mk^4}{4P_M^2\lambda^2(t-t_o)^2(1+P_M)}\right.$$

$$+\left\{\frac{5P_M^2}{(1+P_M)^2}-\frac{3}{2}\right\}\frac{k^6}{P_M\lambda(t-t_o)}+\left\{\frac{P_M^3}{(1+P_M)^3}-\frac{P_M}{1+P_M}\right\}k^8$$

$$-\frac{\delta_o P_M\sqrt{\pi}}{4\lambda^{3/2}(t-t_o+\Delta t)^{3/2}(1+P_M)^{5/2}}\exp\left\{\left(\frac{fs}{\lambda}\right)(t-t_o)\right\}\times$$

$$\exp\left[\frac{-k^2\lambda(1+2P_M)}{1+P_M}\left(t-t_o+\frac{P_M}{1+P_M}\Delta t\right)-2R(t-t_0+\Delta t/2)\right]\times\left[\frac{15P_M k^4}{4\nu^2(t-t_o+\Delta t)^2(1+P_M)}\right.$$

$$\left.+\left\{\frac{5P_M^2}{(1+P_M)^2}-\frac{3}{2}\right\}\frac{k^6}{P_M\lambda(t-t_o+\Delta t)}+\left\{\frac{P_M^3}{(1+P_M)^3}-\frac{P_M}{1+P_M}\right\}k^8\right] \qquad \text{------ (4.11.14)}$$

The series of equation (4.11.14) contains only even power of k and start with k^4 and the equation represents the transfer function arising owing to consideration of magnetic field at three-point and three-times.

If we integrate equation (4.11.14) for $\Delta t=0$ over all wave numbers, we find that

$$\int_0^\infty F\,dk=0 \qquad \text{---------- (4.11.15)}$$

which indicates that the expression for F satisfies the condition of continuity and homogeneity. Physically it was to be expected as F is a measure of the energy transfer and the total energy transferred to all wave numbers must be zero.

The linear equation (4.11.12) can be solved to give

$$E=\exp\left[-2\lambda k^2(t-t_o+\Delta t/2)\int F\exp\left[2\lambda(k^2+R/\lambda)(t-t_o+\Delta t/2)\right]dt\right.$$

$$\left.+J(k)\exp\left[-2\lambda(k^2+R/\lambda(t-t_o+\Delta t/2)\right]\right. \qquad \text{---------- (4.11.16)}$$

where, $J(k)=\frac{N_o k^2}{\pi}$ is a constant of integration and can be obtained as by Corrsin[32].

Substituting the values of F from equation (4.11.14) into equation (4.11.16) gives the equation

$$E=\frac{N_o k^2}{\pi}\exp\left[-2\lambda(k^2+R/\lambda)(t-t_o+\Delta t/2)\right]+\frac{\delta_o P_M\sqrt{\pi}}{4\lambda^{3/2}(1+P_M)^{7/2}}\times\exp[fs(t-t_o)]$$

$$\exp\left[\frac{-k^2\lambda(1+2P_M)}{1+P_M}\left(t-t_o+\frac{1+P_M}{1+2P_M}\Delta t\right)-2R(t-t_0+\Delta t/2)\right]$$

$$\left[\frac{3k^4}{2P_M\lambda^2(t-t_0)^{5/2}}+\frac{(7P_M-6)k^6}{3\lambda(1+P_M)(t-t_0)^{3/2}}-\frac{4(3P_M^2-2P_M+3)k^8}{3(1+P_M)^2(t-t_0)^{1/2}}\right.$$

$$\left.+\frac{8\sqrt{\lambda}(3P_M^2-2P_M+3)k^9}{3(1+P_M)^{5/2}}F(\omega)\right]+\frac{\delta_o P_M\sqrt{\pi}}{4\lambda^{3/2}(1+P_M)^{7/2}}\exp[fs(t-t_o)]$$

$$\exp\left[\frac{-k^2\lambda(1+2P_M)}{1+P_M}\left(t-t_o+\frac{P_M}{1+P_M}\Delta t\right)-2R(t-t_0+\Delta t/2)\right]$$

$$\left[\frac{3k^4}{2P_M\lambda^2(t-t_0+\Delta t)^{5/2}}+\frac{(7P_M-6)k^6}{3\lambda(1+P_M)(t-t_0+\Delta t)^{3/2}}\right.$$

$$\left.-\frac{4(3P_M^2-2P_M+3)k^8}{3(1+P_M)^2(t-t_0+\Delta t)^{1/2}}+\frac{8\sqrt{\lambda}(3P_M^2-2P_M+3)k^9F(\omega)}{(1+P_M)^{5/2}P_M^{1/2}}\right] \quad \text{--------- (4.11.17)}$$

where $F(\omega)=e^{-\omega^2}\int_0^{\omega}e^{x^2}dx$, $\quad \omega=k\sqrt{\frac{\lambda(t-t_0)}{1+P_M}}$ or $\quad k\sqrt{\frac{\lambda(t-t_0+\Delta t)}{1+P_M}}$

By setting $\hat{r}=0$, j=i, $d\hat{k} = -2\pi k^2 d(\cos\theta)dk$ and $E=2\pi k^2\langle\psi_i\psi_j'\rangle$ in equation (4.9.10) we get the expression for magnetic energy decay law as

$$\frac{\langle h_i h_i'\rangle}{2}=\int_0^{\infty}E\,dk \quad \text{--------- (4.11.18)}$$

Substituting equation (4.11.17) into equation (4.11.18) and after integration, we get

$$\frac{\langle h_i h_i'\rangle}{2}=\frac{N_0}{8\sqrt{2\pi}\lambda^{3/2}(T+\Delta T/2)^{3/2}}\exp[-2R(T+\Delta T/2)]$$

$$+\frac{\pi\delta_0}{4\lambda^6(1+P_M)(1+2P_M)^{5/2}}\exp[-2R(T+\Delta T/2)]\exp[(fs)]$$

$$+\frac{5P_M(7P_M-6)}{16(1+2P_M)T^{3/2}\left(T+\frac{1+P_M}{1+2P_M}\Delta T\right)^{7/2}}+\frac{5P_M(7P_M-6)}{16(1+2P_M)(T+\Delta T)^{3/2}\left(T+\frac{P_M}{1+2P_M}\Delta T\right)^{7/2}}$$

$$+\frac{35P_M(3P_M^2-2P_M+3)}{8(1+2P_M)T^{1/2}\left(T+\frac{1+P_M}{1+2P_M}\Delta T\right)^{9/2}}+\frac{35P_M(3P_M^2-2P_M+3)}{8(1+2P_M)(T+\Delta T)^{1/2}\left(T+\frac{P_M}{1+2P_M}\Delta T\right)^{9/2}}$$

$$+\frac{8P_M(3P_M^2-2P_M+3)(1+2P_M)^{5/2}}{3.2^{23/2}(1+P_M)^{11/2}}\sum_{n=0}^{\infty}\frac{1.3.5............(2n+9)}{n!(2n+1)2^{2n}(1+P_M)^n}\times$$

$$\left.\left\{\frac{T^{(2n+1)/2}}{\left(T+\Delta T/2\right)^{(2n+1)/2}}+\frac{(T+\Delta T)^{(2n+1)/2}}{\left(T+\Delta T/2\right)^{(2n+1)/2}}\right\}\right] \quad \text{---------} (4.11.19)$$

where T=t-t_0 .

For $T_m = T + \Delta T/2$, equation (4.11.19) takes the form

$$\frac{\langle h^2\rangle}{2}=\frac{\langle h_i h_i'\rangle}{2}=\exp[-2RT_m]\left[\frac{N_0}{8\sqrt{2\pi}\lambda^{3/2}T_m^{3/2}}+\frac{\pi\delta_0}{4\lambda^6(1+P_M)(1+2P_M)^{5/2}}\exp[fs]\right.$$

$$\times\left[\frac{9}{16\left(T_m-\Delta T/2\right)^{5/2}\left(T_m+\frac{\Delta T}{1+2P_M}\right)^{5/2}}+\frac{9}{16\left(T_m+\Delta T/2\right)^{5/2}\left(T_m-\frac{\Delta T}{2(1+2P_M)}\right)^{5/2}}\right.$$

$$+\frac{5P_M(7P_M-6)}{16(1+2P_M)\left(T_m-\frac{\Delta T}{2}\right)^{3/2}\left(T_m+\frac{\Delta T}{2(1+2P_M)}\right)^{7/2}}$$

$$\left.\left.+\frac{5P_M(7P_M-6)}{16(1+2P_M)\left(T_m+\frac{\Delta T}{2}\right)^{3/2}\left(T_m-\frac{\Delta T}{2(1+2P_M)}\right)^{7/2}}+---\right]\right]. \quad \text{-------}(4.11.20)$$

This is the decay law of magnetic energy fluctuations of concentration of a dilute contaminant undergoing a first order chemical reaction of MHD turbulence before the final period for the case of multi-point and multi-time in presence of dust particle.

4.12. Results and Discussion:

In equation (4.11.20) we obtained the decay law of magnetic energy fluctuations of a dilute contaminant undergoing a first order chemical reaction before the final period considering three-point correlation terms for the case of multi-point and multi-time in MHD turbulence in presence of dust particle.

If the fluid is clean, f=0 then the equation (4.11.20) becomes

$$\frac{\langle h^2\rangle}{2} = \exp[-2RT_m]\left[\frac{N_0}{8\sqrt{2\pi}\lambda^{3/2}T_m^{3/2}} + \frac{\pi\delta_0}{4\lambda^6(1+P_M)(1+2P_M)^{5/2}}\right.$$
$$\times\left[\frac{9}{16\left(T_m-\Delta T/2\right)^{5/2}\left(T_m+\frac{\Delta T}{1+2P_M}\right)^{5/2}} + \frac{9}{16\left(T_m+\Delta T/2\right)^{5/2}\left(T_m-\frac{\Delta T}{2(1+2P_M)}\right)^{5/2}}\right.$$
$$+\frac{5P_M(7P_M-6)}{16(1+2P_M)\left(T_m-\frac{\Delta T}{2}\right)^{3/2}\left(T_m+\frac{\Delta T}{2(1+2P_M)}\right)^{7/2}}$$
$$\left.\left.+\frac{5P_M(7P_M-6)}{16(1+2P_M)\left(T_m+\frac{\Delta T}{2}\right)^{3/2}\left(T_m-\frac{\Delta T}{2(1+2P_M)}\right)^{7/2}}+---\right]\right] \quad \text{----------(4.12.1)}$$

which was obtained earlier by Islam and Sarker [56].

If we put ΔT=0, R=0, in equation (4.12.1) we can easily find out

$$\frac{\langle h^2\rangle}{2} = \frac{\langle h_i h_i'\rangle}{2} = \frac{N_0 T^{-3/2}}{8\sqrt{2\pi}\lambda^{3/2}} + \frac{\pi\delta_0}{4\lambda^6(1+P_M)(1+2P_M)^{5/2}}T^{-5}\left\{\frac{9}{16}+\frac{5}{16}\frac{P_M(7P_M-6)}{1+2P_M}+-----\right\} \quad \text{--------(4.12.2)}$$

which is same as obtained earlier by Sarker and Kishor [120].

This study shows that due to the effect of rotation of fluid in the flow field with chemical reaction of the first order in the concentrarion the magnetic field fluctuation in MHD turbulence in presence of dust particle for the case of multi-point and multi-time i,e.the turbulent energy decays more slowly than the energy for clean fluid and the rate is governed by $\exp[fs]$. Here the chemical reaction ($R\neq0$) in dusty fluid MHD turbulence for the case of multi-point and multi-time causes the concentration to decay more they would for clean fluid and it is governed by $\exp\left[-\{2RT_M - fs\}\right]$

The first term of right hand side of equation (4.11.20) corresponds to the energy of magnetic field fluctuation of concentration for the two-point correlation and the second term represents magnetic energy for the three-point correlation. In equation (4.11.20), the term associated with the three-point correlation die out faster than the two-point correlation. If higher order correlations are considered in the analysis, it appears that more terms of higher power of time would be added to the equation (4.11.20). For large times the last term in the equation (4.11.20) becomes negligible, leaving the -3/2 power decay law for the final period.

CHAPTER-IV

PART-C

FIRST ORDER REACTANT IN MAGNETO-HYDRODYNAMIC TURBULENCE BEFORE THE FINAL PERIOD OF DECAY FOR THE CASE OF MULTI-POINT AND MULTI-TIME IN A ROTATING SYSTEM IN PRESENCE OF DUST PARTICLE

4.13. Introduction:

Funada, Tuitiya and Ohji [47] considered the effect of coriolis force on turbulent motion in presence of strong magnetic field with the assumption that the coriolis force term is balanced by the geostropic wind approximation Sarker and Islam [129] studied the decay of dusty fluid turbulence before the final period in a rotating system. Kishore and Sinha [68] studied the rate of change of vorticity covariance in dusty fluid turbulence. Sinha [134] also studied the effect of dust particles on the acceleration covariance of ordinary turbulence. Deissler [36, 37] developed a theory "decay of homogeneous turbulence for times before the final period".

Using Deissler's theory, Loeffler and Deissler [81] studied the decay of temperature fluctuations in homogeneous turbulence before the final period. In their approach they considered the two and three-point correlation equations and solved these equations after neglecting fourth and higher order correlation terms. Kumar and Patel [73] studied the first-order reactant in homogeneous turbulence before the final period of decay for the case of multi-point and single-time correlation. Kumar and Patel [74] extended their problem [73] for the case of multi-point and multi-time concentration correlation. Patel [106] also studied in detail the same problem to carry out the numerical results. Sarker and Kishore [120] studied the decay of MHD turbulence at time before the final period using chandrasekher's relation [27]. Sarker and Islam [127] studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time. Azad and Sarker[2] studied the Decay of dusty fluid MHD turbulence before the final period in a rotating system for the case of multi-point and multi-time. Islam and Sarker [56] also studied the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time.

In this chapter, following the above theories we have studied the magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction in dusty fluid MHD turbulence before the final period of decay for the case of multi-point and multi-time in a rotating system. Here, we have considered the two-point, two-time and three-point, three-time correlation equations and solved these equations after neglecting the fourth-order correlation terms. Finally we obtained the decay law for magnetic field energy fluctuation of concentration of dilute contaminant undergoing a first order chemical reaction in dusty fluid MHD turbulence for the case of multi-point and multi-time in a rotating system is obtained. If the fluid is clean and the system is non-rotating, the equation reduces to one obtained earlier by [56].

4.14. Basic Equations:

The equations of motion and the equation of continuity for viscous, incompressible dusty fluid MHD turbulent flow in a rotating system are given by

$$\frac{\partial u_i}{\partial t}+\frac{\partial}{\partial x_k}(u_i u_k - h_i h_k) = -\frac{\partial w}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_k \partial x_k} - 2\in_{mkl}\Omega_m u_i + f(u_i - v_i), \qquad \text{-------- (4.14.1)}$$

$$\frac{\partial h_i}{\partial t}+\frac{\partial}{\partial x_k}(h_i u_k - u_i h_k) = \lambda\frac{\partial^2 h_i}{\partial x_k \partial x_k}, \qquad \text{-------- (4.14.2)}$$

$$\frac{\partial v_i}{\partial t}+v_k\frac{\partial v_i}{\partial x_K} = -\frac{K}{m_s}(v_i - u_i) \qquad \text{---------- (4.14.3)}$$

with
$$\frac{\partial u_i}{\partial x_i} = \frac{\partial v_i}{\partial x_i} = \frac{\partial h_i}{\partial x_i} = 0. \qquad \text{----------- (4.14.4)}$$

Here, u_i, turbulence velocity component; h_i, magnetic field fluctuation component; v_i, dust particle velocity component; $w(\hat{x},t) = \frac{P}{\rho} + \frac{1}{2}\langle h^2 \rangle + \frac{1}{2}|\hat{\Omega}\times\hat{x}|^2$, total MHD pressure $p(\hat{x},t)$, hydrodynamic pressure; ρ, fluid density; ν, Kinematic viscosity; $\lambda = \nu/P_M$, magnetic diffusivity; P_M, magnetic prandtl number; x_k, space co-ordinate; the subscripts can take on the values 1, 2 or 3 and the repeated subscripts in a term indicate a summation; Ω_m, constant angular velocity component; $\in_{mkl}$, alternating tensor; $f = \frac{KN}{\rho}$, dimension of frequency ; N, constant number of density of dust particle, K= Stokes's resistance coefficient which for spherical particle of radius r is 6πμr. $m_s = \frac{4}{3}\pi R_s^3 \rho_s$, mass of single spherical dust particle of radius R_s; ρ_s, constant density of the material in dust particle.

4.15. Two-Point, Two-Time Correlation and Spectral Equations:

With the conditions (i) the turbulence and the concentration magnetic field are homogeneous (ii) the chemical reaction has no effect on the velocity field and (iii) the reaction rate and the magnetic diffusivity are constant, the induction equation of a magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction at the points p and p' separated by the vector $\hat{r}$ could be written as

$$\frac{\partial h_i}{\partial t} + u_k \frac{\partial h_i}{\partial x_k} - h_k \frac{\partial u_i}{\partial x_k} = \lambda \frac{\partial^2 h_i}{\partial x_k \partial x_k} - Rh_i \qquad \text{--------- (4.15.1)}$$

and $$\frac{\partial h'_j}{\partial t'} + u'_k \frac{\partial h'_j}{\partial x'_k} - h'_k \frac{\partial u'_j}{\partial x'_k} = \lambda \frac{\partial^2 h'_j}{\partial x'_k \partial x'_k} - Rh'_j \qquad \text{----------- (4.15.2)}$$

where R is the constant reaction rate.

Multiplying equation (4.15.1) by h'_j and equation (4.15.2) by h_i and taking ensemble average, we get

$$\frac{\partial \langle h_i h'_j \rangle}{\partial t} + \frac{\partial}{\partial x_k}\left[\langle u_k h_i h'_j \rangle - \langle u_i h_k h'_j \rangle\right] = \lambda \frac{\partial^2 \langle h_i h'_j \rangle}{\partial x_k \partial x_k} - R\langle h_i h'_j \rangle \qquad \text{----------- (4.15.3)}$$

and $$\frac{\partial \langle h_i h'_j \rangle}{\partial t'} + \frac{\partial}{\partial x'_k}\left[\langle u'_k h_i h'_j \rangle - \langle u'_j h_i h'_k \rangle\right] = \lambda \frac{\partial^2 \langle h_i h'_j \rangle}{\partial x'_k \partial x'_k} - R\langle h_i h'_j \rangle \qquad \text{----------- (4.15.4)}$$

Angular bracket $\langle -----\rangle$ is used to denote an ensemble average.

Using the transformations

$$\frac{\partial}{\partial x_k} = -\frac{\partial}{\partial r_k}, \quad \frac{\partial}{\partial x'_k} = \frac{\partial}{\partial r_k}, \quad \left(\frac{\partial}{\partial t}\right)t' = \left(\frac{\partial}{\partial t}\right)\Delta t - \frac{\partial}{\partial \Delta t}, \quad \frac{\partial}{\partial t'} = \frac{\partial}{\partial \Delta t} \qquad \text{--------- (4.15.5)}$$

into equations (4.15.3) and (4.15.4), we obtain

$$\frac{\partial\langle h_i h'_j\rangle}{\partial t}+\frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle-\langle u'_j h_i h'_k\rangle\right](\hat{r},\Delta t,t)-\frac{\partial}{\partial r_k}\left[\langle u_k h_i h'_j\rangle-\langle u_i h_k h'_j\rangle\right](\hat{r},\Delta t,t)$$

$$=2\lambda\frac{\partial^2\langle h_i h'_j\rangle}{\partial r_k\partial r_k}-2R\langle h_i h'_j\rangle \qquad \text{---------- (4.15.6)}$$

and $$\frac{\partial\langle h_i h'_j\rangle}{\partial\Delta t}+\frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle-\langle u'_j h_i h'_k\rangle\right](\hat{r},\Delta t,t)=\lambda\frac{\partial^2\langle h_i h'_j\rangle}{\partial r_k\partial r_k}-R\langle h_i h'_j\rangle \qquad \text{-------- (4.15.7)}$$

Using the relations of Chandrasekhar [27]

$$\langle u_k h_i h'_j\rangle=-\langle u'_k h_i h'_j\rangle,\quad \langle u'_j h_i h'_k\rangle=\langle u_i h_k h'_j\rangle.$$

Equations (4.15.6) and (4.15.7) become

$$\frac{\partial\langle h_i h'_j\rangle}{\partial t}+2\frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle-\langle u_i h_k h'_j\rangle\right]=2\lambda\frac{\partial^2\langle h_i h'_j\rangle}{\partial r_k\partial r_k}-2R\langle h_i h'_j\rangle \qquad \text{--------- (4.15.8)}$$

and $$\frac{\partial\langle h_i h'_j\rangle}{\partial t}+\frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle-\langle u_i h_k h'_j\rangle\right]=\lambda\frac{\partial^2\langle h_i h'_j\rangle}{\partial r_k\partial r_k}-R\langle h_i h'_j\rangle\ . \qquad \text{--------- (4.15.9)}$$

Now we write equations (4.15.8) and (4.15.9) in spectral form in order to reduce it to an ordinary differential equation by use of the following three-dimensional Fourier transforms:

$$\langle h_i h'_j\rangle(\hat{r},\Delta t,t)=\int_{-\infty}^{\infty}\langle\psi_i\psi'_j\rangle(\hat{K},\Delta t,t)\exp\left[\hat{i}(\hat{K}.\hat{r})\right]d\hat{K} \qquad \text{---------- (4.15.10)}$$

and $$\langle u_i h_k h'_j\rangle(\hat{r},\Delta t,t)=\int_{-\infty}^{\infty}\langle\alpha_i\psi_k\psi'_j\rangle(\hat{K},\Delta t,t)\exp\left[\hat{i}(\hat{K}.\hat{r})\right]d\hat{K} \qquad \text{--------- (4.15.11)}$$

Interchanging the subscripts i and j then interchanging the points p and p′ gives

$$\langle u'_k h_i h'_j\rangle(\hat{r},\Delta t,t)=\langle u_k h_i h'_j\rangle(-\hat{r},-\Delta t,t+\Delta t)$$

$$=\int_{-\infty}^{\infty}\langle\alpha_i\psi_i\psi'_j\rangle(-\hat{K},-\Delta t,t+\Delta t)\exp\left[\hat{i}(\hat{K}.\hat{r})\right]d\hat{K} \qquad \text{--------- (4.15.12)}$$

where, $\hat{K}$ is known as a wave number vector and $d\hat{K}$ =dK_1 dK_2 dK_3. The magnitude of $\hat{K}$ has the dimension 1/length and can be considered to be the reciprocal of an eddy size.

Substituting of equation (4.15.10) to (4.15.12) in to equations (4.15.8) and (4.15.9) leads to the spectral equations

$$\frac{\partial\langle\psi_i\psi'_j\rangle}{\partial t}+2\left[\lambda k^2+R\right]\langle\psi_i\psi'_j\rangle=2ik_k\left[\langle\alpha_i\psi_k\psi'_j\rangle(\hat{K},\Delta t,t)-\langle\alpha_k\psi_i\psi'_j\rangle(-\hat{K},-\Delta t,t+\Delta t)\right]$$

--------------- (4.15.13)

and $$\frac{\partial\langle\psi_i\psi'_j\rangle}{\partial\Delta t}+\left[\lambda K^2+R\right]\langle\psi_i\psi'_j\rangle=ik_k\left[\langle\alpha_i\psi_k\psi'_j\rangle(\hat{K},\Delta t,t)-\langle\alpha_k\psi_i\psi'_j\rangle(-\hat{K},-\Delta t,t+\Delta t)\right]$$

------------------- (4.15.14)

The tensor equations (4.15.13) and (4.15.14) becomes a scalar equation by contraction of the indices i and j

$$\frac{\partial\langle\psi_i\psi'_i\rangle}{\partial t}+2\left[\lambda K^2+R\right]\langle\psi_i\psi'_i\rangle=2ik_k\left[\langle\alpha_i\psi_k\psi'_i\rangle(\hat{K},\Delta t,t)-\langle\alpha_k\psi_i\psi'_i\rangle(-\hat{K},-\Delta t,t+\Delta t)\right]$$

-------(4.15.15)

and $$\frac{\partial\langle\psi_i\psi'_i\rangle}{\partial\Delta t}+\left[\lambda K^2+R\right]\langle\psi_i\psi'_i\rangle=ik_k\left[\langle\alpha_i\psi_k\psi'_i\rangle(\hat{K},\Delta t,t)-\langle\alpha_k\psi_i\psi'_i\rangle(-\hat{K},-\Delta t,t+\Delta t)\right]$$

-------------------(4.15.16)

The terms on the right side of equations (4.15.15) and (4.15.16) are collectively proportional to what is known as the magnetic energy transfer terms.

4.16. Three-Point, Three-Time Correlation and Spectral Equations:

In order to find the three-point correlation equations, similar procedure can be used. For this purpose we take the momentum equation of dusty fluid MHD turbulence in a rotating system at the point P and the induction equations of magnetic field fluctuations, governing the concentration of a dilute contaminant undergoing a first order chemical reaction at p′ and p″ separated by the vector $\hat{r}$ and $\hat{r}'$ as

$$\frac{\partial u_l}{\partial t}+u_k\frac{\partial u_l}{\partial x_k}-h_k\frac{\partial h_l}{\partial x_k}=-\frac{\partial w}{\partial x_l}+\nu\frac{\partial^2 u_l}{\partial x_k\partial x_k}-2\in_{mkl}\Omega_m u_l+f(u_l-v_l)\,, \qquad \text{------------ (4.16.1)}$$

$$\frac{\partial h_i'}{\partial t'}+u_k'\frac{\partial h_i'}{\partial x_k'}-h_k'\frac{\partial u_i'}{\partial x_k'}=\lambda\frac{\partial^2 h_i'}{\partial x_k'\partial x_k'}-Rh_i' \qquad \text{------------- (4.16.2)}$$

$$\frac{\partial h_j''}{\partial t''}+u_k''\frac{\partial h_j''}{\partial x_k''}-h_k''\frac{\partial u_j''}{\partial x_k''}=\lambda\frac{\partial^2 h_j''}{\partial x_k''\partial x_k''}-Rh_j''\,. \qquad \text{------------ (4.16.3)}$$

Multiplying equation (4.16.1) by $h'_i\, h_j''$, equation (4.16.2) by $u_l h_j''$ and equation (4.16.3) by $u_l h_i'$, taking ensemble average, one obtains

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t}+\frac{\partial}{\partial x_k}\left[\langle u_k u_l h_i' h_j''\rangle-\langle h_k h_l h_i' h_j''\rangle\right]=\frac{\partial\langle w h_i' h_j''\rangle}{\partial x_l}+\nu\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k\partial x_k}$$

$$-2\in_{mkl}\Omega_m\langle u_l h_i' h_j''\rangle+f(\langle u_l h_i' h_j''\rangle-\langle v_l h_i' h_j''\rangle \qquad \text{----------- (4.16.4)}$$

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t'}+\frac{\partial}{\partial x_k'}\left[\langle u_l u_k' h_i' h_j''\rangle-\langle u_l u_i' h_k' h_j''\rangle\right]=\lambda\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k'\partial x_k'}-R\langle u_l h_i' h_j''\rangle \qquad \text{------------ (4.16.5)}$$

and $$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t''}+\frac{\partial}{\partial x_k''}\left[\langle u_l u_k'' h_i' h_j''\rangle-\langle u_l u_j'' h_i' h_k''\rangle\right]=\lambda\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k''\partial x_k''}-R\langle u_l h_i' h_j''\rangle \qquad \text{----------- (4.16.6)}$$

Using the transformations

$$\frac{\partial}{\partial x_k}=-\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'}\right),\quad \frac{\partial}{\partial x_k'}=\frac{\partial}{\partial r_k},\quad \frac{\partial}{\partial x_k''}=\frac{\partial}{\partial r_k'}\,,$$

$$\left(\frac{\partial}{\partial t}\right)t',t''=\left(\frac{\partial}{\partial t}\right)\Delta t,\Delta t'-\frac{\partial}{\partial\Delta t}-\frac{\partial}{\partial\Delta t'}\,,$$

$$\frac{\partial}{\partial t'}=\frac{\partial}{\partial\Delta t},\quad \frac{\partial}{\partial t''}=\frac{\partial}{\partial\Delta t'}$$

into equations (4.16.4) to (4.16.6), we have

$$\frac{\partial\left\langle u_l h_i' h_j''\right\rangle}{\partial t}-\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'}\right)\left[\left\langle u_k u_l h_i' h_j''\right\rangle-\left\langle h_k h_l h_i' h_j''\right\rangle\right]+\frac{\partial}{\partial r_k}\left[\left\langle u_l u_k' h_i' h_j''\right\rangle-\left\langle u_l u_i' h_k' h_j''\right\rangle\right]$$

$$+\frac{\partial}{\partial r_k'}\left[\left\langle u_l u_k'' h_i' h_j''\right\rangle-\left\langle u_l u_j'' h_i' h_k''\right\rangle\right]=-\left(\frac{\partial}{\partial r_l}+\frac{\partial}{\partial r_l'}\right)\left\langle w h_i' h_j''\right\rangle+\nu\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'}\right)^2\left\langle u_l h_i' h_j''\right\rangle$$

$$+\lambda\left[\frac{\partial^2\left\langle u_l h_i' h_j''\right\rangle}{\partial r_k \partial r_k}+\frac{\partial^2\left\langle u_l h_i' h_j''\right\rangle}{\partial r_k' \partial r_k'}\right]-2\in_{mkl}\,\Omega_m\left\langle u_l h_i' h_j''\right\rangle+f\left(\left\langle u_l h_i' h_j''\right\rangle-\left\langle v_l h_i' h_j''\right\rangle\right)$$

--------- (4.16.7)

$$\frac{\partial\left\langle u_l h_i' h_j''\right\rangle}{\partial \Delta t}+\frac{\partial}{\partial r_k}\left[\left\langle u_l u_k' h_i' h_j''\right\rangle-\left\langle u_l u_i' h_k' h_j''\right\rangle\right]=\lambda\frac{\partial^2\left\langle u_l h_i' h_j''\right\rangle}{\partial r_k \partial r_k}-R\langle u_l h_i' h_j''\rangle \quad \text{----------- (4.16.8)}$$

and $$\frac{\partial\left\langle u_l h_i' h_j''\right\rangle}{\partial \Delta t'}+\frac{\partial}{\partial r_k'}\left[\left\langle u_l u_k'' h_i' h_j''\right\rangle-\left\langle u_l u_j'' h_i' h_k''\right\rangle\right]=\lambda\frac{\partial^2\left\langle u_l h_i' h_j''\right\rangle}{\partial r_k' \partial r_k'}-R\langle u_l h_i' h_j''\rangle \quad \text{-------------(4.16.9)}$$

In order to convert equations (4.16.7)–(4.16.9) to spectral form, we can define the following six dimensional Fourier transforms:

$$\left\langle u_l h_i' h_j''\right\rangle\langle \hat{r},\hat{r}',\Delta t,\Delta t',t\rangle=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \phi_l \beta_i' \beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}'$$

------------(4.16.10)

$$\left\langle u_l u_k' h_i' h_j''\right\rangle\langle \hat{r},\hat{r}',\Delta t,\Delta t',t\rangle=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \phi_l \phi_K' \beta_i' \beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}'$$

-------(4.16.11)

$$\left\langle w h_i' h_j''\right\rangle\langle \hat{r},\hat{r}',\Delta t,\Delta t',t\rangle=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \gamma \beta_i' \beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}'$$

---------(4.16.12)

$$\left\langle u_k u_l h_i' h_j''\right\rangle\langle \hat{r},\hat{r}',\Delta t,\Delta t',t\rangle=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \phi_K \phi_l \beta_i' \beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}'$$

--------(4.16.13)

$$\left\langle h_k h_l h_i' h_j''\right\rangle\langle \hat{r},\hat{r}',\Delta t,\Delta t',t\rangle=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \beta_K \beta_l \beta_i' \beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}'$$

------ (4.16.14)

$$\langle u_l u_i' h_K' h_j'' \rangle \langle \hat{r}, \hat{r}', \Delta t, \Delta t', t \rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \phi_l \phi_i' \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) \exp[\hat{i}(\hat{K}.\hat{r} + \hat{K}'.\hat{r}')] d\hat{K} d\hat{K}'$$

------ (4.16.15)

$$\langle v_l h_i' h_j'' \rangle \langle \hat{r}, \hat{r}, \Delta t, \Delta t', t \rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \mu_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) \exp[\hat{i}(\hat{K}.\hat{r} + \hat{K}'.\hat{r}')] d\hat{K} d\hat{K}'$$

--------- (4.16.16)

Interchanging the points P′ and P″ along with the indices i and j result in the relations

$$\langle u_l u_k'' h_i' h_j'' \rangle = \langle u_l u_k' h_i' h_j'' \rangle \ .$$

By use of these facts and the equations (4.16.10)-(4.16.16), we can write equations (4.16.7)-(4.16.9) in the form

$$\frac{\partial}{\partial t} \langle \phi_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda \left[(1 + P_M)(k^2 + k'^2) + 2 P_M k k' + \frac{2R}{\lambda} + \frac{1}{\lambda}(2 \in_{mkl} \Omega_m - f) \right]$$

$$\times \langle \phi_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) = [i(k_k + k_k') \langle \phi_k \phi_l \beta_i' \beta'' \rangle - i(k_k + k_k') \langle \beta_k \beta_l \beta_i' \beta_j'' \rangle$$

$$- i(k_k + k_k') \langle \phi_l \phi_k' \beta_i' \beta_j'' \rangle + i(k_k + k_k') \langle \phi_l \phi_i' \beta_k' \beta_j'' \rangle - i(k_l + k_l') \langle \gamma \beta_i' \beta_j'' \rangle$$

$$- f \langle \mu_l \beta_i' \beta_j'' \rangle \ ](\hat{K}, \hat{K}', \Delta t, \Delta t', t)$$

---------- (4.16.17)

$$\frac{\partial}{\partial \Delta t} \langle \phi_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda \left[K^2 + \frac{R}{\lambda} \right] \langle \phi_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t)$$

$$= -ik_k \langle \phi_l \phi_k' \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}' \Delta t, \Delta t', t) + ik_k \langle \phi_l \phi_i' \beta_k' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t)$$

-------- (4.16.18)

and
$$\frac{\partial}{\partial \Delta t'} \langle \phi_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda \left[K^2 + \frac{R}{\lambda} \right] \langle \phi_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t)$$

$$= -ik_k' \langle \phi_l \phi_k' \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}' \Delta t, \Delta t', t) + ik_k' \langle \phi_l \phi_i' \beta_k' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t)$$

--------- (4.16.19)

If the derivative with respect to x_l is taken of the momentum equation (4.16.1) for the point P, the equation multiplied by $h_i'h_j''$ and time average taken, the resulting equation

$$-\frac{\partial^2 \langle wh_i'h_j''\rangle}{\partial x_l \partial x_l} = \frac{\partial^2}{\partial x_l \partial x_k}\left(\langle u_l u_k h_i'h_j''\rangle - \langle h_l h_k h_i'h_j''\rangle\right) \qquad \text{---------- (4.16.20)}$$

Writing this equation in terms of the independent variables $\hat{r}$ and $\hat{r}'$

$$-\left[\frac{\partial^2}{\partial r_l \partial r_l} + 2\frac{\partial^2}{\partial r_l \partial r_l'} + \frac{\partial^2}{\partial r_l' \partial r_l'}\right]\langle wh_i'h_j''\rangle = \left[\frac{\partial^2}{\partial r_l \partial r_k} + \frac{\partial^2}{\partial r_l' \partial r_k} + \frac{\partial^2}{\partial r_l \partial r_k'} + \frac{\partial^2}{\partial r_l' \partial r_k'}\right]\times$$

$$\left(\langle u_l u_k h_i'h_j''\rangle - \langle h_l h_k h_i'h_j''\rangle\right) \qquad \text{---------(4.16.21)}$$

Taking the Fourier transforms of equation (4.16.8)

$$-\langle \gamma\beta_i'\beta_j''\rangle = \frac{(k_l k_k + k_l'k_k + k_l k_k' + k_l'k_k')\left(\langle \phi_l\phi_k\beta_i'\beta_j''\rangle - \langle \beta_l\beta_k\beta_i'\beta_j''\rangle\right)}{k_l k_l + 2k_l k_l' + k_l'k_l'} \qquad \text{----------(4.16.22)}$$

Equation (4.16.22) can be used to eliminate $\langle \gamma\beta_i'\beta_j''\rangle$ from equation (4.16.17)

The tensor equations (4.16.17) to (4.16.19) can be converted to scalar equation by contraction of the indices i and j and inner multiplication by k_l

$$\frac{\partial}{\partial t}k_l\langle \phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) + \lambda\Big[(1+P_M)(k^2+k'^2) + 2P_M kk' + \frac{2R}{\lambda} +$$

$$\frac{1}{\lambda}(2\in_{mkl}\Omega_m - f)\Big]\langle \phi_l\beta_i''\beta_i''\rangle(\hat{K},\hat{K},\Delta t,\Delta t',t) = i(k_k + k_k')\langle \phi_k\phi_l\beta_i'\beta_j''\rangle$$

$$(\hat{K},\hat{K}',\Delta t,\Delta t',t) - i(k_k + k_k')\langle \beta_K\beta_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) - i(k_k + k_k')$$

$$\langle \phi_l\phi_k'\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) + i(k_k + k_k')\langle \phi_l\phi_i'\beta_k'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) - i(k_l + k_l')$$

$$\langle \gamma\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) - f\langle \mu_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) \qquad \text{--------- (4.16.23)}$$

$$\frac{\partial}{\partial \Delta t} k_L \langle \phi_l \beta_i' \beta_i'' \rangle \left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) + \lambda \left[K^2 + \frac{R}{\lambda} \right] \langle \phi_l \beta_i' \beta_i'' \rangle \left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right)$$

$$= -ik_k \langle \phi_l \phi_k' \beta_i' \beta_i'' \rangle \left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) + ik_k \langle \phi_l \phi_i' \beta_k' \beta_i'' \rangle \left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) \text{ --------- } (4.16.24)$$

and $$\frac{\partial}{\partial \Delta t'} k_l \langle \phi_l \beta_i' \beta_i'' \rangle \left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) + \lambda \left[K'^2 + \frac{R}{\lambda} \right] \langle \phi_l \beta_i' \beta_i'' \rangle \left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right)$$

$$= -ik_k' \langle \phi_l \phi_k' \beta_i' \beta_i'' \rangle \left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) + ik_k' \langle \phi_l \phi_i' \beta_i' \beta_i'' \rangle \left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) \text{ --------- } (4.16.25)$$

4.17. Solution for Times Before the Final Period:

It is clear that the equation for final period of decay is obtained by considering the two-point correlations after neglecting third-order correlation terms. To study the decay for times before the final period, the three-point correlations are considered and the quadruple correlation terms are neglected because the quadruple correlation terms decays faster than the lower-order correlation terms. The term $\langle \gamma \beta_i' \beta_j'' \rangle$ associated with the pressure fluctuations should also be neglected. Thus neglecting all the terms on the right hand side of equations (4.16.23) to (4.16.25)

$$\frac{\partial}{\partial t} K_l \langle \phi_l \beta_i' \beta_i'' \rangle \left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) + \lambda \left[(1 + P_M)(k^2 + k'^2) + 2 P_M k k' + \frac{2R}{\lambda} \right.$$

$$\left. + \frac{1}{\lambda}(2 \in_{mkl} \Omega_m - fs) \right] \langle \phi_l \beta_i' \beta_i'' \rangle \left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) = 0 \text{ ---------- } (4.17.1)$$

$$\frac{\partial}{\partial \Delta t} K_l \langle \phi_l \beta_i' \beta_i'' \rangle \left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) + \lambda \left[k^2 + \frac{R}{\lambda} \right] \langle \phi_l \beta_i' \beta_i'' \rangle \left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) = 0 \text{ --------- } (4.17.2)$$

and $$\frac{\partial}{\partial \Delta t'} K_l \langle \phi_l \beta_i' \beta_i'' \rangle \left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) + \lambda \left[k'^2 + \frac{R}{\lambda} \right] \langle \phi_l \beta_i' \beta_i'' \rangle \left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) = 0 \text{ --------- } (4.17.3)$$

where $\langle \mu_l \beta_i' \beta_i'' \rangle = C \langle \phi_l \beta_i' \beta_i'' \rangle$ and 1-C=S, here C and S are arbitrary constant.

Integrating equations (4.17.1) to (4.17.3) between t_o and t, we obtain

$$k_l\langle\phi_l\beta_i'\beta_i''\rangle = f_l \exp\{-\lambda[(1+P_M)(k^2+k'^2)+2P_M kk'\cos\theta+\frac{2R}{\lambda}$$

$$+\frac{1}{\lambda}(2\in_{mkl}\Omega_m - fs)](t-t_o)\},$$

$$k_l\langle\phi_l\beta_i'\beta_i''\rangle = g_l \exp\left[-\lambda\left(K^2+\frac{R}{\lambda}\right)\Delta t\right]$$

and $$k_l\langle\phi_l\beta_i'\beta_i''\rangle = q_l \exp\left[-\lambda\left(k'^2+\frac{R}{\lambda}\right)\Delta t'\right].$$

For these relations to be consistent, we have

$$k_l\langle\phi_l\beta_i'\beta_i''\rangle = k_l\langle\phi_l\beta_i'\beta_i''\rangle_o \exp\{-\lambda[(1+P_M)(k^2+k'^2)(t-t_o)+k^2\Delta t+k'^2\Delta t'$$

$$+2P_M kk'\cos\theta(t-t_o)+\frac{2R}{\lambda}\left(t-t_0+\frac{\Delta t+\Delta t'}{2}\right)+\left(\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{fs}{\lambda}\right)(t-t_o)]\}$$

----------- (4.17.4)

where, θ is the angle between $\hat{K}$ and $\hat{K}'$ and $\langle\phi_l\beta_i'\beta_i''\rangle_o$ is the value of $\langle\phi_l\beta_i'\beta_i''\rangle$ at $t = t_o$, $\Delta t = \Delta t' = 0$, $\lambda = \frac{\nu}{P_M}$

By letting $\hat{r}' = 0$, $\Delta t' = 0$ in the equation (4.16.10) and comparing with equations (4.15.11) and (4.15.12) we get

$$\langle\alpha_i\psi_k\psi_i'\rangle(\hat{K},\Delta t,t) = \int_{-\infty}^{\infty}\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,o,t)d\hat{K}'$$ ----------- (4.17.5)

and $$\langle\alpha_i\psi_k\psi_i'\rangle(-\hat{K},-\Delta t,t+\Delta t) = \int_{-\infty}^{\infty}\langle\phi_l\beta_i'\beta_i''\rangle(-\hat{K},\hat{K}',\Delta t,o,t)d\hat{K}'$$ ---------- (4.17.6)

Substituting equation (4.17.4) to (4.17.6) into equation (4.15.15), one obtains

$$\frac{\partial}{\partial t}\langle\psi_i\psi_i'\rangle\left(\hat{K},\Delta t,t\right)+2\lambda\left[k^2+\frac{R}{\lambda}\right]\langle\psi_i\psi_i'\rangle\left(\hat{K},\Delta t,t\right)=\int_{-\infty}^{\infty}2ik_l\left[\langle\phi_l\beta_i'\beta_i''\rangle\left(\hat{K},\hat{K}',\Delta t,0,t\right)\right.$$

$$-\langle\phi_l\beta_i'\beta_i''\rangle\left(-\hat{K},-\hat{K}',\Delta t,0,t\right)\Big]_o\exp\left[-\lambda\left\{(1+P_M)\left(k^2+k'^2\right)(t-t_o)\right.\right.$$

$$\left.\left.+k^2\Delta t+2P_M(t-t_o)kk'\cos\theta+\frac{2R}{\lambda}(t-t_0+\Delta t)+\left(\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{fs}{\lambda}\right)(t-t_o)\right\}\right]d\hat{k}\ .$$

----------------(4.17.7)

Now, $d\hat{K}'$ can be expressed in terms of k′ and θ as $-2\pi k'd(\cos\theta)dk'$ (cf. Deissler [37])

i.e. $$d\hat{K}'=-2\pi k'd(\cos\theta)dk' \qquad \text{--------- (4.17.8)}$$

Substituting of equation (4.17.8) in equation (4.17.7) yields

$$\frac{\partial}{\partial t}\langle\psi_i\psi_i'\rangle\left(\hat{K},\Delta t,t\right)+2\lambda\left[k^2+\frac{R}{\lambda}\right]\langle\psi_i\psi_i'\rangle\left(\hat{K},\Delta t,t\right)=2\int_0^{\infty}2\pi ik_l\left[\langle\phi_l\beta_i'\beta_i''\rangle\left(\hat{K},\hat{K}'\right)\right.$$

$$-\langle\phi_l\beta_i'\beta_i''\rangle\left(-\hat{K},-\hat{K}'\right)\Big]_o k'^2\left[\int_{-1}^{1}\exp\left\{-\lambda\left[(1+P_M)\left(k^2+k'^2\right)(t-t_o)\right.\right.\right.$$

$$\left.\left.\left.+k^2\Delta t+2P_M(t-t_o)kk'\cos\theta+\frac{2R}{\lambda}(t-t_0+\Delta t/2)+\left(\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{fs}{\lambda}\right)(t-t_o)\right]\right\}d(\cos\theta)\right]dk'$$

----------(4.17.9)

The quantity $[\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}')-\langle\phi_l\beta_i'\beta_i''\rangle(-\hat{K},-\hat{K}')]_0$ depends on the initial conditions of the turbulence.

In order to find the solution completely and following Loeffler and Deissler [81] we assume that

$$ik_l\left[\langle\phi_l\beta_i'\beta_i''\rangle\left(\hat{K},\hat{K}'\right)-\langle\phi_l\beta_i'\beta_i''\rangle\left(-\hat{K},-\hat{K}'\right)\right]_o=\frac{-\delta_0}{(2\pi)^2}\left(k^2k'^4-k^4k'^2\right) \qquad \text{---------- (4.17.10)}$$

where, δ_0 is a constant determined by the initial conditions. The negative sign is placed in front of δ_0 in order to make the transfer of energy from small to large wave numbers for positive value of δ_0.

Substituting equation (4.17.10) into equation (4.17.9) we get

$$\frac{\partial}{\partial t}2\pi\langle\psi_i\psi_i'\rangle(\hat{K},\Delta t,t)+2\lambda\left[k^2+R/\lambda\right]2\pi\langle\psi_i\psi_i'\rangle(\hat{K},\Delta t,t)=-2\delta_0\int_0^\infty(k^2k'^4-k^4k'^2)k'^2$$

$$\left[\int_{-1}^{1}\exp\{-\lambda[(1+P_M)(k^2+k'^2)(t-t_o)+k^2\Delta t+2P_M(t-t_o)kk'\cos\theta\right.$$

$$\left.+\frac{2R}{\lambda}(t-t_0+\Delta t/2)+\left(\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{fs}{\lambda}\right)(t-t_o)\Big]\Big\}d(\cos\theta)\right]d\hat{k}' \qquad \text{---------} (4.17.11)$$

Multiplying both sides of equation (4.17.11) by k^2, we get

$$\frac{\partial E}{\partial t}+2\lambda k^2E=F \qquad \text{----------} (4.17.12)$$

where, $E=2\pi k^2\langle\psi_i\psi_i'\rangle$, E is the magnetic energy spectrum function and F is the magnetic energy transfer term and is given by

$$F=-2\delta_0\int_0^\infty(k^2k'^4-k^4k'^2)k^2k'^2\times\left[\int_{-1}^{1}\exp\{-\lambda[(1+P_M)(k^2+k'^2)(t-t_o)\right.$$

$$+k^2\Delta t+2P_M(t-t_o)kk'\cos\theta$$

$$\left.+\frac{2R}{\lambda}(t-t_0+\Delta t/2)+\left(\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{fs}{\lambda}\right)(t-t_o)\Big]\Big\}d(\cos\theta)\right]dk' \qquad \text{----- ---} (4.17.13)$$

Integrating equation (4.17.13) with respect to cosθ and k' we have

$$F=-\frac{\delta_oP_M\sqrt{\pi}}{4\lambda^{3/2}(t-t_o)^{3/2}(1+P_M)^{5/2}}\exp\left\{-\left(\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{fs}{\lambda}\right)(t-t_o)\right\}\times$$

$$\exp\left[\frac{-k^2\lambda(1+2P_M)}{1+P_M}\left(t-t_o+\frac{1+P_M}{1+2P_M}\Delta t\right)-2R(t-t_0+\Delta t/2)\right]\times\left[\frac{15P_Mk^4}{4P_M^2\lambda^2(t-t_o)^2(1+P_M)}\right.$$

$$+\left\{\frac{5P_M^2}{(1+P_M)^2}-\frac{3}{2}\right\}\frac{k^6}{P_M\lambda(t-t_o)}+\left\{\frac{P_M^3}{(1+P_M)^3}-\frac{P_M}{1+P_M}\right\}k^8$$

$$-\frac{\delta_o P_M \sqrt{\pi}}{4\lambda^{3/2}(t-t_o+\Delta t)^{3/2}(1+P_M)^{5/2}} \exp\left\{-\left(\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{fs}{\lambda}\right)(t-t_o)\right\}\times$$

$$\exp\left[\frac{-k^2\lambda(1+2P_M)}{1+P_M}\left(t-t_o+\frac{P_M}{1+P_M}\Delta t\right)-2R(t-t_0+\Delta t/2)\right]\times\left[\frac{15P_M k^4}{4\nu^2(t-t_o+\Delta t)^2(1+P_M)}\right.$$

$$\left.+\left\{\frac{5P_M^2}{(1+P_M)^2}-\frac{3}{2}\right\}\frac{k^6}{P_M\lambda(t-t_o+\Delta t)}+\left\{\frac{P_M^3}{(1+P_M)^3}-\frac{P_M}{1+P_M}\right\}k^8\right] \quad \text{------ (4.17.14)}$$

The series of equation (4.17.14) contains only even power of k and start with k^4 and the equation represents the transfer function arising owing to consideration of magnetic field at three-point and three-times.

If we integrate equation (4.17.14) for $\Delta t=0$ over all wave numbers, we find that

$$\int_0^\infty F dk = 0 \quad \text{---------- (4.17.15)}$$

which indicates that the expression for F satisfies the condition of continuity and homogeneity. Physically it was to be expected as F is a measure of the energy transfer and the total energy transferred to all wave numbers must be zero.

The linear equation (4.17.12) can be solved to give

$$E=\exp\left[-2\lambda k^2(t-t_o+\Delta t/2)\int F\exp\left[2\lambda(k^2+R/\lambda)(t-t_o+\Delta t/2)\right]dt\right.$$

$$\left.+J(k)\exp\left[-2\lambda(k^2+R/\lambda\right.(t-t_o+\Delta t/2)\right] \quad \text{---------- (4.17.16)}$$

where $J(k)=\frac{N_o k^2}{\pi}$ is a constant of integration and can be obtained as by Corrsin[32].

Substituting the values of F from equation (4.17.14) into equation (4.17.16) gives the equation

$$E=\frac{N_o k^2}{\pi}\exp\left[-2\lambda(k^2+R/\lambda)(t-t_o+\Delta t/2)\right]+\frac{\delta_o P_M\sqrt{\pi}}{4\lambda^{3/2}(1+P_M)^{7/2}}\times$$

$$\exp\left[-\left(2\in_{mkl}\Omega_m-fs\right)(t-t_o)\right]$$

$$\exp\left[\frac{-k^2\lambda(1+2P_M)}{1+P_M}\left(t-t_o+\frac{1+P_M}{1+2P_M}\Delta t\right)-2R(t-t_0+\Delta t/2)\right]$$

$$\left[\frac{3k^4}{2P_M\lambda^2(t-t_0)^{5/2}}+\frac{(7P_M-6)k^6}{3\lambda(1+P_M)(t-t_0)^{3/2}}-\frac{4(3P_M^2-2P_M+3)k^8}{3(1+P_M)^2(t-t_0)^{1/2}}\right.$$

$$\left.+\frac{8\sqrt{\lambda}(3P_M^2-2P_M+3)k^9}{3(1+P_M)^{5/2}}F(\omega)\right]+\frac{\delta_o P_M\sqrt{\pi}}{4\lambda^{3/2}(1+P_M)^{7/2}}\exp\left[-\left(2\in_{mkl}\Omega_m-fs\right)(t-t_o)\right]$$

$$\exp\left[\frac{-k^2\lambda(1+2P_M)}{1+P_M}\left(t-t_o+\frac{P_M}{1+P_M}\Delta t\right)-2R(t-t_0+\Delta t/2)\right]$$

$$\left[\frac{3k^4}{2P_M\lambda^2(t-t_0+\Delta t)^{5/2}}+\frac{(7P_M-6)k^6}{3\lambda(1+P_M)(t-t_0+\Delta t)^{3/2}}\right.$$

$$\left.-\frac{4(3P_M^2-2P_M+3)k^8}{3(1+P_M)^2(t-t_0+\Delta t)^{1/2}}+\frac{8\sqrt{\lambda}(3P_M^2-2P_M+3)k^9F(\omega)}{(1+P_M)^{5/2}P_M^{1/2}}\right] \quad \text{--------- (4.17.17)}$$

where $F(\omega)=e^{-\omega^2}\int_0^{\omega}e^{x^2}dx$, $\quad \omega=k\sqrt{\frac{\lambda(t-t_0)}{1+P_M}}$ or $k\sqrt{\frac{\lambda(t-t_0+\Delta t)}{1+P_M}}$.

By setting $\hat{r}=0$, j=i, $d\hat{k}=-2\pi k^2 d(\cos\theta)dk$ and $E=2\pi k^2\langle\psi_i\psi'_j\rangle$ in equation (4.15.10) we get the expression for magnetic energy decay law as

$$\frac{\langle h_i h'_i\rangle}{2}=\int_0^{\infty}Edk \quad \text{--------- (4.17.18)}$$

Substituting equation (4.17.17) into equation (4.17.18) and after integration, we get

$$\frac{\langle h_i h_i' \rangle}{2} = \frac{N_0}{8\sqrt{2\pi}\,\lambda^{3/2}(T+\Delta T/2)^{3/2}} \exp[-2R(T+\Delta T/2)]$$

$$+ \frac{\pi\delta_0}{4\lambda^6(1+P_M)(1+2P_M)^{5/2}} \exp[-2R(T+\Delta T/2)]\exp[-(2\in_{mkl}\Omega_m - fs)]$$

$$\times \left[\frac{9}{16T^{5/2}\left(T+\frac{1+P_M}{1+2P_M}\Delta T\right)^{3/2}} + \frac{9}{16(T+\Delta T)^{5/2}\left(T+\frac{P_M}{1+2P_M}\Delta T\right)^{5/2}} \right.$$

$$+ \frac{5P_M(7P_M-6)}{16(1+2P_M)T^{3/2}\left(T+\frac{1+P_M}{1+2P_M}\Delta T\right)^{7/2}} + \frac{5P_M(7P_M-6)}{16(1+2P_M)(T+\Delta T)^{3/2}\left(T+\frac{P_M}{1+2P_M}\Delta T\right)^{7/2}}$$

$$+ \frac{35P_M(3P_M^2-2P_M+3)}{8(1+2P_M)T^{1/2}\left(T+\frac{1+P_M}{1+2P_M}\Delta T\right)^{9/2}} + \frac{35P_M(3P_M^2-2P_M+3)}{8(1+2P_M)(T+\Delta T)^{1/2}\left(T+\frac{P_M}{1+2P_M}\Delta T\right)^{9/2}}$$

$$+ \frac{8P_M(3P_M^2-2P_M+3)(1+2P_M)^{5/2}}{3.2^{23/2}(1+P_M)^{11/2}} \sum_{n=0}^{\infty} \frac{1.3.5.\ldots\ldots\ldots\ldots(2n+9)}{n!(2n+1)2^{2n}(1+P_M)^n} \times$$

$$\left. \left\{ \frac{T^{(2n+1)/2}}{\left(T+\Delta T/2\right)^{(2n+1)/2}} + \frac{(T+\Delta T)^{(2n+1)/2}}{\left(T+\Delta T/2\right)^{(2n+1)/2}} \right\} \right] \quad \text{---------} \ (4.17.19)$$

where T=t-t$_0$.

For $T_m = T + \Delta T/2$, equation (4.17.19) takes the form

$$\frac{\langle h^2\rangle}{2}=\frac{\langle h_i h_i'\rangle}{2}=\exp[-2RT_m]\left[\frac{N_0}{8\sqrt{2\pi}\lambda^{3/2}T_m^{3/2}}+\frac{\pi\delta_0}{4\lambda^6(1+P_M)(1+2P_M)^{5/2}}\exp[-(2\in_{mkl}\Omega_m-fs)]\right.$$

$$\times\left[\frac{9}{16\left(T_m-\Delta T/2\right)^{5/2}\left(T_m+\frac{\Delta T}{1+2P_M}\right)^{5/2}}+\frac{9}{16\left(T_m+\Delta T/2\right)^{5/2}\left(T_m-\frac{\Delta T}{2(1+2P_M)}\right)^{5/2}}\right.$$

$$+\frac{5P_M(7P_M-6)}{16(1+2P_M)\left(T_m-\frac{\Delta T}{2}\right)^{3/2}\left(T_m+\frac{\Delta T}{2(1+2P_M)}\right)^{7/2}}$$

$$\left.\left.+\frac{5P_M(7P_M-6)}{16(1+2P_M)\left(T_m+\frac{\Delta T}{2}\right)^{3/2}\left(T_m-\frac{\Delta T}{2(1+2P_M)}\right)^{7/2}}+---\right]\right] \qquad \text{-------(4.17.20)}$$

This is the decay law of magnetic energy fluctuations of concentration of a dilute contaminant undergoing a first order chemical reaction before the final period for the case of multi-point and multi-time in dusty fluid MHD turbulence in a rotating system.

4.18. Results and Discussion:

In equation (4.17.20) we obtained the decay law of magnetic energy fluctuations of a dilute contaminant undergoing a first order chemical reaction before the final period considering three-point correlation terms for the case of multi-point and multi-time in MHD turbulence in presence of dust particle in a rotating system.

For clean fluid, f=0 then the equation (4.17.20) reduces to the equation (4.5.20) of this chapter in part-A.

For non-rotating system, $\Omega_m=0$, then the equation (4.17.20) reduces to the equation (4.11.20) of this chapter in part-B.

If the fluid is non-rotating and clean then $\Omega_m=0$, f=0, the equation (4.17.20) becomes

$$\frac{\langle h^2\rangle}{2}=\exp[-2RT_m]\left[\frac{N_0}{8\sqrt{2\pi}\lambda^{3/2}T_m^{3/2}}+\frac{\pi\delta_0}{4\lambda^6(1+P_M)(1+2P_M)^{5/2}}\right.$$

$$\times\left[\frac{9}{16\left(T_m-\Delta T/2\right)^{5/2}\left(T_m+\frac{\Delta T}{1+2P_M}\right)^{5/2}}+\frac{9}{16\left(T_m+\Delta T/2\right)^{5/2}\left(T_m-\frac{\Delta T}{2(1+2P_M)}\right)^{5/2}}\right.$$

$$+\frac{5P_M(7P_M-6)}{16(1+2P_M)\left(T_m-\frac{\Delta T}{2}\right)^{3/2}\left(T_m+\frac{\Delta T}{2(1+2P_M)}\right)^{7/2}}$$

$$\left.\left.+\frac{5P_M(7P_M-6)}{16(1+2P_M)\left(T_m+\frac{\Delta T}{2}\right)^{3/2}\left(T_m-\frac{\Delta T}{2(1+2P_M)}\right)^{7/2}}+---\right]\right] \quad \text{----------(4.18.1)}$$

which was obtained earlier by Islam and Sarker [56].

If we put ΔT=0, R=0, in equation (4.18.1) we can easily find out

$$\frac{\langle h^2\rangle}{2}=\frac{\langle h_i h_i'\rangle}{2}=\frac{N_0 T^{-3/2}}{8\sqrt{2\pi}\lambda^{3/2}}+\frac{\pi\delta_0}{4\lambda^6(1+P_M)(1+2P_M)^{5/2}}T^{-5}\left\{\frac{9}{16}+\frac{5}{16}\frac{P_M(7P_M-6)}{1+2P_M}+-----\right\}$$

--------(4.18.2)

which is same as obtained earlier by Sarker and Kishor [120].

This study shows that due to the effect of rotation of fluid in the flow field with chemical reaction of the first order in the concentrarion the magnetic field fluctuation in dusty fluid MHD turbulence in rotating system for the case of multi-point and multi-time i,e.the turbulent energy decays more rapidly than the energy for non-rotating clean fluid and the faster rate is governed by $\exp[-(2\in_{mkl}\Omega_m - fs)]$. Here the chemical reaction (R≠0) in MHD turbulence for the case of multi-point and multi-time causes the concentration to decay more they would for non-rotating clean fluid and it is governed by $\exp\left[-\left\{2RT_M+2\in_{mkl}\Omega_m-fs\right\}\right]$.

The first term of right hand side of equation (4.17.20) corresponds to the energy of magnetic field fluctuation of concentration for the two-point correlation and the second term represents magnetic energy for the three-point correlation. In equation (4.17.20), the term associated with the three-point correlation die out faster than the two-point correlation.

CHAPTER-V

EFFECT OF CORIOLIS FORCE ON DUSTY VISCOUS FLUID BETWEEN TWO HORIZONTAL PARALLEL PLATES IN MHD FLOW

5.1. Introduction:

The subject of magneto-hydrodynamics (MHD) has developed in many directions and the industry has exploited the use of magnetic fields in controlling a range of fluid and thermal processes. The study of fluids having uniform distribution of solid spherical particles is of interest in a wide range of areas of technical importance. These areas include fluidization (flow through packed feds), flow in rocket, tubes, where small carbon or metallic fuel particles are present, environmental pollution, the process by which rain drops are formed by the coalescence of small droplets, which might be considered as solid particles for the purpose of examining their movement prior to coal scene, combustion and more recently blood flow in capillaries. The coriolis force due to rotation plays an important role in a rotating system of the flow, while the centrifugal force with the potential is incorporated into the pressure. Ohji [98] considered the effect of coreolis force on turbulent motion in the presence of strong magnetic field with the assumption that Coriolis force term ($-2\Omega\times U$) is balanced by the geostropic wind approximation. Saffman [118] worked on the stability of the flow of a dusty gas, which is very useful for this work. Reddy [116] studied about the flow of dusty viscous liquid through rectangular channel. Azad, Aziz and Sarker [3] studied on first order reactant in MHD turbulence before the final period of decay in a rotating system. Hazem Attia [54] studied unsteady flow of a dusty conducting fluid between parallel porous plates with temperature dependent viscosity. Bhargava and Takhar [21] studied the effect of Hall currents on the MHD flow and heat transfer of a second order fluid between two parallel porous plates. Varma and Mathur [150] studied in this field. Sreeharireddy, Nagarajan and Sivaiah [139] also studied on MHD flow of a viscous conducting liquid between two parallel plates.

In the present investigation, the MHD flow of a dusty viscous incompressible fluid in a rotating frame between two parallel flat plates in presence of a uniform transverse magnetic field with pressure gradient is studied. The velocities of the fluid and the dust particles for rotating frame are obtained and the effect of magnetic field on these velocities is investigated. The variation in the magnetic parameters causes significant changes in the velocity profiles of fluid particles as well as of dust particles and these changing levels of velocity profiles are comparatively higher than that of the non-rotating frame. The effects of the coriolis force on velocity profiles of the fluid and the dust particles are graphically discussed. It is observed that the velocities of fluid and dust particles increase with the increase of coroilis force.

5.2. Formulation and Solution of the problem:

In the present discussions, we consider the flow of dusty viscous fluid between two rotating parallel plates in the presence of a uniform transverse magnetic field. It is assumed that the fluid is of small electrical conductivity with magnetic Reynolds number much less than unity, so that the induced magnetic field can be neglected in comparison with applied magnetic field.

The x-axis is taken along the mid way of the channel and a straight line perpendicular to that is taken as the y-axis. Let the distance between the two plates be 2h and the magnetic field of intensity H_0 is introduced in the y-direction as shown in fig. A.

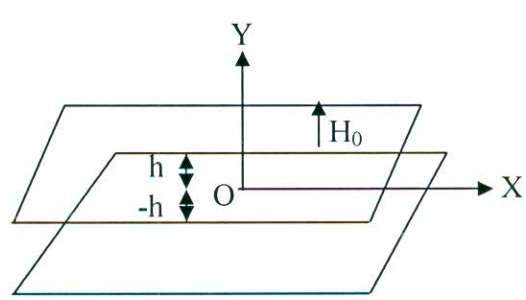

Fig-A

The equations of motion of a dusty conducting viscous, unsteady and incompressible fluid in the absence of input electric field are [117, 151]:

$$\frac{\partial \bar{u}}{\partial t} + (\bar{u}.\nabla)\bar{u} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\bar{u} + \frac{KN}{\rho}(\bar{v} - \bar{u}) + \frac{\mu_e}{\rho}(\bar{J} \times \bar{H}) - 2 \in \Omega\, \bar{u}\,, \qquad \text{------------(5.2.1)}$$

$$m\left[\frac{\partial \bar{v}}{\partial t}+(\bar{v}.\nabla)\bar{v}\right]=K[\bar{u}-\bar{v}] \ , \qquad \text{---------------(5.2.2)}$$

$$div\ \bar{u}=0 \ , \qquad \text{---------------(5.2.3)}$$

$$\frac{\partial N}{\partial t}+div(N\,\bar{v})=0 \ , \qquad \text{--------------(5.2.4)}$$

where, u and v are velocities of fluid and dust particles respectively; t, the time; p, the fluid pressure; ρ, the fluid density; υ, the kinematic coefficient of viscosity; K, the Stokes's resistance coefficient which for spherical particle of radius r is 6πμr; N represents the number of density of the dust particles; μ_e the magnetic permeability; J, the current density; H, the magnetic field; m, the mass of the dust particles; $\in$, the unit alternating tensor and Ω, the angular velocity vector of uniform rotation.

For the present problem the velocity distribution of fluid and dust particles are defined respectively as

$$u_1 = u_1(y, t), \quad u_2 = 0, \quad u_3 = 0$$
$$v_1 = v_1(y, t), \quad v_2 = 0, \quad v_3 = 0 \qquad \text{------------(5.2.5)}$$

where, (u_1, u_2, u_3) and (v_1, v_2, v_3) are the velocity field of fluid and dust respectively.

Using these relations, equations (5.2.1) and (5.2.2) become

$$\frac{\partial u_1}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u_1}{\partial y^2}+\frac{KN}{\rho}(v_1-u_1)-\frac{\sigma\mu_e^{\,2}H_0^{\,2}}{\rho}u_1-2\in\Omega\,u_1 \qquad \text{--------------(5.2.6)}$$

and $$\frac{\partial v_1}{\partial t}=\frac{k}{m}(u_1-v_1) \qquad \text{--------------(5.2.7)}$$

i.e. $$\frac{\partial u_1}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u_1}{\partial y^2}+\frac{l}{\tau}(v_1-u_1)-\frac{\sigma\mu_e^{\,2}H_0^{\,2}}{\rho}u_1-2\in\Omega\,u_1 \qquad \text{--------------(5.2.8)}$$

and $$\frac{\partial v_1}{\partial t}=\frac{1}{\tau}(u_1-v_1) \qquad \text{---------------(5.2.9)}$$

where $l = \frac{mN}{\rho}$ (Mass concentration) and $\tau = \frac{m}{K}$ (Time relaxation).

The initial and boundary conditions as:

$$\left.\begin{array}{lll} \text{at } t = 0, & u_1 = 0, & v_1 = 0 \\ \text{at } > 0, & u_1 = 0, & v_1 = 0 \text{ at } y = -h \\ & u_1 = 0, & v_1 = 0 \text{ at } y = +h \end{array}\right\} \qquad \text{--------------(5.2.10)}$$

Introducing the non-dimensional quantities u_1', v_1', p', t', x', y' and z' as follows:

$$u_1' = \frac{u_1}{u_0}, \quad v_1' = \frac{v_1}{u_0}, \quad p' = \frac{p}{\rho u_0^2}, \quad t' = \frac{\nu t}{h^2}$$

$$x' = \frac{x}{h}, \quad y' = \frac{y}{h}, \quad \tau' = \frac{\tau\nu}{h^2} = \frac{m\nu}{kh^2} \qquad \text{-------------(5.2.11)}$$

where u_0 is the characteristic velocity and h the half distance between two plates.

In view of Eq.(5.2.11), Eqs. (5.2.8) and (5.2.9), after removing the primes reduce to,

$$\frac{\partial u_1}{\partial t} = -\frac{\mu_0 h}{\nu}\frac{\partial p}{\partial x} + \frac{\partial^2 u_1}{\partial y^2} + \frac{l}{\tau}(v_1 - u_1) - \frac{\sigma\mu_e^{\,2} H_0^{\,2} h^2}{\mu} u_1 - 2 \in \Omega\, u_1 \qquad \text{--------------(5.2.12)}$$

and $$\frac{\partial v_1}{\partial t} = \frac{1}{\tau}(u_1 - v_1) \qquad \text{--------------(5.2.13)}$$

i.e. $$\frac{\partial u_1}{\partial t} = -R\frac{\partial p}{\partial x} + \frac{\partial^2 u_1}{\partial y^2} + \frac{l}{\tau}(v_1 - u_1) - M u_1 - 2 \in \Omega\, u_1 \qquad \text{--------------(5.2.14)}$$

and $$\frac{\partial v_1}{\partial t} = \frac{1}{\tau}(u_1 - v_1) \qquad \text{---------------(5.2.15)}$$

where, $R = \frac{\mu_0 h}{\nu}$ (Reynolds number) and $M = \frac{\sigma\mu_e^{\,2} H_0^{\,2} h^2}{\mu}$ (Hartman number).

The boundary conditions in the non-dimensional form are

$$\left.\begin{aligned} &\text{at } t=0, && u_1=0,\ v_1=0 \\ &\text{at } >0, && u_1=0,\ v_1=0 \text{ at } y=-1 \\ & && u_1=0,\ v_1=0 \text{ at } y=1 \end{aligned}\right\} \quad \text{--------------(5.2.16)}$$

Eliminating v_1 from the Eqs. (5.2.14) and (5.2.15), we get

$$\frac{\partial^2 u_1}{\partial t^2}+\frac{\partial}{\partial t}\left(R\frac{\partial p}{\partial x}\right)-\frac{\partial}{\partial t}\left(\frac{\partial^2 u_1}{\partial y^2}\right)+\left[\frac{l+1}{\tau}+M-2\in\Omega\right]\frac{\partial u_1}{\partial t}+\frac{1}{\tau}\left[R\frac{\partial p}{\partial x}-\frac{\partial^2 u_1}{\partial y^2}\right]=0 \quad \text{------------(5.2.17)}$$

Consider that, the sum of the residues of the poles as [35]

$$u_1= f(y)\,(a_0+at) + a\, g(y) \quad \text{--------------(5.2.18)}$$

and $$-R\frac{\partial p}{\partial x}=a_0+at \quad \text{-------------(5.2.19)}$$

where, f and g are functions of y only.

In view of Eqs. (5.2.18) and (5.2.19), Eq. (5.2.17) reduces to

$$-a\left[1+f''(y)-\left\{\frac{l+1}{\tau}+M-2\in\Omega\right\}f(y)-\frac{g''(y)}{\tau}\right]-\left(\frac{a_0+at}{\tau}\right)\left[1+f''(y)\right]=0 \quad \text{----------(5.2.20)}$$

where primes denote differentiation with respect to y.

By equating the coefficients of (a_0+at) and a to zero, we can obtain the expressions for f(y) and g(y) from Eq. (5.2.20), Thus

$$f(y)=\frac{1}{2}\left(1-y^2\right) \quad \text{--------------(5.2.21)}$$

$$g(y)=\frac{\tau}{24}\left[\frac{l+1}{\tau}+M-2\in\Omega\right]\left(6y^2-y^4-5\right) \quad \text{--------------(5.2.22)}$$

From the Eqs. (5.2.18), (5.2.21) and (5.2.22), we obtain the velocity of the fluid

$$u_1 = \frac{1}{2}(a_0 + at)(1-y^2) + \frac{a\tau}{24}\left[\frac{l+1}{\tau} + M - 2\in\Omega\right](6y^2 - y^4 - 5) \qquad \text{-------------(5.2.23)}$$

From the Eqs. (5.2.15) and (5.2.23), we obtain the velocity of the dust particle

$$v_1 = \frac{\tau}{l}\left[\begin{array}{l}\frac{1}{2}a(1-y^2) - \frac{a\tau}{2}\left\{\frac{l+1}{\tau} + M - 2\in\Omega\right\}(1-y^2) \\ +\left(\frac{l}{\tau} + M\right)\left[\frac{1}{2}(a_0 + at)(1-y^2) + \frac{a\tau}{24}\left\{\frac{l+1}{\tau} + M - 2\in\Omega\right\}(6y^2 - y^4 - 5)\right]\end{array}\right]$$

i.e $$v_1 = \frac{\tau}{l}\left[\frac{1}{2}a(1-y^2) - \frac{a\tau}{2}\left\{\frac{l+1}{\tau} + M - 2\in\Omega\right\}(1-y^2) + \left(\frac{l}{\tau} + M\right)u_1\right] \qquad \text{--------------(5.2.24)}$$

The Eqs. (5.2.23) and (5.2.24) represent the velocities of the fluid and dust particles respectively in a rotating frame i.e. in presence of coriolis force.

5.3. Results and Discussion:

In Eqs. (5.2.23) and (5.2.24), we obtained the velocities of the fluid and dust particles respectively in MHD flow of a dusty viscous incompressible fluid in a rotating frame with the transverse magnetic field.

If the frame is non-rotating (absence of coriolis force), i.e. $\Omega = 0$, then the Eqs. (5.2.23) and (5.2.24) become

$$u_1 = \frac{1}{2}(a_0 + at)(1-y^2) + \frac{a\tau}{24}\left[\frac{l+1}{\tau} + M\right](6y^2 - y^4 - 5) \qquad \text{-------------(5.3.1)}$$

and $$v_1 = \frac{\tau}{l}\left[\frac{1}{2}a(1-y^2) - \frac{a\tau}{2}\left\{\frac{l+1}{\tau} + M\right\}(1-y^2) + \left(\frac{l}{\tau} + M\right)u_1\right] \qquad \text{-------------(5.3.2)}$$

which were obtained earlier by Sreehareddy, Nagarajan and Sivaiah [139].

This study obtained the velocity profiles of the fluid particles (u_1) and dust particles (v_1) in presence of coriolis force due to the variation of the parameters M, l, τ and t under the influence of the magnetic field. These results are graphically shown in Figs. 1(A)–4(B) and also are discussed.

In Fig. 1(A)&1(B), we observe that the variation in Hartman number (M) causes significant changes in the velocity of fluid particles as well as in the velocity of dust particles. It is mentioned that the velocity of the fluid particles decreases with the Hartman number M increases and the velocity of the dust particles increases with M increases.

In Fig. 2(A) & 2(B), we notice that the variation in mass concentration *l* on the velocities of fluid and dust particles is shown. It is observed that increase in mass concentration *l* leads to decrease in the velocity of fluid particles and also dust particles.

In Fig. 3(A)&3(B), we observe that the variation in time relaxation τ on the velocities of fluid and dust particles is shown. We mention that the velocity of the fluid particles and also dust particles increases with the increasing values of time relaxation τ.

In Fig. 4(A) & 4(B), we mention that the variation in time t on the velocities of fluid and dust particles is shown. We observe that velocity of the fluid and also dust particles increases as time t increases.

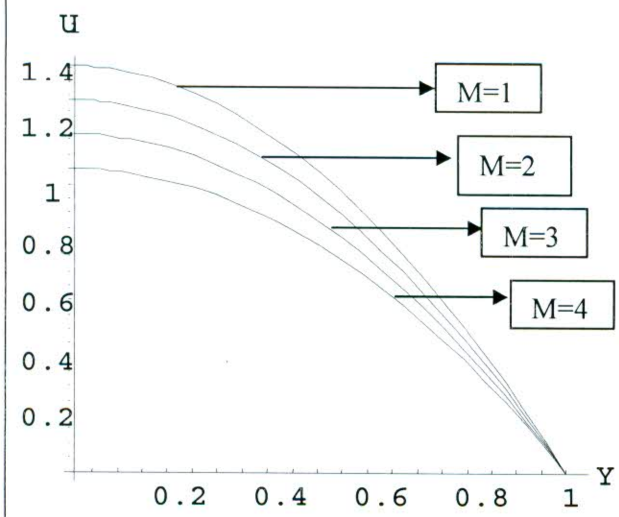

Fig-1(A): u_l-Y graph for velocity profile of fluid particle with coriolis force for different values of M.

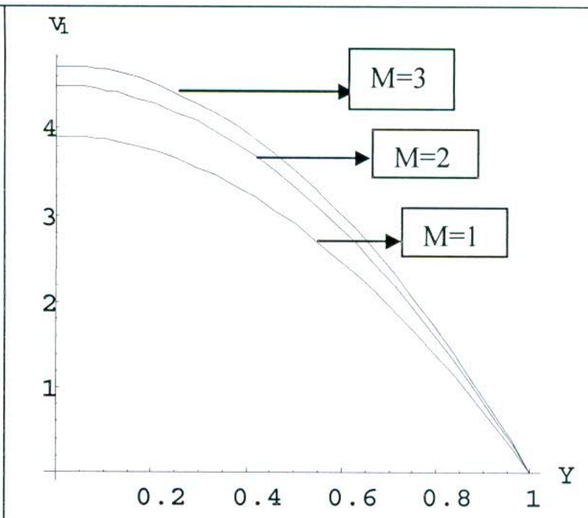

Fig-1(B): v_l-Y graph for velocity profile of dust particle with coriolis force for different values of M.

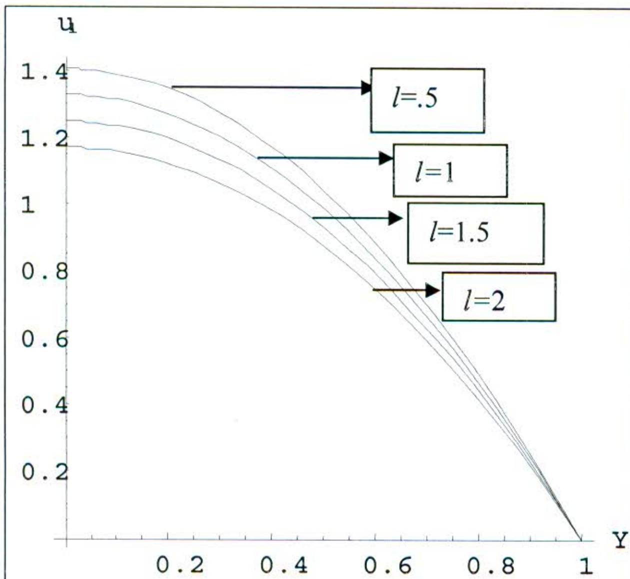

Fig-2(A): u_l-Y graph for velocity profile of fluid particle with coriolis force for different values of *l*.

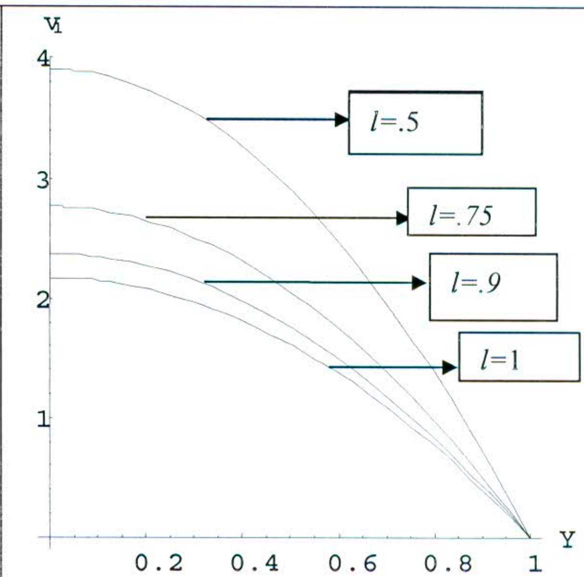

Fig-2(B): v_l-Y graph for velocity profile of dust particle with coriolis force for different values of *l*.

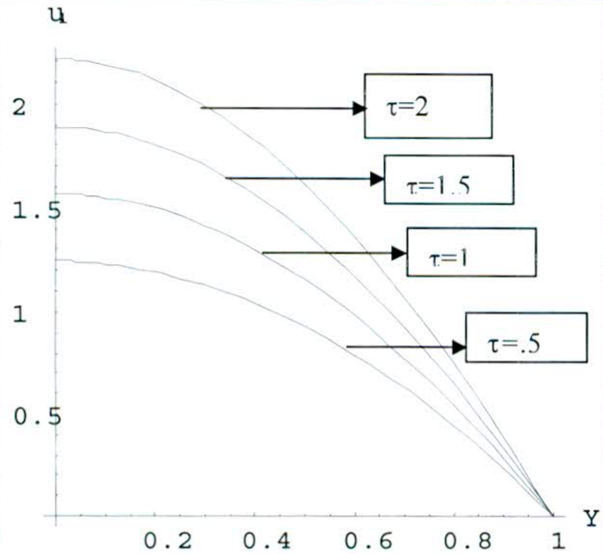

Fig-3(A): u_1-Y graph for velocity profile of fluid particle with coriolis force for different values of τ.

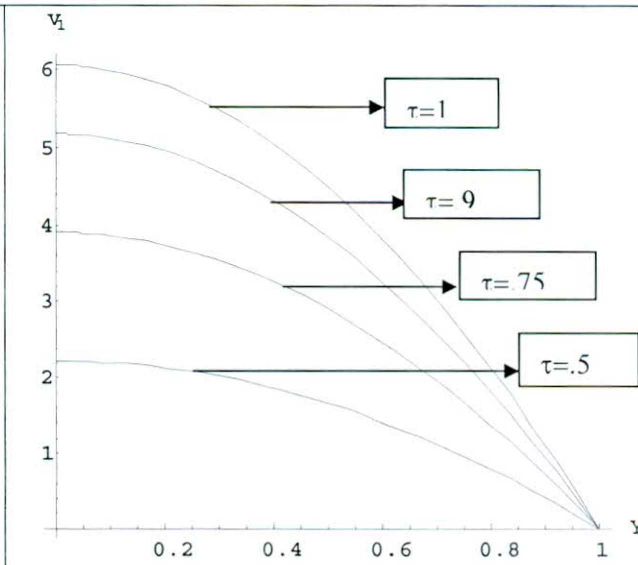

Fig-3(B): v_1-Y graph for velocity profile of dust particle with coriolis force for different values of τ.

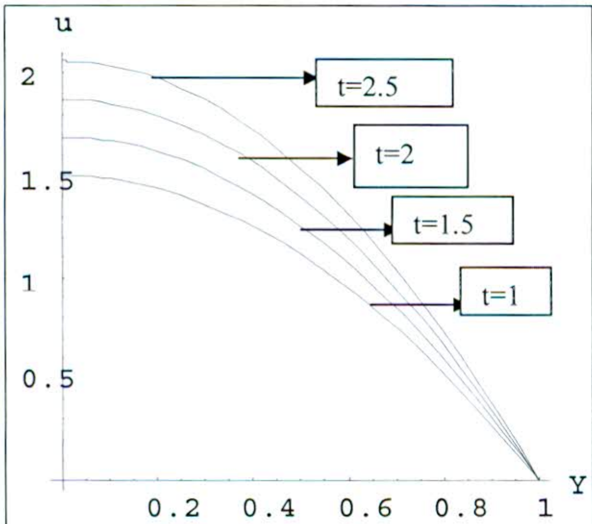

Fig-4(A): u_1-Y graph for velocity profile of fluid particle with coriolis force for different values of t.

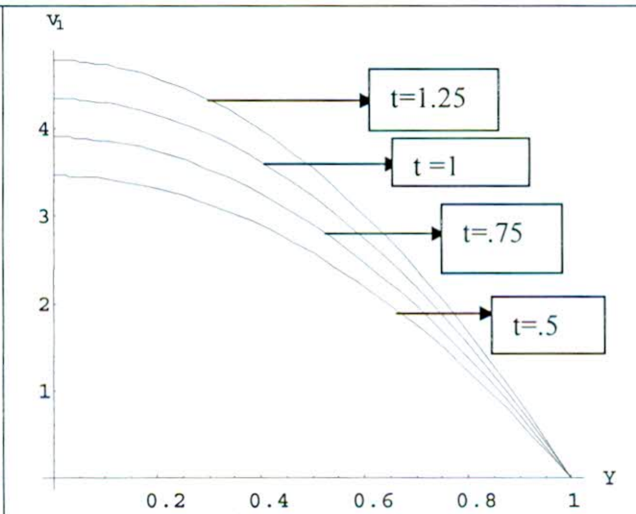

Fig-4(B): v_1-Y graph for velocity profile of dust particle with coriolis force for different values of t.

In the absence of coriolis force, the velocity profiles of the fluid particles (u_l) and dust particles (v_l) due to the variation of the parameters M, l, τ and t under the influence of the magnetic field are graphically shown in Figs. 5(A)-8(B) which were obtained earlier in ref. [139]:

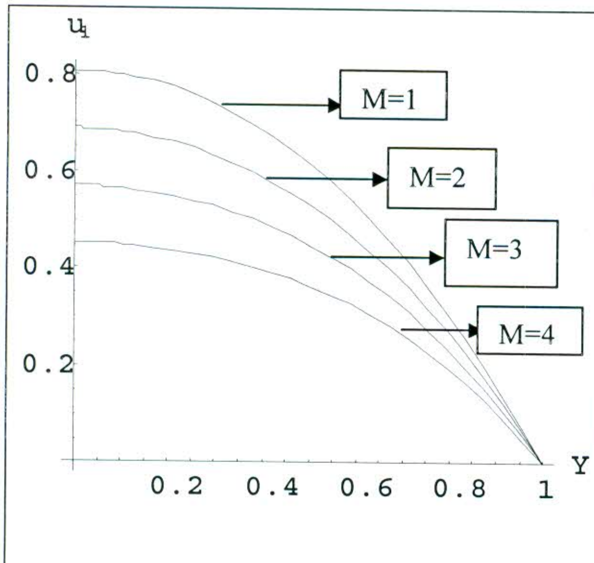

Fig-5(A): u_l-Y graph for velocity profile of fluid particle for different values of M.

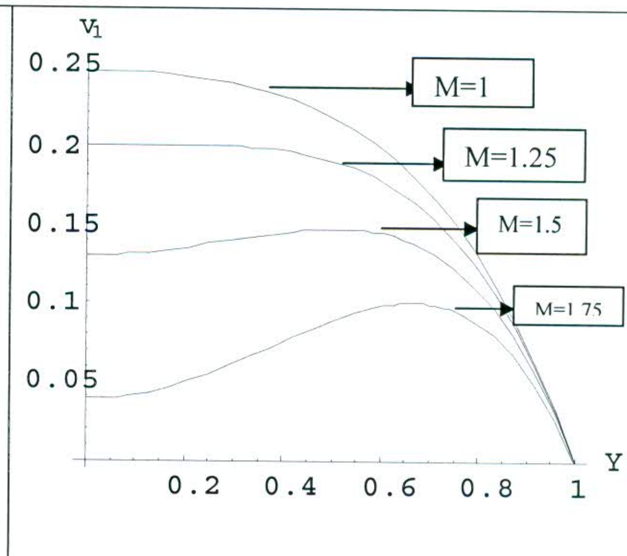

Fig-5(B): v_l-Y graph for velocity profile of dust particle for different values of M.

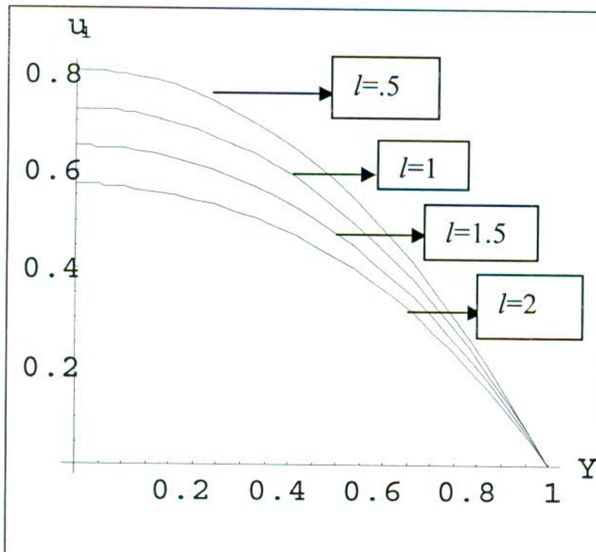

Fig-6(A): u_l-Y graph for velocity profile of fluid particle for different values of *l*.

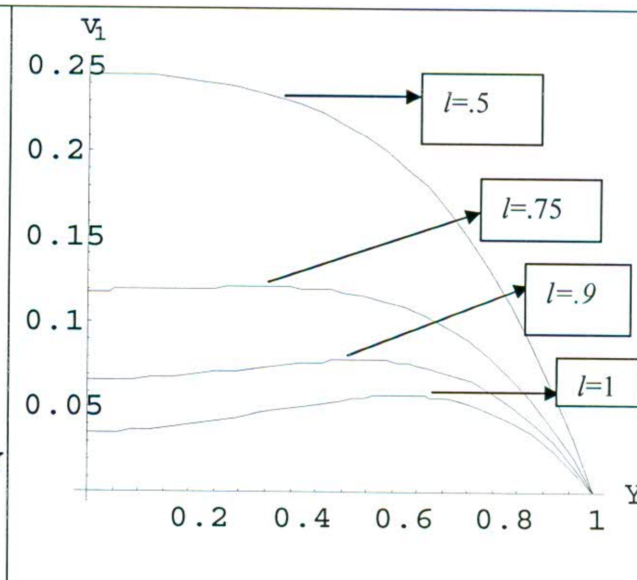

Fig-6(B): v_l-Y graph for velocity profile of dust particle for different values of *l*.

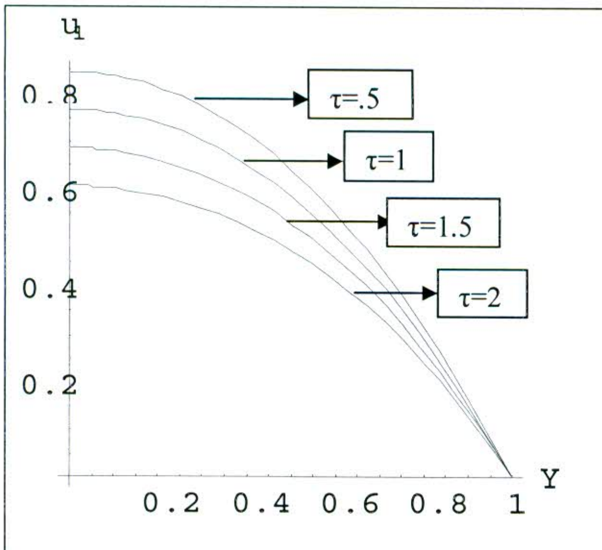

Fig-7(A): u_1-Y graph for velocity profile of fluid particle for different values of τ.

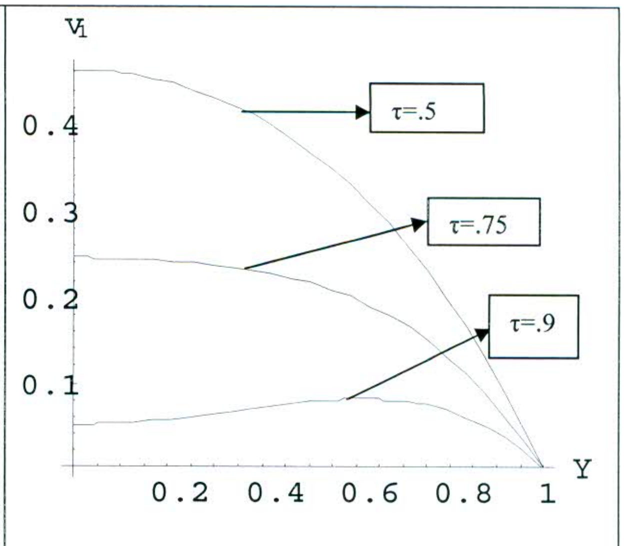

Fig-7(B): v_1-Y graph for velocity profile of dust particle for different values of τ.

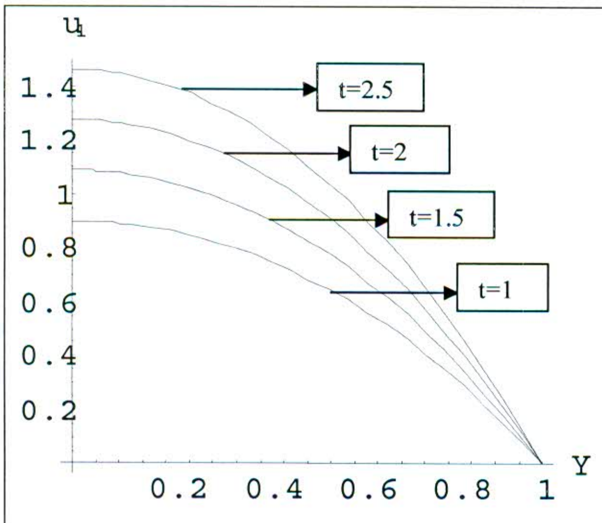

Fig-8(A): u_1-Y graph for velocity profile of fluid particle for different values of t.

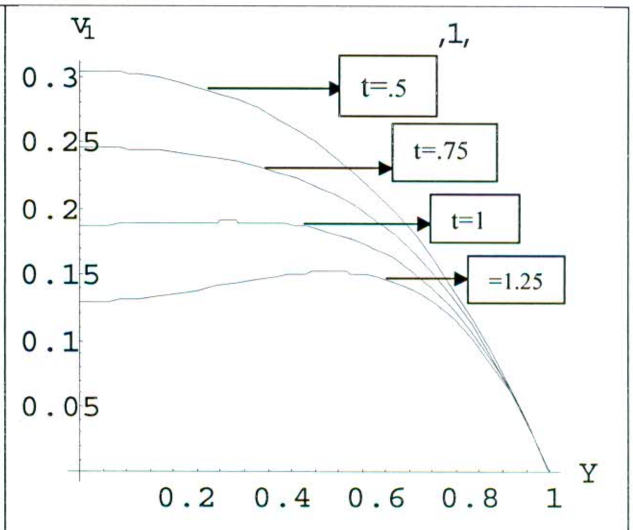

Fig-8(B): v_1-Y graph for velocity profile of dust particle for different values of t.

For both rotating and non-rotating frame, the velocity profiles of the fluid particles (u_l) and dust particles (v_l) due to the variation of the parameters M, l, τ and t under the influence of the magnetic field are graphically shown (comparatively) in Figs. (9-16). Here, the variation level of the velocity profiles of fluid and dust particles between rotating and non-rotating frame are clearly observed. The levels of velocity profiles of both fluid and dust particles are increased for rotating system.

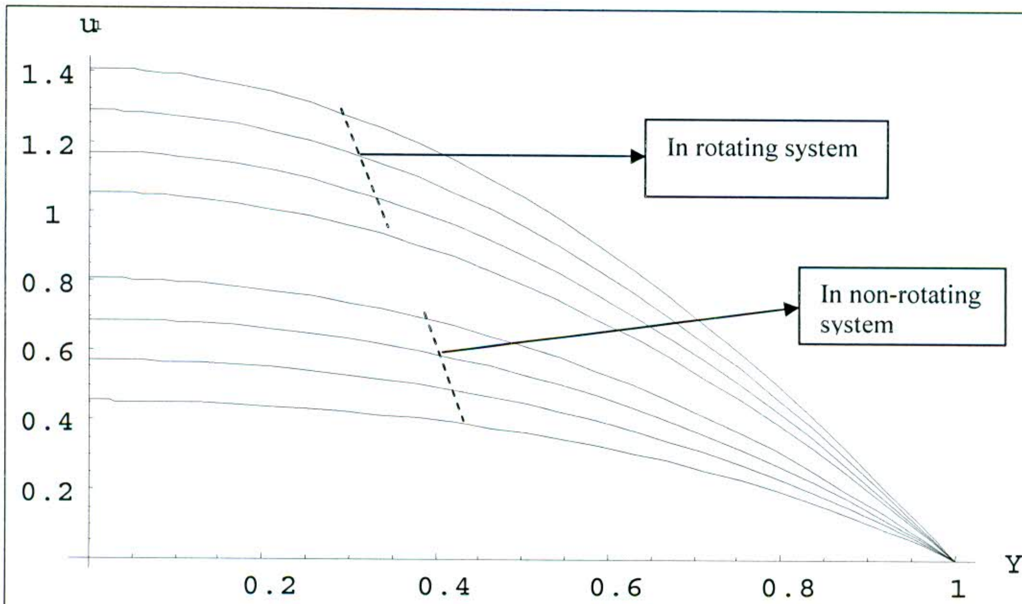

Fig-9: u_l-Y graph for velocity profiles of fluid particles in rotating and non-rotating system for M=1, 2, 3, 4

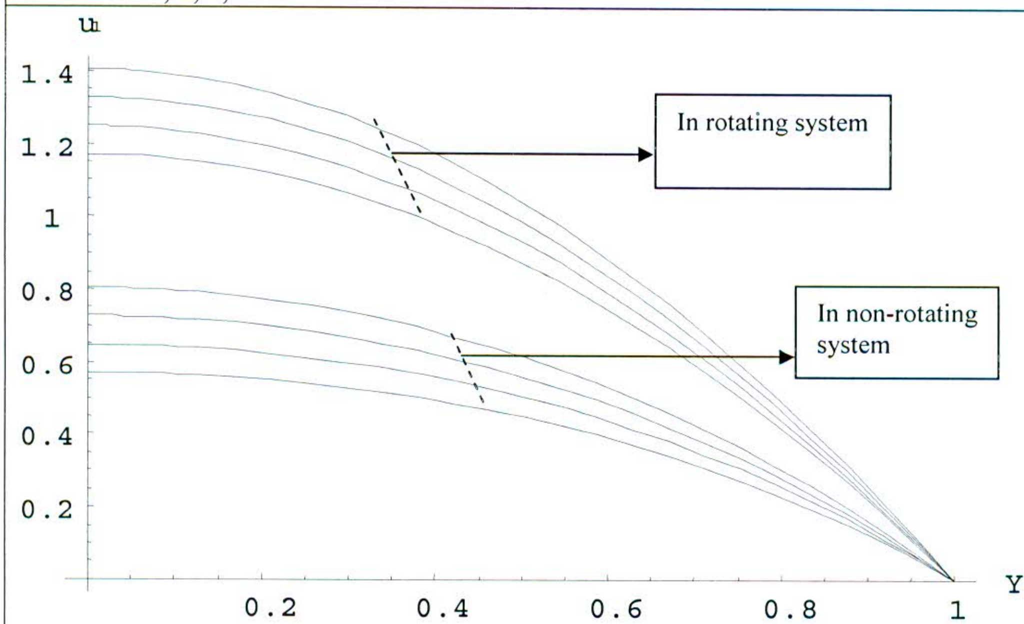

Fig-10: u_l-Y graph for velocity profiles of fluid particles in rotating and non-rotating system for l=.5, 1, 1.5, 2

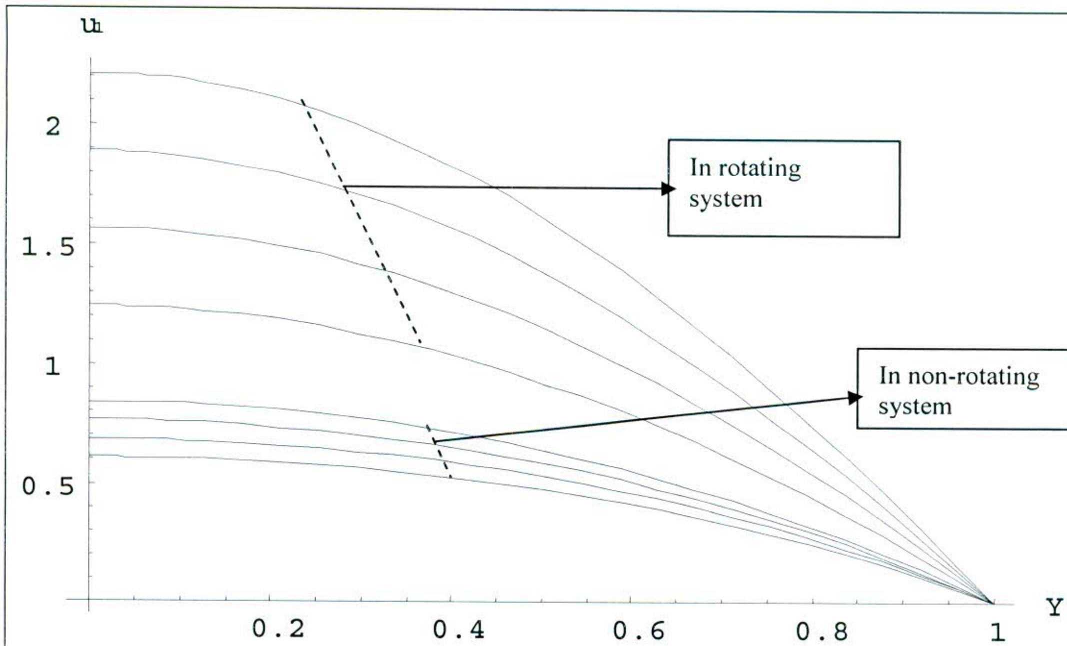

Fig-11: u_1-Y graph for velocity profiles of fluid particles in rotating and non-rotating system for $\tau = .5, 1, 1.5, 2$

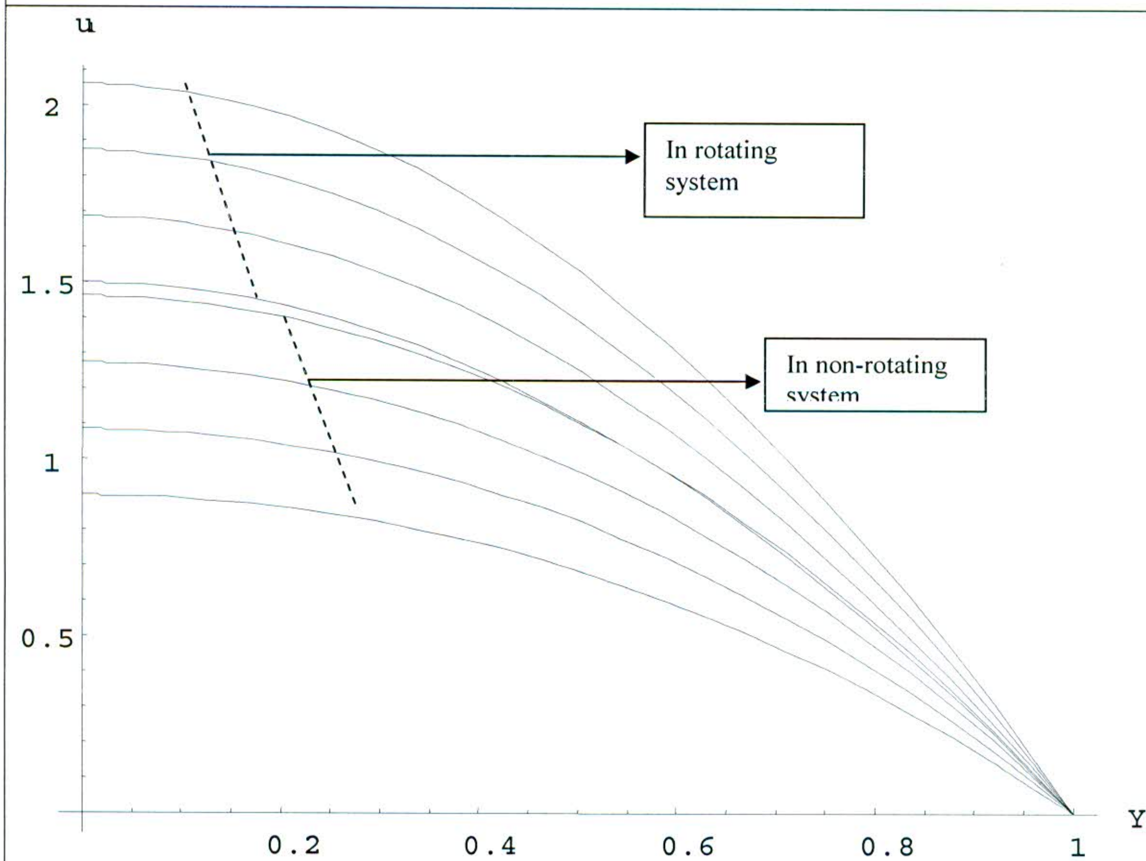

Fig-12: u_1-Y graph for velocity profiles of fluid particles in rotating and non-rotating system for $t = 1, 1.5, 2, 2.5$

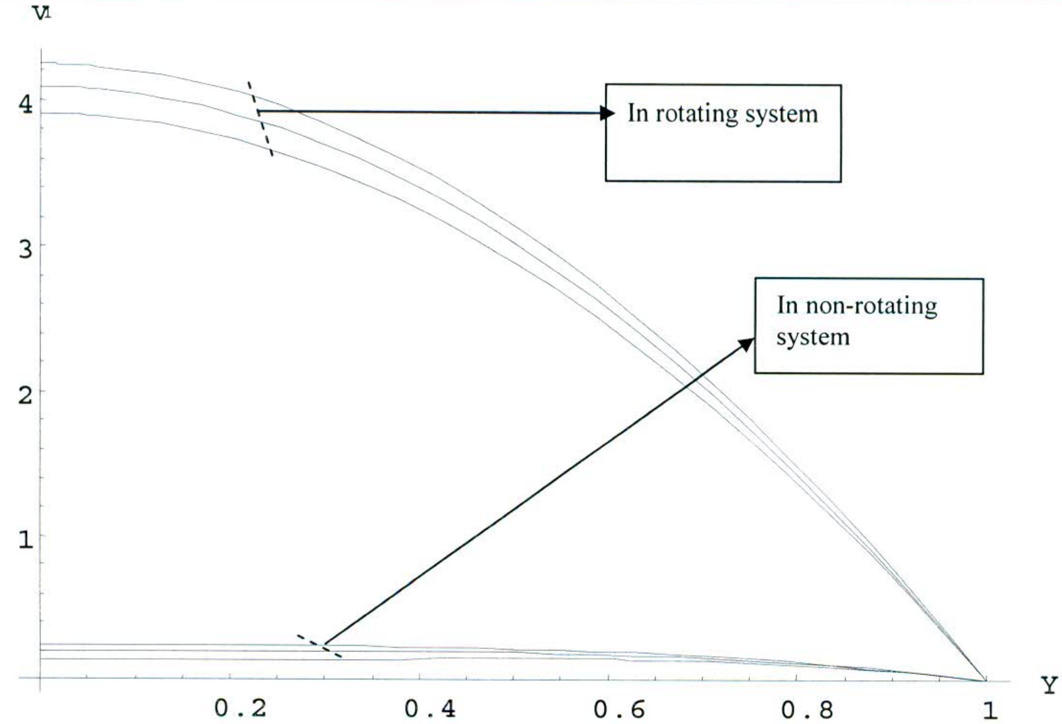

Fig-13: v_1-Y graph for velocity profiles of dust particles in rotating and non-rotating system for M = 1, 1.25, 1.5

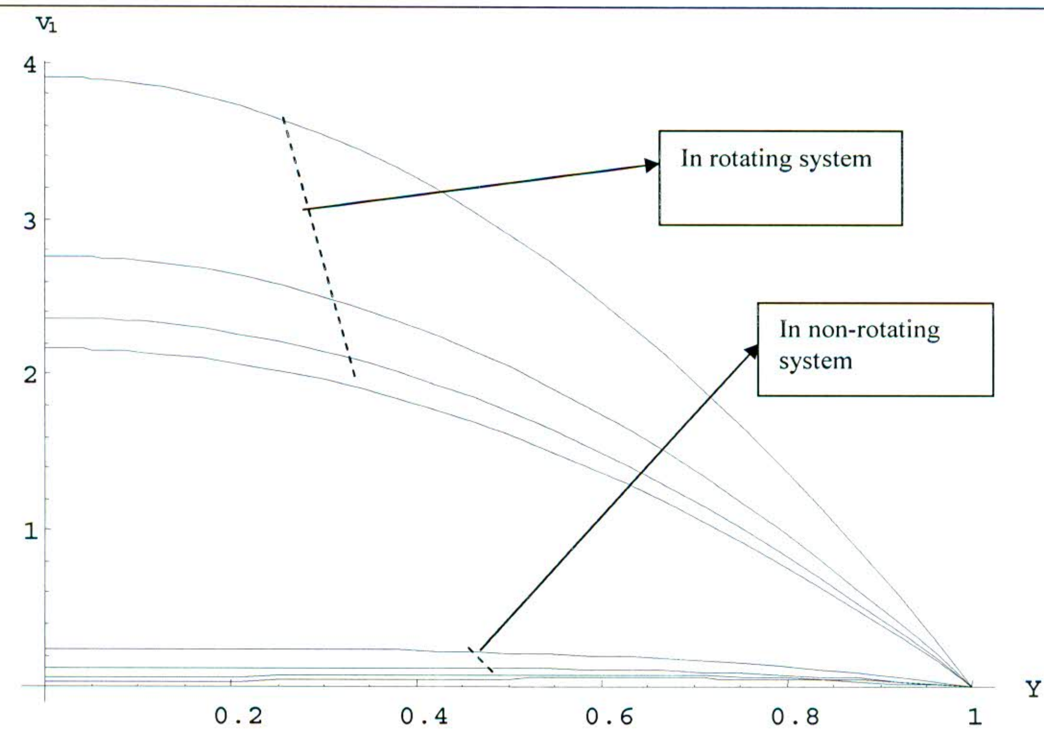

Fig-14: v_1-Y graph for velocity profiles of dust particles in rotating and non-rotating system for l = .5, .75, .9, 1

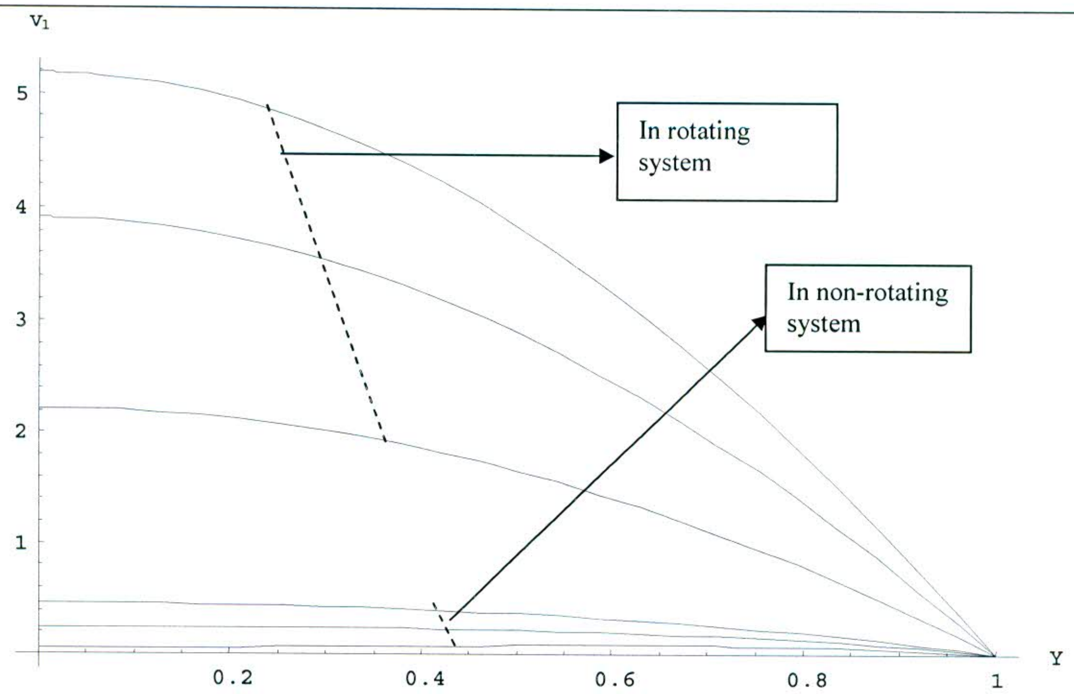

Fig-15: v_1-Y graph for velocity profiles of dust particles in rotating and non-rotating system for $\tau = .5, .75, .9$

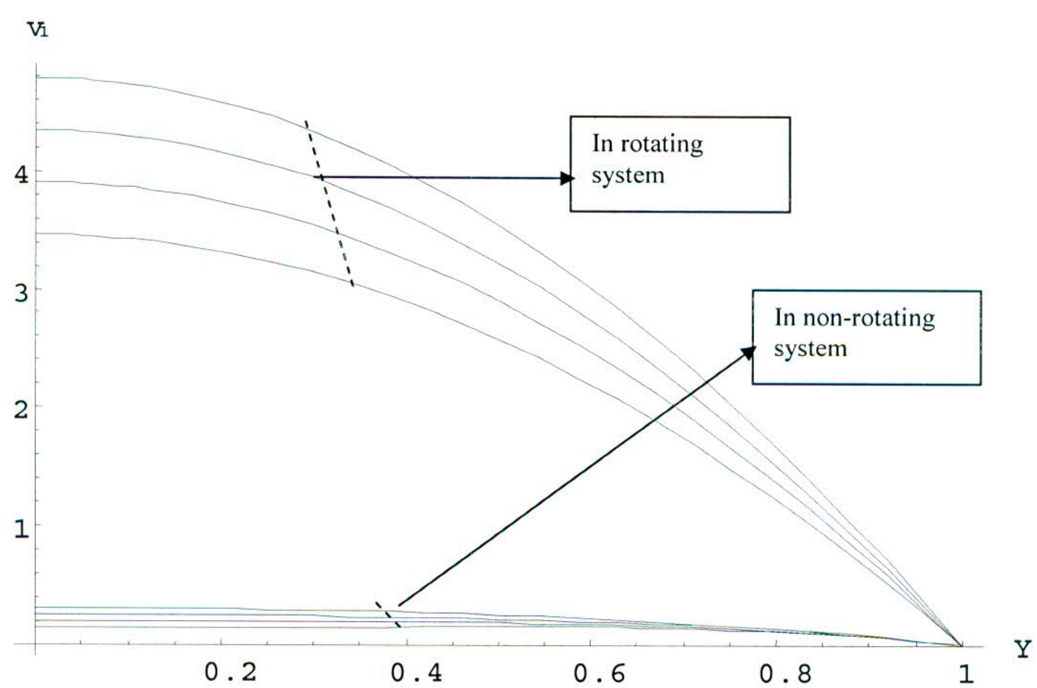

Fig-16: v_1-Y graph for velocity profiles of dust particles in rotating and non-rotating system for $t = .5, .75, 1, 1.25$

CHAPTER-VI

A REVIEW OF THE THESIS WITH CONCLUSIONS

The thesis entitled "**A study on turbulence and MHD turbulence**" has been divided into five chapters.

First Chapter :

In the first chapter, we have studied the definition and concept of turbulence, Reynolds number and its effect of turbulence, method of taking averages, Reynolds rules averages, Reynolds equations, spectral representation of turbulence, correlation functions, historical back ground of early works of turbulence.

In this chapter, we have also discussed the first order reaction, rotating system, equation of motion of dust particles, decay law of turbulence before the final period and in the final period, statistical theory of distribution functions in turbulence, Fourier transformations of Navier-stokes equation and their principal conceptions, Magneto-hydrodynamic(MHD) turbulence and finally, a brief review of the past researches related to this thesis.

The Second Chapter has been divided in to Three Parts:

In Part-A, the first order reactant in Magneto-hydrodynamic Turbulence before the final period of decay in a rotating system is studied. In this part we studied the magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction in MHD turbulence before the final period of decay in a rotating system. Here, we have considered the two-point and three-point correlation equations and solved these equations after neglecting the fourth-order correlation terms. Finally we obtained the decay law for magnetic field energy fluctuation of concentration of dilute contaminant undergoing a first order chemical reaction in MHD turbulence in a rotating system. Equation (2.5.18) denotes this decay law for magnetic energy fluctuation of MHD turbulence governing the concentration of a dilute contaminant undergoing a first order chemical reaction before the final period in a rotating system considering three-point correlation after neglecting quadruple correlation terms.

If the system is non-rotating, i.e. $\Omega_m = 0$, then the equation (2.5.18) becomes the equation (2.6.1) which was obtained earlier by Sarker and Islam [128]

In absence of chemical reaction, i.e, R=0 then the equation (2.6.1) becomes the equation (2.6.2) which was obtained earlier by Sarker and Kishor [120].

This study shows that due to the effect of rotation of fluid in MHD turbulence in a rotating system with chemical reaction of the first order in the concentrarion the magnetic field fluctuation i,e.the turbulent energy decays more rapidly than the energy for non-rotating fluid and the faster rate is governed by $\exp[-\{2 \in_{mkl} \Omega_m\}]$. Here the chemical reaction ($R\neq0$) in MHD turbulence causes the concentration to decay more they would for non-rotating system and it is governed by $\exp\left[-\{2RT_M + \in_{mkl} \Omega_m\}\right]$.

The first term of right hand side of equation (2.5.18) corresponds to the energy of magnetic field fluctuation of concentration for the two-point correlation and the second term represents magnetic energy for the three-point correlation. In equation (2.5.18), the terms associated with the three-point correlation die out faster than the two-point correlation. For large times the last term in the equation (2.5.18) becomes negligible, leaving the -3/2 power decay law for the final period.

In Part-B, the first order reactant in Magneto-hydrodynamic Turbulence before the final period of decay in presence of dust particle is studied. In this part, the same procedure is followed as in part II-A. In equation (2.11.18) we obtained the decay law for magnetic energy fluctuation of dusty MHD turbulence governing the concentration of a dilute contaminant undergoing a first order chemical reaction before the final period.

If the the fluid is clean, i.e. f=0 then the equation (2.11.18) becomes the equation (2.12.1). which was obtained earlier by Sarker and Islam [128]. In absence of chemical reaction, i.e, R=0 then the equation (2.12.1) becomes the equation (2.12.2) which was obtained earlier by Sarker and Kishor [120].

This study shows that due to the effect of dust particles in the magnetic field with chemical reaction of the first order in the concentrarion the magnetic field fluctuation i,e.the turbulent energy decays more rapidly than the energy for clean fluid.

In Part-C, we have studied the first order reactant in Magneto-hydrodynamic Turbulence before the final period of decay under the effect of rotation with an angular velocity Ω_m in presence of dust particles and we obtained the equation (2.17.18). This equation indicates that the decay law for magnetic energy fluctuation of dusty fluid MHD turbulence governing the concentration of a dilute contaminant undergoing a first order chemical reaction before the final period in a rotating system more rapidly.

If the fluid is clean and the system is non-rotating then $f = 0$ and $\Omega_m = 0$, the equation (2.17.18) becomes equation (2.18.1) which was obtained earlier by Sarker and Islam [128]. In absence of chemical reaction, i.e, R=0 then the equation (2.18.1) becomes (2.18.2) which was obtained earlier by Sarker and Kishor [120].

In equation (2.17.18), the terms associated with the three-point correlation die out faster than the two-point correlation. For large times the last term in the equation (2.17.18) becomes negligible, leaving the -3/2 power decay law for the final period. If higher order correlations are considered in the analysis, it appears that more terms of higher power of time would be added to the equation (2.17.18).

The Third Chapter consists of Three Parts:

In part-A, we have studied the statistical theory of distribution function for simultaneous velocity, magnetic, temperature, concentration fields and reaction in MHD turbulence in a rotating system. We have derived the transport equations (3.6.17) and (3.6.18) for evolution of one point distribution function $F_1^{(1)}$ and two point distribution function $F_2^{(1,2)}$ in MHD turbulent flow under the effect of coriolis force and various properties of the distribution function have been discussed. We can also derive the equations for evolution of $F_3^{(1,2,3)}, F_4^{(1,2,3,4)}$ and so on. The resulting one-point equation is compared with the first equation of BBGKY hierarchy of equations and the closure difficulty is to be removed as in the case of ordinary turbulence.

But it is a great difficulty that the N-point distribution function depends upon the N+1-point distribution function and thus result is an unclosed system. This is so-called "closer problem". In this chapter, the closure difficulty is to be removed as in the case of ordinary turbulence and some properties of distribution functions have been discussed.

If the system is non rotating then $\Omega m=0$, the transport equation for one point distribution function in MHD turbulent flow equation(3.6.17) becomes the equation (3.7.1) which was obtained earlier by [57].

In part-B, we studied the statistical theory of distribution function for simultaneous velocity, magnetic, temperature, concentration fields and reaction in MHD turbulence in presence of dust particles. Finally, the transport equations (3.13.17) and (3.13.18) for evolution of one point distribution function $F_1^{(1)}$ and two point distribution function $F_2^{(1,2)}$ in MHD turbulent flow for evolution of distribution functions have been derived and various properties of the distribution function have been discussed. The resulting one-point equation is compared with the first equation of BBGKY hierarchy of equations and the closure difficulty is to be removed as in the case of ordinary turbulence.

If the fluid is clean then f=0 , the transport equation for one point distribution function in MHD turbulent flow (3.13.17) becomes the equation (3.14.1) which was obtained earlier by [57].

In Part-C, we have studied the statistical theory of distribution function for simultaneous velocity, magnetic, temperature,concentration fields and reaction in MHD turbulence in a rotating system in presence of dust particles. Here, the transport equations (3.20.18) and(3.20.19) for evolution of one point distribution function $F_1^{(1)}$ and two point distribution function $F_2^{(1,2)}$ in dusty fluid MHD turbulent flow under the effect of coriolis force have been derived and various properties of the distribution function have been discussed. The resulting one-point equation is compared with the first equation of BBGKY hierarchy of equations and the closure difficulty is to be removed as in the case of ordinary turbulence.

The Fourth Chapter also consists of Three Parts:

In Part-A, we have studied the magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction in MHD turbulence before the final period of decay for the case of multi-point and multi-time in a rotating system. Here, we have considered the two-point, two-time and three-point, three-time correlation equations and solved these equations after neglecting the fourth-order correlation terms. Finally the decay law for magnetic field energy fluctuation of concentration of dilute contaminant undergoing a first order chemical reaction in MHD turbulence considering three-point correlation terms for the case of multi-point and multi-time in a rotating system is obtained in equation (4.5.20).

If the system is non-rotating then $\Omega_m = 0$, the equation (4.5.20) becomes the equation (4.6.1) which was obtained earlier by [56].

If we put $\Delta T=0$, $R=0$, in equation (4.6.1), we can easily find out the equation (4.6.2) which is same as obtained earlier by [120].

This study shows that due to the effect of rotation of fluid in the flow field with chemical reaction of the first order in the concentrarion the magnetic field fluctuation in MHD turbulence in a rotating system for the case of multi-point and multi-time i,e.the turbulent energy decays more rapidly than the energy for non-rotating fluid and the faster rate is governed by $\exp[-\{2\in_{mkl}\Omega_m\}]$. Here the chemical reaction ($R\neq0$) in MHD turbulence for the case of multi-point and multi-time causes the concentration to decay more they would for non-rotating system and it is governed by $\exp\left[-\{2RT_M+\in_{mkl}\Omega_m\}\right]$

The first term of right hand side of equation (4.5.20) corresponds to the energy of magnetic field fluctuation of concentration for the two-point correlation and the second term represents magnetic energy for the three-point correlation. In equation (4.5.20), the term associated with the three-point correlation die out faster than the two-point correlation. If higher order correlations are considered in the analysis, it appears that more terms of higher power of time would be added to the equation (4.5.20). For large times the last term in the equation (4.5.20) becomes negligible, leaving the -3/2 power decay law for the final period.

In Part-B, we have studied the magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction in MHD turbulence before the final period of decay for the case of multi-point and multi-time in presence of dust particle. Here, we have considered the two-point, two-time and three-point, three-time correlation equations and solved these equations after neglecting the fourth-order correlation terms. Finally we obtained the decay law for magnetic field energy fluctuation of concentration of dilute contaminant undergoing a first order chemical reaction in MHD turbulence for the case of multi-point and multi-time in presence of dust particle is obtained in equation (4.11.20).

If the fluid is clean then $f=0$, the equation (4.11.20) becomes the equation (4.12.1) which was obtained earlier by [56].

If we put $\Delta T=0$, $R=0$, in equation (4.12.1) we can easily find out the equation (4.12.2) which is same as obtained earlier by [120].

This study shows that due to the effect of rotation of fluid in the flow field with chemical reaction of the first order in the concentrarion the magnetic field fluctuation in MHD turbulence in presence of dust particle for the case of multi-point and multi-time i,e.the turbulent energy decays more slowly than the energy for clean fluid and the rate is governed by $\exp[fs]$. Here the chemical reaction (R≠0) in dusty fluid MHD turbulence for the case of multi-point and multi-time causes the concentration to decay more they would for clean fluid and it is governed by $\exp\left[-\left\{2RT_M - fs\right\}\right]$.

In part-C, the magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction in dusty fluid MHD turbulence before the final period of decay for the case of multi-point and multi-time in a rotating system is studied. Here, we have considered the two-point, two-time and three-point, three-time correlation equations and solved these equations after neglecting the fourth-order correlation terms. In equation (4.17.20) we obtained the decay law of magnetic energy fluctuations of a dilute contaminant undergoing a first order chemical reaction before the final period considering three-point correlation terms for the case of multi-point and multi-time in MHD turbulence in presence of dust particle in a rotating system.

If the fluid is non-rotating and clean then $\Omega_m = 0$, f=0, the equation (4.17.20) becomes the equation (4.18.1) , which was obtained earlier by [56].
If we put ΔT=0, R=0, in equation (4.18.1) we can easily find out the equation (4.18.2), which is same as obtained earlier by [120].

The first term of right hand side of equation (4.17.20) corresponds to the energy of magnetic field fluctuation of concentration for the two-point correlation and the second term represents magnetic energy for the three-point correlation. In equation (4.17.20), the terms associated with the three-point correlation die out faster than the two-point correlation. If higher order correlations are considered in the analysis, it appears that more terms of higher power of time would be added to the equation (4.17.20). For large times the last term in the equation (4.17.20) becomes negligible, leaving the -3/2 power decay law for the final period.

Fifth Chapter :

In Chapter V, we have studied the MHD flow of a dusty viscous incompressible fluid in a rotating frame between two parallel flat plates in presence of a uniform transverse magnetic field with pressure gradient. The velocities of the fluid and the dust particles for rotating frame are obtained and the effect of magnetic field on these velocities is investigated. The variation in the magnetic parameters causes significant changes in the velocity profiles of fluid particles as well as of dust particles and these changing levels of velocity profiles are comparatively higher than that of the non-rotating frame. The effects of the coriolis force on velocity profiles of the fluid and the dust particles are graphically discussed. It is observed that the velocities of fluid and dust particles increase with the increase of coroilis force.

In Eqs. (5.2.23) and (5.2.24), we obtained the velocities of the fluid and dust particles respectively in MHD flow of a dusty viscous incompressible fluid in a rotating frame with the transverse magnetic field.

If the frame is non-rotating (absence of coriolis force), i.e. $\Omega = 0$, then the Eqs. (5.2.23) and (5.2.24) become the equations (5.3.1) and(5.3.2), which was obtained earlier by Sreehareddy, Nagarajan and Sivaiah [139].

This study obtained the velocity profiles of the fluid particles (u_1) and dust particles (v_1) in presence of coriolis force due to the variation of the parameters M, l, τ and t under the influence of the magnetic field. These results are graphically shown in Figs. 1(A)–4(B) and also are discussed.

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