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Laminar Flow of Incompressible Viscous Newtonian Fluid

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LAMINAR FLOW OF INCOMPRESSIBLE VISCOUS NEWTONIAN FLUID

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STATEMENT Of ORIGINALITY

I declare that the contents in my Ph. D. thesis entitled "Laminar Flow of Incompressible Viscous Newtonian Fluid" are original and accurate to the best of my knowledge. I also certify that the materials contained in my thesis have not been previously published or written by any person for a degree or diploma.

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ABSTRACT

The thesis entitled "**Laminar Flow of Incompressible Viscous Newtonian Fluid**" is being presented for the award of the degree of Doctor of Philosophy in Applied Mathematics. It is the outcome of my researches conducted in the Department of Applied Mathematics, University of Rajshahi, Bangladesh.

The whole thesis consists of nine chapters. The first chapter is a general introductory chapter, giving the general information about Laminar Flow of Incompressible Viscous Newtonian Fluid. **In chapter II**, we have described about the basic concepts of incompressible viscous Newtonian fluid. Some fundamental equations are presented in this chapter.

In Chapter III a laminar flow of incompressible viscous fluid has been considered. Here two numerical methods for solving boundary layer equation have been discussed; (i)Keller Box scheme,(ii) shooting method. Runge-Kutta method is used to solve the initial value problem. The shooting method is supported by a suitable example.

The Chapter IV is divided into two parts. **In Part: A** an attempt has been made to investigate the velocity profile of unsteady laminar flow of incompressible viscous fluid. The method of separation of variable is used to determine the solutions of the governing differential equations. Time varying pressure gradient is considered for poiseuille flow. The velocity profiles for the various types of flow are shown by the figures.

In Part: B a fully developed conducting flow of incompressible viscous Newtonian fluid between two parallel plates under the action of a parallel Lorentz force is considered. Analytic solutions for this type of flow are developed. The velocity profiles are presented in figures.

In Chapter V, an attempt has been made to study the flow of a viscous incompressible fluid between two parallel porous plates. In case I, we have considered the flow of conducting fluid between two fixed porous plates in presence of a transverse magnetic field. Small suction and injection are imposed on the plates. The velocity of the fluid has been obtained under the three different cases, when pressure gradient is (i) varying linearly with time (ii) decreasing exponentially with time and (iii) varying periodically with time. In case II of this chapter an attempt has been made to study the flow of a conducting viscous incompressible fluid between two porous plates in absence of pressure gradient force. One plate is at rest and the other plate is oscillating with a constant frequency. A small suction is imposed on the oscillating plate. A transverse magnetic field is also placed on the fluid. The velocity distribution has been investigated numerically with the help of finite difference method.

In Chapter VI the laminar flow of Newtonian conducting fluid produced by a moving plate in presence of transverse magnetic field is investigated. The basic equation governing the motion of such flow is expressed in non-dimensional form. Analytic solution of the governing equation is obtained by Laplace transformation. Numerical solution of the dimensionless equation is also obtained with the help of Crank-Nicholson implicit scheme. Velocity profiles of the corresponding problem are shown in the graphs.

The Chapter VII is also divided into two parts. **In part: A,** the temperature distributions of various types of parallel flow of incompressible viscous fluid have been considered. Temperature distribution near a heated plate is also discussed. Coefficients of heat transfer for various types flow has been investigated. **In part: B** of this chapter, we have considered unsteady MHD flows of an incompressible viscous fluid past an infinite vertical plate. The uniform flow is subject to a transverse applied magnetic field. We have also considered small magnetic Reynolds number so that induced magnetic field is neglected.

In Chapter VIII a steady laminar flow of viscous incompressible Newtonian flow through a uniform circular tube is considered. Two cases related to this type of flow have been considered here; i) circular tube flow past across a transverse magnetic field, ii)circular tube flow past a solid narrow obstacle .The distributions of radial velocity are shown for various Hartmann number. The pressure drop across the obstacle is shown by graphs.

CHAPTER III

Steady Laminar Flow of Incompressible Fluid Over a Flat Plate

CHAPTER IV

Laminar Flow of Incompressible Fluid Between Two Parallel Plates

Part: A

CHAPTER IV

Part: B

Laminar Flow of Incompressible Fluid Due to Lorentz Force

CHAPTER V

Laminar Flow of Incompressible Fluid between Two Parallel Porous Plates

CHAPTER VI

Laminar Flow of Incompressible Fluid over a Suddenly Accelerated Flat Plate

CHAPTER VII

Part: A

Temperature Distribution of Laminar Flow of Incompressible Fluid

CHAPTER VII

Part: B

Unsteady MHD Flow Past an Infinite Vertical Plate

CHAPTER VIII

Steady Laminar Flow of Incompressible Fluid Through a Circular Tube

CHAPTER IX

NOMENCLATURE

$\overline{F}$	: Body force per unit volume
$\overline{V}$	: Fluid velocity
Re	: Reynolds number
τ	: Shear stress
$\overline{B}$	: Magnetic field
$\overline{E}$	: Electric field
$\overline{J}$	: Current density
ε	: Electrical permittivity of the medium
ρ_e	: Charge density
$\overline{D}$	: Electric displacement
c_p	: Specific heat at constant pressure
μ	: Coefficient of viscosity
k	: Thermal conductivity of a fluid
α	: Thermal diffusivity
ν	: Kinematic coefficient of viscosity.
U_∞	: Some reference velocity
h_L	: Local heat transfer coefficient
Pr	: Prandtl number

Nu	: Nusselt number
Gr	: Grashof number
E_c	: Eckert number
δ	: Boundary layer thickness
M	: Hartmann number
λ	: Constant
B_O	: Uniform magnetic field
C_f	: Skin friction
g	: Acceleration due to gravity
ΔT	: Temperature difference
p	: Fluid pressure
t	: Time
ρ	: Density of the fluid
μ_e	: Magnetic permeability
H_0	: Magnetic field intercity
u	: Velocity of the fluid in x direction
υ	: Velocity of the fluid in y direction
T_∞	: Temperature of the fluid outside the boundary layer
σ	: Electrical conductivity of the fluid
φ	: The dissipation function

CHAPTER I

INTRODUCTION

Fluid dynamics is the science treating the study of fluid in motion. By the term fluid is meant a substance that flows. It is thus more convenient to treat the fluid as having continuous structure so that at each point we can prescribe a unique velocity, a unique pressure, a unique density, etc. Fluid may be classified as Newtonian, non-Newtonian; incompressible, compressible; viscous, non-viscous etc.

Suppose two fluid particles, having at different velocities, have a common boundary. Then across the boundary there will be interchange of momentum. The normal transport of molecules across the boundary will lead to a direct or normal force. In the case of viscous fluid there is a friction between the particles: this will manifest itself in the form of equal and opposite tangential or shearing force on each particle at the common boundary.

A fluid is a substance that deforms continuously when subject to a shear stress, no matter how small that shear stress may be. A shear force is the force component tangent to the surface. In laminar flow shear stress is caused by internal friction. Shear stress at a point is the limiting value of shear force to area as the area reduces to a point.

Flow may be classified in many ways such as laminar, turbulent; steady, unsteady; uniform, non-uniform; rotational, irrotational etc. In laminar flow the fluid moves in a layer, each fluid particle follows a smooth and continuous path. The fluid particles in each layer remain in an orderly sequence without crossing one another even when they turn a corner or pass an obstacle. There is no transfer of masses between adjacent layers. The momentum transfer between adjacent layers is molecular.

The essential problems of hydrodynamic theory were enlarged by the interest in the hydrodynamics flow of electrically conducting fluids in presence of magnetic field. Although vast problems in the hydrodynamic and hydro-magnetic flows are now available, yet many problems in the hydrodynamic and hydro-magnetic flows are to be analyzed. Navier in 1827 established the equation of viscous fluid motion taking into consideration the effects of intermolecular forces. After some times G.G. Stokes in 1845 derived the same equations based on the assumption that the normal and shearing stresses are linear functions of the rate of deformation, in relation with Newton law of friction. This equations are historically Navier-Stokes equations.

In this doctoral thesis, two types of parallel flow of Newtonian fluid have been considered; namely (i) incompressible isothermal flow between two infinite flat plates or over a flat plate, (ii) incompressible isothermal flow through a circular duct. The thesis is mainly devoted to solve the Navier-Stokes equations for laminar flow problems arising from the incompressible viscous Newtonian fluid. Some problems have been solved in absence of magnetic field and some problems have been solved in presence of magnetic field. One of the most important contributions in this research work is that earlier investigations have been extended to the case of incompressible viscous Newtonian fluid. The steady and unsteady flow of viscous incompressible fluid between two parallel plates have been presented in the standard books of Bachelor [6], Chorlton [10], Ferraro and Plumpton [18], Lamb [38], Landau and Lifshitz

[39], Milne Thomson [45], Pai [48], and others. In this thesis some problems of laminar flow of conducting fluid in presence of magnetic field have been considered. We have also investigated the case when the effect of magnetic field is small or large.

In velocity boundary layer, when a fluid flows with a free stream velocity U_∞ past a flat plate it is assumed that the velocity of fluid adjacent to the surface is zero. At increasing distances perpendicular to the plate, the stream velocity approaches the free stream velocity U_∞ asymptotically. In this case the whole flow region is divided into two parts; one in which velocity varies from zero to U_∞ the other region, called the free stream, lies outside the boundary layer. It is assured that the velocity of fluid outside the boundary layer will be U_∞ everywhere. The process for solving boundary layer equation has been described analytically in many standard books of Bansal [7], Rogers & Mayhew [58], Schlichting [62]. Keller [29,32] solved boundary layer problems numerically using some suitable transformations of variables. In solving boundary layer equations we have considered Box scheme method and shooting method. We have transferred the Blasius equation to equivalent system of linear equations and solved them by fourth order Runge-Kutta method.

In two dimensional parallel flow, as the velocity profile does not change in the direction of motion x, the shear stress can only be a function of y (i.e. perpendicular direction of the flow). We also note that the pressure varies hydrostatically in the y direction and can be a function additionally of x and t; t being time. For steady flow if we delete hydrostatic increase of pressure in y direction then the pressure gradient is a function of x only.

Sengupta, Bazlur & Kander [63] considered a unsteady flow problem between two parallel plates. They solved the problem by Laplace transformation. We have solved the problem by the method of variation of parameters considering suitable functions for pressure gradient force.

Gailitis & Lielausis [20] introduced the idea of Lorentz force to control the flow of an electrically conducting fluid over a flat plate. The problem for a non conducting fluid was considered by Sinha & Chaudhury [69] for periodic moment of the plate. Sengupta & Kumar [64] considered MHD flow problems of a viscous incompressible fluid near a moving porous flat plate. Poria, Mamaloukas , Layek & Mazumdar [52] solved some problems of transverse magnetic field on the flow of a viscous conducting fluid produced by an oscillating plane wall. Recently Pantokratoras [50] described the effect of parallel magnetic field to the flow of conducting fluid. We have extended some MHD flow problems for transverse and parallel magnetic field. In recent years a number of papers have been published on incompressible viscous fluid through porous medium. These types of problems are usually known as the problem of transpiration cooling. Transpiration cooling is a very effective process in reducing heat transfer between the fluid and the boundary. Two problems related to porous medium have been discussed in the thesis. Panton [51], Von, Kerezek & Davis [74], Erdogan [17] derived the solutions of some problems related to a oscillating plate. Taking inverse Laplace transformation and using complex inversion formula we have solved the laminar flow problem analytically. We have also solved the same problem numerically with the help of Crank Nicholson Implicit Scheme.

Various problems related to thermal boundary layer over a flat plate have been presented in many standard books and papers Bansal [7], Hossain [24], Milne [45], Rogers [58], Schlichting [62] and Sparrow & Eichhorn[70]. We have discussed the various properties of energy equation and finally established a few relationships between velocity and temperature of the fluid.

Laminar flow through a circular tube is a common phenomenon of fluid dynamics. Kapur [28] described about flow of Newtonian fluid through human blood vessels. Das and Sengupta [12] discussed the unsteady flow of conducting viscous fluid through a straight tube. Mazumdar, Ganguly & Venkatesan [43] described the effects of magnetic field on the flow of Newtonian fluid through a circular tube. We have considered the laminar flow of incompressible fluid through a uniform circular tube in presence of magnetic field. For steady case the pressure drop across the length of obstacle has been discussed. For steady case the flow of Newtonian fluid through circular tube has been investigated. Here we have transferred the equation of flow from Cartesian coordinate to cylindrical polar coordinates and solved them by Bessel function.

CHAPTER II

Available Information on Laminar Flow of Incompressible Viscous Fluid

2.1 Laminar and turbulent flows

A flow, in which each fluid particle traces out a definite curve and the curves traced out by any two different fluid particles do not intersect, is said to be laminar. On the other hand, a flow, in which each fluid particle does not trace out a definite curve and the curves traced out by the fluid particles intersect, is said to be turbulent. At low velocities, a fluid flow may be laminar but at high velocities the tendency of fluid particles is to mix and to become turbulent. Highly viscous fluids tend to flow laminar because the shear forces due to viscosity tend to oppose motion and inhibit free mixing whereas low viscosity fluids tend to flow turbulent. It is observes that low-density fluids are more likely to flow laminar than are denser fluids. This is because the exchange of momentum and hence the effectiveness of mixing is less for low-density fluids than for denser fluids.

2.2 Newtonian and non-Newtonian fluids

Fluid may be classified as Newtonian and Non-Newtonian. In Newtonian fluid there is a linear relation between the magnitude of applied shear stress and the resulting rate of deformation. Most common fluids fall into this category. In Non-Newtonian fluid there is a nonlinear relation between the magnitude of applied shear stress and the rate of deformation. Gases, water, thin liquids etc. are Newtonian fluid, while thick long chained hydrocarbons, colloidal solutions, clay etc. are Non-Newtonian fluid. A short description of Newtonian and Non-Newtonian of fluids is listed in the following table:

Newtonian fluid	Non-Newtonian fluid
$\tau = \mu \frac{du}{dy}$	i) Plastic fluid $\tau = A + B\ (\frac{du}{dy})^n$ where A , B and n are constants. If n=1 the material is known as a Bingham Plastic. ii) Pseudoplastic fluid $\tau = \mu (\frac{du}{dy})^n, \quad n < 1$ iii) Dilatant fluid $\tau = \mu (\frac{du}{dy})^n, n > 1$ iv) Viscoelastic fluid $\tau = \mu \frac{du}{dy} + E$ where E is the modulus of elasticity. v) Thirotropic fluid $\tau = \mu (\frac{du}{dy})^n + f(t)$ where $f(t)$ is a decreasing function of time (the dynamical viscosity decreases with the time for which shearing forces are applied). vi) Rheopectic fluid $\tau = \mu (\frac{du}{dy})^n + f(t)$ where f (t) is a increasing function of time (the dynamical viscosity increase with the time for which shears forces are applied.)

A Newtonian fluid having velocity u, the rate of deformation $\frac{du}{dy}$ exhibits shear stress obtained by the following relation

$$\tau = \mu \frac{du}{dy}$$

where μ is the proportionality constant and is called the coefficient of viscosity.

2.3 Magneto Hydrodynamics

Magneto Hydrodynamics is that branch of continuum mechanics, which deals with the flow of electrically conducting fluids in electric and magnetic fields. The motion of the conduction fluid across the magnetic field generates electric currents, which change the magnetic field, and the action of the magnetic field on these currents gives rise to mechanical forces, which modify the flow of the fluid. Thus there is a two-way interaction between the flow field and the magnetic field; the magnetic field exerts force on the fluid by producing induced currents, and the induced currents change the original magnetic field.

The study of hydromantic goes back to Faraday who predicted induced currents in the ocean due to the earth's magnetic field. Faraday (1832) carried out experiments with the flow of mercury in glass placed between poles of a magnet, and discovered that a voltage was induced across the tube due to the motion of the mercury across the magnetic field, perpendicular to the direction of flow and to the magnetic field. He observed that the current generated by this induced voltage interacted with the magnetic field to slow down the motion of the fluid, and this current produced its own magnetic field that obeyed Ampere's right hand rule and thus, in turn distorted the magnetic field.

In the resent time, Hartmann (1938) was first to discuss both experimentally and theoretically the magneto-hydrodynamics flow between two parallel plates. But the real boost was given by Alfren in 1942 when he established transverse waves in electrically conducting fluids and explained many astrophysical phenomena with it. Since then the literature on magneto-hydrodynamics has increased many fold.

2.4 Electromagnetic equations

Since in MHD we are mainly concerned with conducting fluids in motion, it is necessary to consider first the electrodynamics of moving media. Magneto-hydrodynamics equations are the ordinary electromagnetic and hydro-magnetic equations modified to take account of the interaction between the motion of the fluid and electromagnetic field; formulation of electromagnetic theory in mathematical form is known as Maxwell's equations. The basic laws of electromagnetic theory are all contained in special theory of relativity. But here we will always assume that all velocities are small in comparison with the speed of light. The well-known electromagnetic equations are as follows:

(a) *Charge Continuity*:

$$\nabla \cdot \overline{D} = \rho_c$$

(b) Current continuity:

$$\nabla \cdot \bar{J} = -\frac{\partial \rho_e}{\partial t}$$

(c) Magnetic Field continuity:

$$\nabla \cdot \overline{B} = 0$$

(d) Ampere's Law:

$$\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t}$$

(e) Faraday's Law:

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

(f) Constitutive Equations for $\overline{D}$ and $\overline{B}$:

$$\overline{D} = \varepsilon \overline{E}$$

$$\overline{B} = \mu_e \overline{H}$$

(g) Lorentz force on a charge:

$$\overline{F} = q(\overline{E} + \overline{V} \times \overline{B})$$

(h) Current density:

$$\overline{J} = \sigma(\overline{E} + \overline{V} \times \overline{B})$$

Here $\overline{D}$ is the electric displacement, ρ_e the charge density, $\overline{E}$ the electric field, $\overline{H}$ the magnetic field, $\overline{J}$ the current density, ε the electrical permittivity of the medium, μ_e the magnetic permeability of the medium, $\overline{V}$ the fluid velocity.

2.5 Fundamental equations of fluid dynamics of incompressible viscous fluid

In the study of fluid flow, one determines the velocity distribution as well as the states of the fluid over the whole space for all time. For incompressible viscous fluid, the velocity where $\frac{\partial Q}{\partial t}$ is the rated of heat produced per unit volume by the external agencies, c_p the specific heat at constant pressure, k the thermal conductivity of the fluid, φ the dissipation function, T the temperature of the fluid, $\overline{V}$ the fluid velocity.

2.6 The important non-dimensional parameters

Reynolds Number Re

For laminar flow of incompressible viscous Newtonian fluid having velocity u, density ρ and kinematics coefficient of viscosity ν the motion is governed by the well known Navier-Stokes' equations

$$\frac{\partial \overline{u}}{\partial t}+(\overline{u}.\nabla)\overline{u} =-\frac{1}{\rho}\nabla p + \nu \nabla^2 \overline{u}$$

and the continuity equation

$$\nabla.\overline{u} = 0.$$

If we denote the characteristic velocity by the amounts of order U over characteristic distances of order L, then the velocity components, such as $\frac{\partial u}{\partial x}$ will typically be of order $\frac{U}{L}$. So the second derivative such as $\frac{\partial^2 u}{\partial x^2}$ will be of order $\frac{U}{L^2}$. In this way we obtain the following order of magnitude estimates for two of the terms in Navier-Stokes' equations:

Inertia term: $|(\overline{u}.\nabla)\overline{u}| = 0(U^2/L)$

Viscous term: $|\nu \nabla^2\overline{u}| = 0(\nu U/L^2)$.

Hence

$$\frac{|inertia\ term|}{|Viscous\ term|} = 0(\frac{U^2 / L}{\nu U / L^2})$$

$$= 0(\frac{UL}{\nu}).$$

The quantity $\frac{UL}{\nu}$ is known as Reynolds number and is denoted by

$$\mathrm{Re} = \frac{UL}{\nu}.$$

The British scientist Osborne Reynold, 1883, introduced this number while discussing boundary layer theory.

Hartmann Number M

When the magnetic field is applied to the flow field then another non-dimensional number, called Hartmann number, plays a very important role. It is the ratio of magnetic force to viscous force and is denoted by

$$M = \mu_e H_0 L \sqrt{\sigma / \mu}$$

where μ_e is the magnetic permeability, σ the electrical conductivity, μ the dynamic viscosity of fluid, H_0 the is magnetic field intensity, L the characteristic length. Hartman number is the most important dimensionless number in magneto-hydrodynamics. The hydro-magnetic effects are important when the Hartmann number is significant.

Prandtl Number Pr

The Prandtl number is a dimensionless number approximating the ratio of momentum diffusivity(kinematic viscosity) and thermal diffusivity. It is named after the German physicist Ludwig Prandtl and is defined by

$$\Pr = \frac{\nu}{\alpha} = \frac{c_p \mu}{k}$$

where c_p = specific heat at constant pressure

μ = coefficient of viscosity

k = thermal conductivity of a fluid

α = thermal diffusivity, $\alpha = \dfrac{k}{\rho c_p}$

$\nu = \dfrac{\mu}{\rho}$ = kinematic coefficient of viscosity.

The Prandtl number may be written as follows

$$\Pr = \frac{\nu}{\dfrac{k}{\rho c_p}}.$$

The value of ν shows the effect of viscosity of the fluid. The smaller the value of ν is, the narrower the region which is affected by viscosity and which is known as the boundary layer region when ν is very small. The value of $\dfrac{k}{\rho c_p}$ shows the thermal diffusivity due to heat conduction. The Prandtl number shows the relative importance of heat conduction and viscosity of a fluid. Prandtl number depends on the properties of the fluid. For air $\Pr = 0.7$ approx. and for water (at $60^0 F$) $\Pr = 7$ approx., whereas for oil it is of the order of 1000 due to large values of μ. In heat transfer problem, the Prandtl number controls the relative thickness of the momentum and thermal boundary layers. When Prandtl number is small, it means that the heat diffuses very quickly compared to the velocity. This means that for liquid metals the thickness of the thermal boundary layer is much bigger than the velocity boundary layer.

Eckert Number E_c

The Eckert number can be interpreted as the addition of heat due to viscous dissipation and is very small for incompressible fluid and for low motion. It may be defined as follows:

$$E_c = \frac{U^2}{c_p \Delta T}$$

where $\Delta T =$ temperature difference between the wall and the fluid at a large distance from the wall

$U =$ some reference velocity

$c_p =$ specific heat at constant pressure

Nusselt Number Nu

The Nusselt Number is a dimensionless number named after Wilhelm Nusselt and is defined as

$$Nu = \frac{Lh_L}{k}$$

where h_L = local heat transfer coefficient at section L

L = characteristic length

k = thermal conductivity

Nusselt number is the measure of the rate of heat transfer by convection. It can be expressed as the function of two dimensionless groups, the Reynolds number that describes the flow, and the Prandtl number which is the property of the fluid. Thus

$$Nu = l p_r^{\frac{1}{3}} \mathrm{Re}^{\frac{1}{2}}$$

where l is a constant.

Grashof Number *Gr*

The Grashof number is a dimensionless number in fluid dynamics. It is named after the German engineer Franz Grashof. This number is defined as the ratio of product of inertia force and buoyancy force to the square of viscous force i.e.,

$$Gr = \frac{\rho \upsilon^2 \times \rho \beta g(\Delta T) L^3}{(\mu \upsilon)^2}$$

$$= \frac{\rho^2 \beta g(\Delta T) L^3}{\mu^2}$$

where υ = the velocity of the fluid caused due to buoyancy force

g = acceleration due to gravity

β = volumetric thermal expansion coefficient

ΔT = temperature difference

L = characteristic length

$\rho =$ fluid density

$\mu =$ viscosity of fluid

$\rho\beta g(\Delta T)L^3 =$ the buoyancy force for the total volume.

2.7 Boundary layer approximation

When a fluid flows past a solid surface, the velocity of the fluid at the solid surface must be same as that of the solid surface. If the solid surface is stationary, the velocity of the fluid at the surface must be zero. As a result there is a region closed to the surface through which the velocity increase from zero velocity at the solid surface to the velocity of the main stream. This region in the vicinity of the solid surface is generally a narrow region where the velocity gradients are large. The origin of the large velocity gradient is the viscous action and the large shear stresses in that region. This narrow region is known as boundary layer region. Boundary layer phenomenon occurs when the Reynolds number is large and flow is considered near the bodies. The boundary layers are then the velocity and the thermal or magnetic boundary layers; and each thickness is inversely proportional to the square root of the associated number. L. Prandtl made an important contribution to fluid dynamics in 1904 by introducing the concept of boundary layer. He classified the essential influence of viscosity in flows at high Reynolds number, that for large Reynolds number, the viscosity and thermal conductivity appreciably influenced the flow only near a wall. When distant measurements in the flow direction are compared with a characteristic dimension in that direction, transverse measurements compared with the boundary layer thickness, and velocities compared with the free stream velocity, the

Navier-Stokes and energy equations can be considerably simplified by neglecting small quantities. The number of component equations is reduced to those in the flow direction and pressure changes across the boundary layer are negligible. The pressure is then only a function of the flow direction and can be determined from the inviscid flow solution. Also the number of viscous terms is reduced to the dominant term and the heat conduction in the flow direction is negligible.

2.8 The MHD boundary layer equations for two-dimensional flow in the case of small magnetic Reynolds number

If the velocity distribution in a moving fluid depends on only two coordinates (x and y say) and the velocity is everywhere parallel to the x-y plane, the flow is said to be two dimensional. For simplicity, we derive boundary layer equation for the flow over a semi-infinite flat plate. We take rectangular cartesian coordinates (x, y) with x measured in the plate in the direction of the two dimensional laminar incompressible flow, and y measured normal to the plate and (u, υ) are the velocity components. Let the viscosity of the fluid be small. With constant fluid properties, transversely applied uniform magnetic field Ho, the MHD boundary layer equations for incompressible fluid flow under the boundary layer assumptions are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial x}=0$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial x}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma}{\rho}\mu_e{}^2 H_0{}^2 u$$

$$\frac{\partial p}{\partial y} = 0$$

$$\rho c_p \left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + \upsilon\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma\mu_e{}^2 H_0{}^2 u^2$$

where p = fluid pressure

t = time

ρ = density of the fluid

μ_e = magnetic permeability

σ = electrical conductivity

μ = dynamic viscosity of fluid

H_0 = magnetic field intercity

c_p = specific heat at constant pressure

k = thermal conductivity of a fluid

ν = kinematic coefficient of viscosity

T = temperature

CHAPTER III

Steady Laminar Flow of Incompressible Fluid Over a Flat Plate

3.1 Introduction

In many flow problems the partial differential equations governing the motion of the fluid are nonlinear. These nonlinear equations cannot be solved easily. A method is to obtain similarity solution by employing transformations that reduces the system of partial differential equation to a system of ordinary differential equations. L Prandtl made an important contribution to fluid dynamics in 1904 by introducing the concept of boundary layer. According to him if a slightly viscous fluid flows over a body in such a way that the Reynold number is very large, then there is a thin layer near the body where the viscous forces are important; and outside the layer the viscous forces are unimportant. There is a variety of numerical methods that are used to solve the boundary layer problems. Two particular methods, the Crank Nicolson Scheme and the Box scheme, seem to dominate in most practical applications. Of them Box scheme method is easy to adapt to new classes of problems. The Box scheme was derived by Keller [29] for solving diffusion problems. The Box scheme to a variety of boundary layer flow problems was given by Keller [29,30,31]. Here we have engaged in numerical simulation of fluid dynamics using computer. Here we have modified the Blasius equation into the equivalent first order system of equations. The shooting method is used to convert the boundary value problem into equivalent initial value problem. Finally Runge-Kutta method is used to find out the solution of the problem.

3.2 Mathematical formulation

For steady laminar flow of incompressible viscous fluid the basic equation of mass conservation and momentum are :

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{3.1}$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \rho}{\partial x_i} + \nu\nabla^2 u_i . \tag{3.2}$$

Suppose the fluid flows in the x direction only and varies perpendicularly to the x axis(i.e. y direction). According to the concepts of Prandtl, we obtain the following equations for two dimensional boundary layer flow:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial \rho}{\partial x} + \nu\frac{\partial^2 u}{\partial x^2} \tag{3.3}$$

$$\frac{\partial p}{\partial y} = 0 \tag{3.4}$$

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0 \tag{3.5}$$

with boundary conditions

$$u = 0, \ \upsilon = 0 \ \text{ when } \ y = 0$$

$$u = U_\infty(x,t) \quad when \quad y \to \infty \ . \tag{3.6}$$

3.3 Tri-diagonal system

An efficient technique sometimes called Thomas algorithm can be used to solve a linear system with a tri-diagonal matrix defined by the following equations

$$b_1u_1 + c_1u_2 = d_1$$

$$a_2u_1 + b_2u_2 + c_2u_3 = d_2$$

$$a_3u_2 + b_3u_3 + c_3u_4 = d_3 \tag{3.7}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$a_nu_{n-1} + b_nu_n = d_n$$

where u_i are the unknowns and a_i, b_i, c_i, d_i are known.

It is possible to calculate the unknown u_i using the following recurrence formula

$$u_i = \beta_i - \frac{c_i\, u_{i+1}}{\alpha_i}, \quad i = n-1,\, n-2,\, \ldots\ldots\ldots\ldots\ldots\ldots 1 \tag{3.8}$$

where $u_n = \beta_n$.

Setting $\alpha_1 = b_1$ and calculating, we have

$$\alpha_i = b_i - \frac{a_1\; c_{i-1}}{\alpha_{i-1}}, \quad i = 2,3,,\ldots\ldots\ldots\ldots\ldots n \; . \tag{3.9}$$

Again setting $\beta_1 = \dfrac{d_1}{b_1}$ and calculating, we have

$$\beta_i = \frac{d_i - a_i\; \beta_{i-1}}{\alpha_i}, \quad i = 2,\, 3, \ldots\ldots\ldots\ldots\ldots\ldots , n \; . \tag{3.10}$$

Finally the recurrence formula (3.8) is used to calculate all u_i successively for

$i = n-1, n-2, \ldots\ldots 1$.

3.4 Box scheme

Let τ be the shear stress of the flowing fluid whose viscosity is μ .For Newtonian fluid, we

have

$$\tau = \mu \frac{\partial u}{\partial y}. \tag{3.11}$$

Eq.(3.3),Eq.(3.4),Eq.(3.5) and Eq.(3.11) yield the following equations:

$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \frac{1}{\rho}\frac{\partial \tau}{\partial y} \tag{3.12}$$

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0. \tag{3.13}$$

Boundary conditions:

$$\begin{aligned} &u\,(x,\ 0) = 0 \\ &\upsilon(x,\ 0) = 0 \\ &u[x,\ y_\delta(x)] = U_\infty \end{aligned} \tag{3.14}$$

where U_∞ represents the free stream velocity , δ the boundary layer thickness. We impose the following additional condition

$$\partial u[x, y_\delta(x)]/\partial y = 0. \tag{3.15}$$

Outside the boundary layer, we have

$$U_\infty \frac{dU_\infty}{dx} = -\frac{1}{\rho}\frac{dp}{dx}. \tag{3.16}$$

We now introduce the following transformations:

$$\eta(x,y) = y[U_\infty(x)/|\nu_0 x]^{1/2} \tag{3.17.a}$$

$$f(x,\eta) = \psi(x,y)|[U_\infty(x)\nu_0 x]^{1/2} \tag{3.17.b}$$

$$P(x) = \frac{x}{U_\infty(x)}\frac{dU_\infty(x)}{dx} \tag{3.17.c}$$

$$b(x,\eta) = \frac{\nu(x,y)}{\nu_o}. \tag{3.17.d}$$

Here ν_o is some reference kinematics coefficient of viscosity and ψ is the stream function for which

$$u(x,y) = \frac{\partial \psi}{\partial y}$$

$$\upsilon(x,y) = -\frac{\partial \psi}{\partial x}.$$

Using Eq.(3.16) and Eq.(3.17) in Eq.(3.12), we get

$$\frac{\partial}{\partial \eta}(b\frac{\partial^2 f}{\partial \eta^2}) + \frac{1}{2}(P+1)\frac{\partial^2 f}{\partial \eta^2} + P\left[1-(\frac{\partial f}{\partial \eta})^2\right] = x\left[\frac{\partial f}{\partial \eta}\frac{\partial^{2f}}{\partial x \partial \eta} - \frac{\partial^2 f}{\partial \eta^2}\frac{\partial f}{\partial x}\right] \tag{3.18}$$

Eq.(3.18) is equivalent to the following first order system of equations:

$$\frac{\partial f}{\partial \eta} = U \tag{3.19.a}$$

$$\frac{\partial U}{\partial \eta} = V \tag{3.19.b}$$

$$\frac{\partial}{\partial \eta}(bV) = x\left[U\frac{\partial U}{\partial x} - V\frac{\partial f}{\partial x}\right] - \frac{P+1}{2}fV - P\left[1-U^2\right]. \tag{3.19.c}$$

The boundary conditions (3.14) becomes

$$\begin{aligned} f(x,0) &= 0 \\ U(x,0) &= 0 \\ U(x, \eta_\delta(x) &= 1. \end{aligned} \tag{3.20}$$

Let the (x, y) plane be divided into a network of rectangles of sides k_n ($n = 1,2,\ldots\ldots$ N) and h_j ($j = 1,2,3,\ldots\ldots.J$) by drawing the sets lines

$$x_0 = 0, \quad x_n = x_{n-1} + k_n, \quad 1 \le n \le N;$$

$$y_0 = 0 \quad y_j = y_{j-1} + h_j \qquad 1 \le j \le J, y_J = y_\delta . \tag{3.21}$$

With the help of Eq. (3.21) we draw the following nets, some points of intersection of these families of times are shown by following circles indicated in Fig-3.1 .

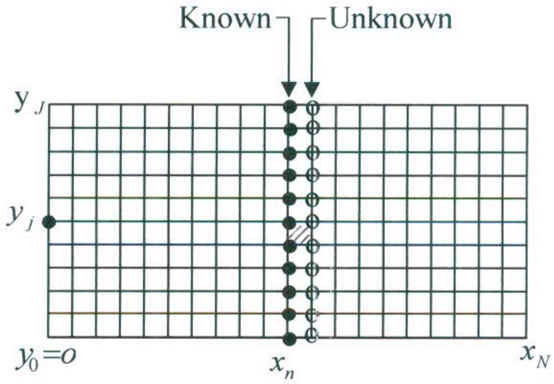


Figure –3. 1

Now we draw the above indicated box as:

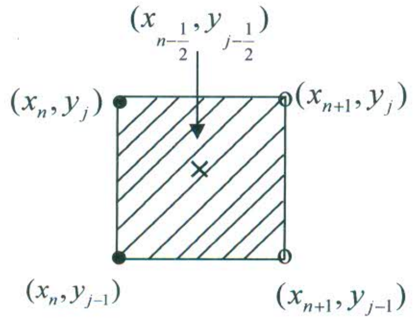


Figure –3.2

Since the boundary layer equations have been formulated as a first order system, all the derivatives can be approximated by the backward difference approximation for first derivatives and two point averages for dependent variables. Thus any net quantity w may be expressed by the following notations.

$$[w]^n_{j-\frac{1}{2}} = \frac{1}{2}(w^n_j + w^n_{j-1}) \tag{3.22.a}$$

$$\left[\frac{\partial w}{\partial y}\right]^n_{j-\frac{1}{2}} = \frac{1}{h_j}((w^n_j - w^n_{j-1}) + o(h_j) \tag{3.22.b}$$

$$\left[\frac{\partial w}{\partial x}\right]^{n-\frac{1}{2}}_{j-\frac{1}{2}} = \frac{1}{k_n}([w]^n_{j-\frac{1}{2}} - [w]^{n-1}_{j-\frac{1}{2}}) + o(k_{n)} \tag{3.22.c}$$

$$\left[\frac{\partial w}{\partial y}\right]^{n-\frac{1}{2}}_{j-\frac{1}{2}} = \frac{1}{2}(\left[\frac{\partial w}{\partial y}\right]^n_{j-\frac{1}{2}} + \left[\frac{\partial w}{\partial y}\right]^{n-1}_{j-\frac{1}{2}}) \tag{3.22.d}$$

$$[w]^{n-\frac{1}{2}}_{j-\frac{1}{2}} = \frac{1}{2}([w]^n_{j-\frac{1}{2}} + [w]^{n-1}_{j-\frac{1}{2}}) \tag{3.22.e}$$

It may be mentioned that $\left[\frac{\partial w}{\partial y}\right]^{n-\frac{1}{2}}_{j-\frac{1}{2}}$ is the best difference approximation to $\partial w(x_{n-\frac{1}{2}}, y_{j-\frac{1}{2}})/\partial y$

.If we omit local truncation error and apply the Box scheme in Eq.(3.19),we get

$$\left[\frac{\partial f}{\partial \eta}\right]^n_{j-\frac{1}{2}} = [U]^n_{j-\frac{1}{2}} \tag{3.23.a}$$

$$\left[\frac{\partial U}{\partial \eta}\right]^n_{j-\frac{1}{2}} = [V]^n_{j-\frac{1}{2}} \tag{3.23.b}$$

$$\left[\frac{\partial(bV)}{\partial\eta}\right]_{j-\frac{1}{2}}^{n-\frac{1}{2}} = x_{n-\frac{1}{2}}\left\{[U]_{j-\frac{1}{2}}^{n-\frac{1}{2}}\right\}\left[\frac{\partial U}{\partial x}\right]_{j-\frac{1}{2}}^{n-\frac{1}{2}}$$

$$-[V]_{j-\frac{1}{2}}^{n-\frac{1}{2}}\left[\frac{\partial f}{\partial x}\right]_{j-\frac{1}{2}}^{n-\frac{1}{2}} - \left[\frac{p+1}{2}fV - P(1-U^2)\right]_{j-\frac{1}{2}}^{n-\frac{1}{2}} \qquad (3.23.c)$$

For example we use the following identity

$$[V\tau/\nu]_{j-\frac{1}{2}}^{n-\frac{1}{2}} \text{ or } [V\tau]_{j-\frac{1}{2}}^{n-\frac{1}{2}}[\nu]_{j-\frac{1}{2}}^{n-\frac{1}{2}} \text{ or } [V]_{j-\frac{1}{2}}^{n-1}\ [\tau]_{j-\frac{1}{2}}^{n-\frac{1}{2}}\ [V]_{j-\frac{1}{2}}^{n-\frac{1}{2}}$$

There are numerous other ways in which the last term in Eq.(3.23.c) could have been written while retaining the proper centering. Keller [29] solved the Eq.(3.23.c) by using Newton's method. The resulting system of linear algebraic equations has tri-diagonal from but with different elements in the vector and matrices. Using the procedure for solving tri-diagonal system the resulting equation or equivalent system can be easily solved by the Box scheme.

3.5 Shooting method

Let us consider a thin infinite flat plate submerged in steady two dimensional flow whose undisturbed velocity is U_∞. Again suppose that the fluid is incompressible with low viscosity. We introduce the stream function ψ, such that

$$u = \frac{\partial\psi}{\partial y}$$

$$\upsilon = -\frac{\partial\psi}{\partial x}.$$

We introduce the following new variables:

$$\eta = y\sqrt{\frac{U_\infty}{\nu x}} \qquad (3.24.a)$$

$$f(\eta)=\frac{\psi(x,y)}{\sqrt{\nu x U_{\infty}}}. \tag{3.24.b}$$

Using Eq.(3.24) in Eq.(3.3) we get

$$2f''' + ff'' = 0. \tag{3.25}$$

The Eq.(3.25) is the well-known as Blasius equation.

We consider the following boundary conditions:

$$f(0)=0, \quad f'(0)=0, \quad f'(\infty)=1. \tag{3.26}$$

Analytic solution:

Blasius obtained the solution of Eq.(3.25) with the help of Eq.(3.26) in the form of power series expansion about $\eta=0$ assuming the following form:

$$f(\eta)=A_0+A_1\eta+\frac{A_2}{2!}\eta^2+\frac{A^3}{3!}\eta^3+\cdots\cdots\cdots\cdots\cdots\cdots. \tag{3.27}$$

The boundary conditions $f\ (0)=0$ *and* $f'\ (0)=0$, reduces Eq.(3.27) to

$$f(\eta)=A_2^{1/3}\,F\,(A_2^{1/3}\,\eta). \tag{3.28}$$

The third boundary condition $f'(\infty)=1$ yields

$$A_2=\left[\frac{1}{\lim_{\eta\to\alpha} F'(\eta)}\right]^{3/2}. \tag{3.29}$$

The value of A_2 can be obtained numerically from Eq.(3.29). Howarth found that A_2=0.332.

Numerical Solution :

Eq.(3.25) is equivalent to the following system of equations:

$$\frac{df}{d\eta}=u \tag{3.30.a}$$

$$\frac{du}{d\eta}=\upsilon \tag{3.30.b}$$

$$\frac{d\upsilon}{d\eta}=-\frac{1}{2}f\upsilon\ . \tag{3.30.c}$$

The boundary conditions Eq.(3.26) becomes

$$\begin{aligned} &f(0)=0 \\ &u(0)=0 \\ &u(\infty)=1. \end{aligned} \tag{3.31}$$

It we find out $\upsilon(0)$ with the help of $u(\infty)=1$ then Eq.(3.30) is said to be a initial value problem . To convert Eq. (3.30) as initial value problem, we may use shooting method. Let $\upsilon(0)=M_1$. Then the above system can be solved for f, u and υ using any initial value method until the solution at $\eta=\infty$ is reached. Let $u(\infty)=B_1$. If $B_1=1$, then we have obtained the required solution. In practices, it is very unlikely that our initial guess $\upsilon(0)=M_1$ is correct. If $B_1\neq 1$, then we obtain the solution with another guess, say $\upsilon(0)=M_2$. Let $u(\infty)=B_2$. If B_2 is not equal to 1 then we can calculate the third approximate value $\bar{\upsilon}(0)=M_3(say)$ by the following formula:

$$\bar{\upsilon}(0)=M_2-\frac{B_2-1}{B_2-B_1}\times(M_2-M_1). \tag{3.32}$$

We make the same kind of calculation as above by using $\bar{\upsilon}(0)$ and take the better of the two initial values $\upsilon(0)$. In this way we can find another improved value of $\upsilon(0)$. This process may be continued. The process is carried out until the change of $\upsilon(0)$ at successive computations is less than some small prescribed value ε, i, e,

$$\frac{[\upsilon(0)]_{k+1} - [\upsilon(0)]_k}{[\upsilon(0)]_k} < \varepsilon . \tag{3.33}$$

After getting the approximate value of $\upsilon(0)$ we may represent this initial value problem by following form:

$$\frac{d\bar{u}}{d\eta} = \bar{f}(\eta, f, u, \upsilon) \tag{3.34.a}$$

$$\bar{u}(0) = \bar{\eta} . \tag{3.34.b}$$

The initial value problem may be solved by fourth order Runge-Kutta method as

$$\bar{u}_{j+1} = \bar{u}_j + \frac{1}{6}(\bar{k}_1 + 2\bar{k}_2 + 2\bar{k}_3 + \bar{k}_4) \tag{3.35}$$

where

$$\bar{k}_1 = \begin{bmatrix} k_{11} \\ k_{21} \\ k_{31} \end{bmatrix} \qquad \bar{k}_2 = \begin{bmatrix} k_{12} \\ k_{22} \\ k_{32} \end{bmatrix}$$

$$\bar{k}_3 = \begin{bmatrix} k_{13} \\ k_{23} \\ k_{33} \end{bmatrix} \qquad \bar{k}_4 = \begin{bmatrix} k_{14} \\ k_{24} \\ k_{34} \end{bmatrix}$$

and

$$k_{i1} = hf_i(\eta_j, u_{1j}, u_{2j}, u_{3j}) \tag{3.36.a}$$

$$k_{i2} = hf_i(\eta_j + \frac{h}{2}, u_{1j} + \frac{1}{2}k_{11}, u_{2j} + \frac{1}{2}k_{21}, u_{3j} + \frac{1}{2}k_{31}) \tag{3.36.b}$$

$$k_{i3} = hf_i\,(\eta_j + \frac{h}{2},\, u_{1j} + \frac{1}{2}k_{12},\, u_{2j} + \frac{1}{2}k_{22}, u_{3j} + \frac{1}{2}k_{32}) \tag{3.36.c}$$

$$k_{i4} = hf_i\,(\eta_j + h,\, u_{1j} + k_{13,}\, u_{2j} + k_{23}, u_{3j} + k_{33}) \tag{3.36.d}$$

$i = 1(1)3$.

3.6 Example

$$\frac{df}{d\eta} = u \tag{3.37.a}$$

$$\frac{du}{d\eta} = \upsilon \tag{3.37.b}$$

$$\frac{dv}{d\eta} = -\frac{1}{2}f\upsilon \tag{3.37.c}$$

$$f(0) = 0, \quad u(0) = 0, \quad \upsilon(2.1) = 1, \quad h = .1. \tag{3.38}$$

By shooting method we have

v (0)= .32.

Table-3.1

η	f	u	υ
0	0	0	.32
.2	.0064	.06399	.319932
.4	.02559	0.12794	.319454
.6	.046634	.172615	.31866
.8	.06145	.19812	.321893
1.0	.07833	.223945	.32539
1.2	0.21101	.36819	.32818
1.4	.41029	.53395	.368238
1.6	0.6874	.70244	.34322
1.8	1.0346	.84325	.28336
2.0	1.4630	.96043	.21105
2.1	1.7090	1	.17316

3.7 Results and discussion

Complete solution of Navier-Stokes equation is not possible and so the flow region is split up into two regions; the boundary layer region and the potential flow region. To establish the accuracy of the solutions of boundary layer problem the Keller-Box scheme and the shooting method are employed. Numerical computation for shooing method is carried out with the help of Runge-Kutta method. By the two methods new phenomena have been discovered via this route before experimentation. This can result in significant savings when one can replace the expensive and time-consuming experimentation that would otherwise be needed.

CHAPTER IV

Laminar Flow of Incompressible Fluid Between Two Parallel Plates

Part: A

4.1.1 Introduction:

The steady and unsteady flow of viscous incompressible fluid between two parallel plates with constant pressure gradient have been presented in the standard books of Bachelor [6], Chorlton [10], Milne Thomson [45], Pai [48]. In describing the unsteady flow of viscous fluid between two parallel plates Sengupta, Rahman & Kandar [63] have considered time varying pressure gradients. They used Laplace transformation to determine the solutions of the differential equations. In this chapter an attempt has been made to study the velocity profile of various types of unsteady two-dimensional flow of incompressible viscous fluid between two parallel plates. In describing time varying pressure gradient for poiseuille flow some suitable functions are considered here. A special case is considered in this chapter. Crank Nicholson method is used to determine the figures of the special case.

4.1.2 Mathematical formulation

Conservation of mass:

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0 \qquad (4.1.1)$$

Conservation of momentum:

x-direction

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \qquad (4.1.2)$$

y-direction

$$\frac{\partial \upsilon}{\partial t} + u\frac{\partial \upsilon}{\partial x} + \upsilon\frac{\partial \upsilon}{\partial y} = \frac{1}{\rho}\frac{\partial p}{\partial y} + \nu(\frac{\partial^2 \upsilon}{\partial x^2} + \frac{\partial^2 \upsilon}{\partial y^2}). \qquad (4.1.3)$$

Let x be the direction of the flow, y the direction perpendicular to the flow. Suppose there is no velocity component perpendicular to the direction of the flow. As a result the equation of conservation of mass reduces to $\frac{\partial u}{\partial x} = 0$, and this leads to $u = u(y)$. Then the Eq. (4.1.2) and Eq.(4.1.3) reduce to

$$0 = \frac{\partial p}{\partial y}. \qquad (4.1.4.a)$$

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} \qquad (4.1.4.b)$$

Let L, U_0, p denote the characteristic length , velocity , pressure and $x', y', u', \upsilon', p'$ be the dimensionless number such that

$$x' = \frac{x}{L}, u' = \frac{u}{U_0}, p' = \frac{p}{\rho U_0^2}, t' = \frac{U_0}{L}t.$$

Then the Eq.(4.1.4.a)and Eq. (4.1.4.b) reduces to

$$\frac{\partial u'}{\partial t} = -\frac{\partial p'}{\partial x'} + \frac{1}{\text{Re}}\frac{\partial^2 u'}{\partial y'^2} \qquad (4.1.5)$$

$$0 = \frac{\partial p'}{\partial y'} \qquad (4.1.6)$$

where Re represents the Reynolds number and

$$\text{Re} = \frac{U_0 h}{\nu}.$$

Dropping the superscripts, we have

$$\frac{\partial u}{\partial t} = - \frac{\partial p}{\partial x} + \frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial y^2} \tag{4.1.7}$$

$$0 = \frac{\partial p}{\partial y} . \tag{4.1.8}$$

4.1.3 Plane Poiseuille Flow

Let us consider the unsteady two dimensional flow of incompressible viscous fluid flowing between two fixed parallel plates $y = 0$ and $y = 1$. Eq.(4.1.8) shows that p does not depends on y. Hence p is a function of x and t.

Suppose

$$-\frac{\partial p}{\partial y} = F(t) . \tag{4.1.9}$$

Eq.(4.1.7) becomes

$$\frac{\partial u}{\partial t} = \mathrm{F(t)} + \frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial y^2} . \tag{4.1.10}$$

Boundary conditions:

$u = 0$ at $y = 0$ and $u = U(t)$ at $y = 1$ and initially when

$$t = 0, u(y,0) = -\frac{\mathrm{Re}}{2}F(0)y^2 + \{U(0) + \frac{\mathrm{Re}}{2}F(0)\}y . \tag{4.1.11}$$

Case I

We choose a nonlinear function of time as

$$F(t) = \alpha + \beta t + \gamma t^2 .$$

For the above function, we have

$$\frac{\partial u}{\partial t} = \alpha + \beta t + \gamma t^2 + \frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial y^2} . \tag{4.1.12}$$

General solution of Eq.(4.1.12) is

$$u = (k_1 \cos\sqrt{\mu}y + k_2 \sin\sqrt{\mu}y)e^{-\frac{\mu t}{R}} + \alpha t + \frac{\beta t^2}{2} + \frac{\gamma t^3}{6} \qquad (4.1.13)$$

where k_1 and k_2 are arbitrary constants.

Boundary conditions:

$u = 0$ at $y = 0$ and at $y = 1$.

Applying the boundary condition in (4.1.13), we have

$$u = (\alpha t + \frac{\beta t^2}{2} + \frac{\gamma t^3}{6})(1 + \frac{\sin y\sqrt{\mu}}{\sin\sqrt{\mu}} - \cos y\sqrt{\mu}) + \frac{\sin y\sqrt{\mu}}{\sin\sqrt{\mu}} U(0)e^{-\frac{\mu t}{\text{Re}}} \qquad (4.1.14)$$

Velocity profile in this case for Reynolds number 1,3 and 5 is given by the Fig-4.1.1

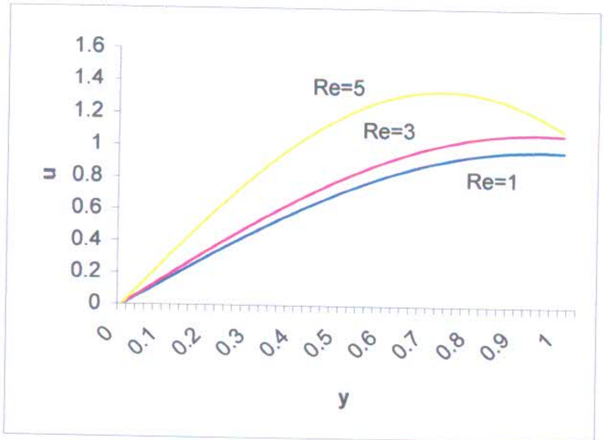


Figure-4.1.1

Case II

We choose a transient function of time as the following form

$F(t) = \alpha e^{-t\beta}$.

For the above function, we have

$$\frac{\partial u}{\partial t} = \alpha e^{-t\beta} + \frac{1}{\text{Re}} \frac{\partial^2 u}{\partial y^2}. \tag{4.1.15}$$

General solution of Eq.(4.1.15) is

$$u = (k_1 \cos\sqrt{\mu} y + k_2 \sin\sqrt{\mu} y) e^{-\frac{\mu t}{\text{Re}}} - \frac{\alpha}{\beta} e^{-t\beta} \tag{4.1.16}$$

where k_1 and k_2 are arbitrary constants.

Boundary conditions:

$u = 0$ at $y = 0$ and at $y = 1$.

Applying the boundary condition in Eq.(4.1.16), we have

$$u = \frac{\alpha}{\beta} \cos\sqrt{\mu} y e^{-t\beta} + [-\frac{\text{Re}}{2} \alpha y^2 + \{U(0) + \frac{\text{Re}\,\alpha}{2}\} y + \frac{\alpha}{\beta}(1 - \cos\sqrt{\mu} y)] e^{-\frac{\mu t}{\text{Re}}} - \frac{\alpha}{\beta} e^{-t\beta}. \tag{4.1.17}$$

Velocity profile in this case for Reynolds number 1,3,5 is given by the Fig-4.1.2.

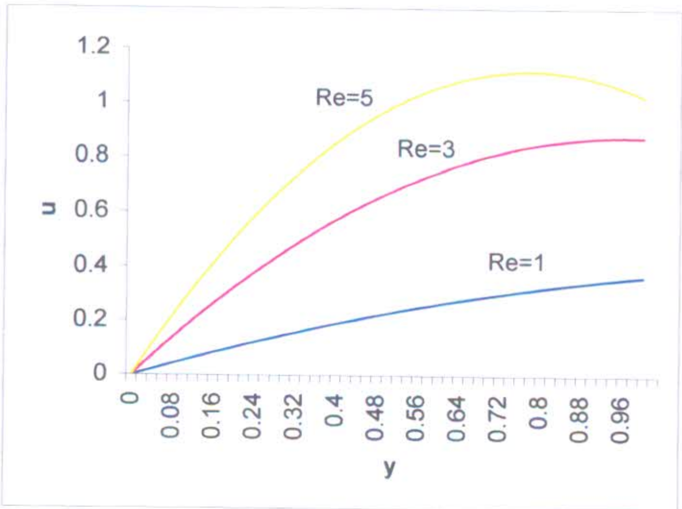


Figure-4.1.2

Case III

We choose a function for which the pressure gradient varies as a periodic function of time, i. e.

$$F(t) = \alpha \cos t\beta .$$

For the above function , we have

$$\frac{\partial u}{\partial t} = \alpha \cos t\beta + \frac{1}{\text{Re}} \frac{\partial^2 u}{\partial y^2} . \qquad (4.1.18)$$

General solution of the Eq.(4.1.18) is

$$u = (k_1 \cos\sqrt{\mu}y + k_2 \sin\sqrt{\mu}y)e^{-\frac{\mu t}{\text{Re}}} + \frac{\alpha}{\beta}\sin t\beta \qquad (4.1.19)$$

where k_1 and k_2 are arbitrary constants.

Applying the boundary condition in Eq.(4.1.19), we have

$$u = [-\frac{\text{Re}}{2}\alpha y^2 + \{U(0) + \frac{\text{Re}\,\alpha}{2}\}y]e^{-\frac{\mu t}{\text{Re}}} + \frac{\alpha}{\beta}\sin t\beta. \qquad (4.1.20)$$

Velocity profile in this case for Reynolds number 1,3,5 is given by the Fig-4.1.3.

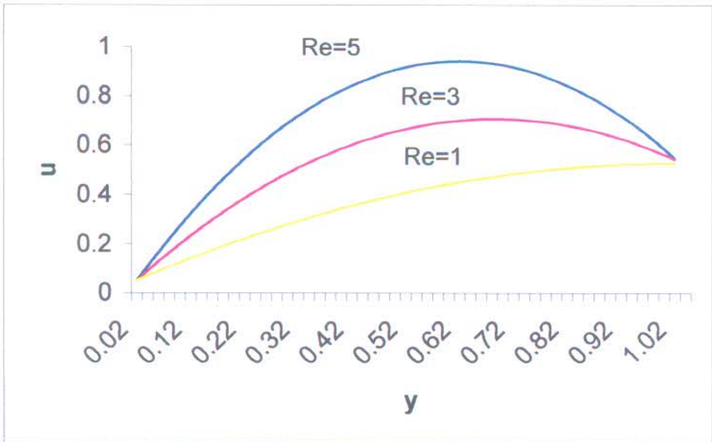


Figure-4.1.3

4.1.4 Plane Couette flow: $F(t) = 0$

Let us consider the unsteady two-dimensional flow of incompressible viscous fluid between two parallel plates. Suppose one plate y=0 is fixed and the other plate is moving with speed U. Again suppose the distance between the plates is one unit. In this case the differential equation for the flow will be

$$\frac{\partial u}{\partial t} = \frac{1}{\text{Re}} \frac{\partial^2 u}{\partial y^2} \ . \tag{4.1.21}$$

To solve the above equation we consider the following three cases:

Case I:

$$U(t) = u_0 + u_1 t$$

General solution of Eq.(4.1.21) is

$$u = (k_1 \cos\sqrt{\mu} y + k_2 \sin\sqrt{\mu} y) e^{-\frac{\mu t}{\text{Re}}} \tag{4.1.22}$$

where k_1 and k_2 are arbitrary constants.

Boundary condition:

$u = 0$ at $y = 0$ and $u = u_0 + u_1 t$ at $y = 1$ and $u(0) = u_0 y$.

Applying the above boundary condition in Eq.(4.1.22), we have

$$u = (u_0 + tu_1)) \frac{\sin\sqrt{\mu} y}{\sin\sqrt{\mu}} \ . \tag{4.1.23}$$

Velocity profile in this case is given by the Fig-4.1.4.

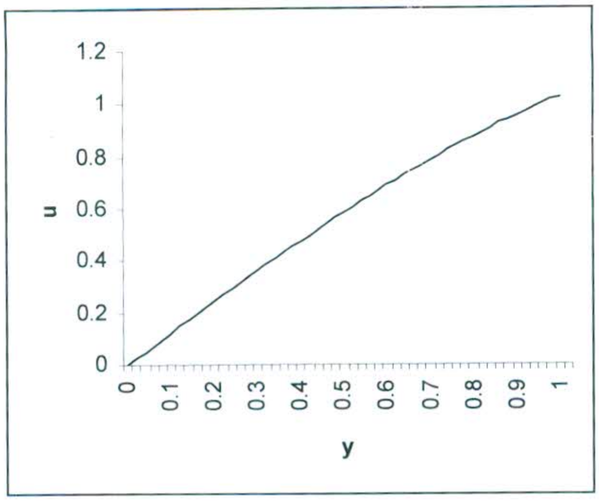


Figure-4.1.4

Case II:

$$U(t) = u_0 e^{tu_1}.$$

Boundary condition:

$$u = 0 \quad \text{at } y = 0 \text{ and } u = u_0 e^{tu_1} \text{ at } \quad y = 1 \quad \text{and} \quad u(0) = u_0 y.$$

Applying the above boundary condition, the solution of Eq.(4.1.21) is

$$u = \frac{u_0 \sin\sqrt{\mu}\, y}{\sin\sqrt{\mu}} e^{tu_1}. \tag{4.1.24}$$

Velocity profile in this case is given by the Fig-4.1.5.

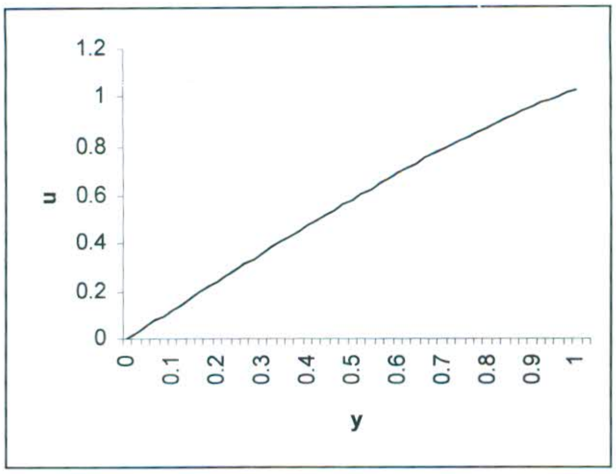


Figure-4.1.5

Case III:

$$U(t) = u_0 \cos u_1 t.$$

Applying the boundary condition, the solution of Eq.(4.1.21) is

$$u = (u_0 \sin \sqrt{\mu} y) \frac{\cos u_1 t}{\sin \sqrt{\mu}} \quad . \tag{4.1.25}$$

Velocity profile in this case is given by the Fig-4.1.6.

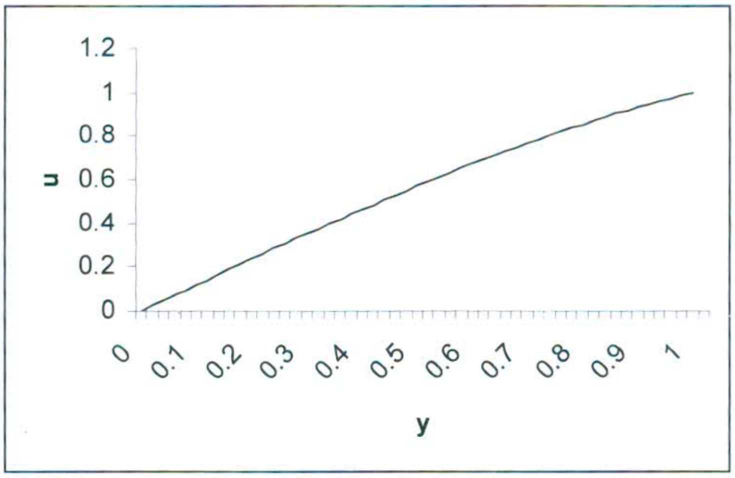


Figure-4.1.6

4.1.5 Special case: $\mathrm{Re} = 1$

Boundary condition:

For Poiseuille flow

$u = \sin \pi x$ at $t = 0$ for $0 \le x \le 1$ and $u = 0$ at $x = 0$ and $x = 1$ for $t > 0$.

To solve this problem by finite difference method, we let

$$y = ih, i = 0,1,2,3,\ldots\ldots\ldots\ldots$$
$$t = jk, j = 0,1,2,3,\ldots\ldots\ldots\ldots$$

Setting $r = \dfrac{k}{h^2}$ and using Crank –Nicolson method, the solution of the problem is given by

$$u_i = \frac{r}{2(1+r)}(u_{i-1} + u_{i+1}) + \frac{c_i}{(r+1)} \tag{4.1.26}$$

where

$$c_i = u_{i,j} + \frac{1}{2}r(u_{i-1,j} - 2u_{i,j} + u_{i+1,j} \ . \tag{4.1.27}$$

Finally, using Jacobi's iteration formula we have the following velocity profile:

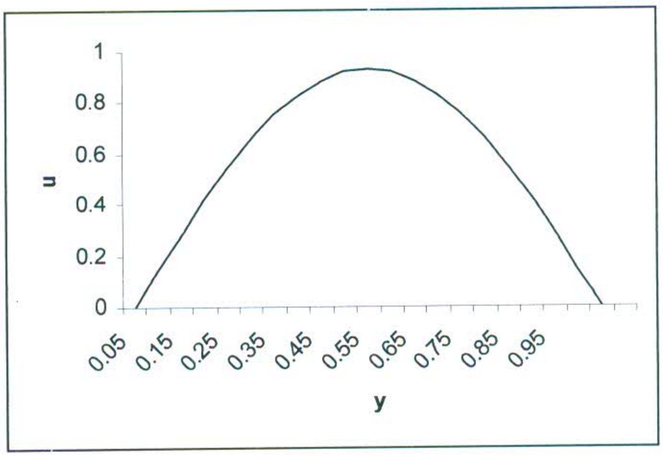


Figure-4.1.7

4.1.6 Results and discussion

It is clear from the Fig-4.1.2 to Fig-4.1.7 that the velocity profiles are very similar to the parabolic nature of the flow, firstly starts with zero velocity and then gradually increases and attain a maximum velocity. From Fig-4.1.1 to Fig-4.1.3 the velocity profiles depend on the Reynolds number and the velocity increases with the increasing value of Reynolds number.

CHAPTER IV

Part: B

Laminar Flow of Incompressible Fluid Due to Lorentz Force

4.2.1 Introduction

Magneto-hydrodynamics is the study of the interaction between magnetic field and moving conducting fluid. The motion of conducting fluid across the magnetic field generates electric currents which change the magnetic field, and the action of the magnetic field on these currents gives rise to mechanical forces which modify the flow of the fluid. A force may be produced inside a flowing fluid by the application of an externally applied magnetic field as well as an externally applied electric field. This force is called Lorentz force. Gailitis and Lielausis [20] introduced the idea of using Lorentz force to control the flow of an electrically conducting fluid over a flat plate. This is achieved by a strip wise arrangement of flush mounted electrodes and permanent magnets of alternating polarity and magnetization. The Lorentz force which acts parallel to the plate can either assist or oppose the flow. Pantokratoras [49,50] obtained the solution of boundary layer flow problem by applying parallel Lorentz force to the flow direction of conducting fluid. The purpose of the present chapter is to analyze the boundary layer flow between two parallel plates in presence of Lorentz force acting parallel to the plates.

4.2.2 Mathematical formulation

Consider a steady laminar two dimensional flow between two horizontal infinite parallel plates. Again suppose that the fluid flows along the x direction and varies perpendicular direction y; u and v being the velocity components along x and y directions respectively. It is assumed that an electromagnetic field exists at the lower plate and therefore a Lorentz force parallel to the plates is produced. The fluid is forced to move due to the action of Lorentz force. For steady two dimensional flow the boundary layer equations with constant fluid properties (Tsinober and Shtern[73],Albrecht and Grundmann [2], Pantokratoroas[49,50]) are

Continuity equation:

$$\frac{\partial u'}{\partial x'}+\frac{\partial \upsilon'}{\partial y'}=0 \qquad (4.2.1)$$

Momentum equation:

$$u'\frac{\partial u'}{\partial x'}+\upsilon'\frac{\partial u'}{\partial y'}=-\frac{1}{\rho}\frac{\partial p'}{\partial x'}+\nu\frac{\partial^2 u'}{\partial y'^2}+\frac{\pi j_o M_O}{8\rho}e^{-\frac{\pi}{a}y'} \qquad (4.2.2)$$

where p' is the pressure , ν is the kinematic coefficient of viscosity , j_o is the applied current density in the electrodes , M_O is the magnetization of the permanent magnets , a is the width of magnets and electrodes and ρ is the fluid density .The last term in the momentum equation is the Lorentz force which decreases exponentially with y' and is independent of the flow . For fully developed conditions the flow is parallel, the transverse velocity is zero , and the flow is described only by the following momentum equation:

$$-\frac{1}{\rho}\frac{dp'}{dx'}+\nu\frac{d^2u'}{dy'^2}+\frac{\pi j_o M_O}{8\rho}e^{-\frac{\pi}{a}y'}=0\ . \qquad (4.2.3)$$

We introduce the following dimensionless quantities :

$$L=\frac{\pi j_o M_O a^3}{8\rho\nu^2} \qquad (4.2.4)$$

$$x=\frac{x'}{a} \qquad (4.2.5)$$

$$y = \frac{y'}{a} L^{\frac{1}{4}} \tag{4.2.6}$$

$$u = \frac{u'a}{\nu} L^{\frac{1}{2}} \tag{4.2.7}$$

$$p = \frac{p'}{\rho U^2} \ . \tag{4.2.8}$$

Putting the above dimensionless quantities in equation (4.2.3), we get

$$-\frac{dp}{dx} + \frac{d^2u}{dy^2} + L\exp(-yL^{-\frac{1}{4}}\pi) = 0 \ . \tag{4.2.9}$$

The quantity L is dimensionless number and is called Lorentz number. This number expresses the balance between the electromagnetic force to viscous force.

4.2.3 Couette flow

Suppose that the lower plate is fixed and the upper plate is moving with velocity U .If h represents the distance between the plates then we get the following boundary conditions:

$$\text{when } y = 0 \text{ then } u = 0 \tag{4.2.10}$$

$$\text{when } y = \frac{h}{a} L^{\frac{1}{4}} \text{ then } u = \frac{Ua}{\nu} L^{\frac{1}{2}} . \tag{4.2.11}$$

Using the boundary conditions(4.2.10) and (4.2.11) we get

$$u = -\frac{L}{\lambda^2}(e^{\lambda y} - 1) + \frac{L}{\lambda^2 l}(e^{\lambda l} - 1)y + \frac{\eta}{l} y \tag{4.2.12}$$

where

$$\lambda = -\pi L^{-\frac{1}{4}} \ , \quad l = \frac{h}{a} L^{\frac{1}{4}} \quad \text{and } \eta = \frac{Ua}{\nu} L^{\frac{1}{2}} \ .$$

The Table-4.2.1 is drown for $h = 1, U = 1, a = 2.1, \nu = 1$.

Table-4.2.1

y	u	u	u
---	L=1	L=3	L=5
0	0	0	0
.1	.454162	.637398	.7711172
.2	.899085	1.244717	1.490616
.3	1.337556	1.829141	2.16938
.4	1.771524	2.396142	2.816151
.5	2.2.2347	2.94988	3.437763
.6	2.630975	3.493525	4.03959
.7	3.058069	4.029489	4.625858
.8	3.484094	4.559606	5.19989
.9	3.909372	5.085273	5.764298
1	4.334128	5.607553	6.32114

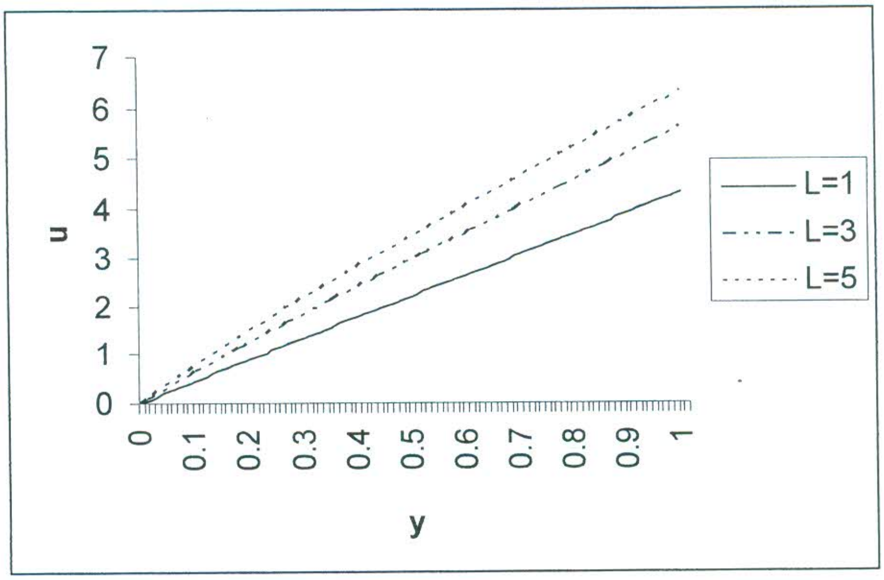


Figure-4.2.1

Velocity profile for various values of Lorentz number

4.2.4 Poiseuille flow

Another kind of flow between parallel plates is the Poiseuille flow(Poiseuille,1840) which is caused by a constant pressure gradient along the plates while the plates are fixed. If h is the distance between the plates then we have the following boundary conditions:

when $y = 0$ then $u = 0$ (4.2.13)

when $y = \frac{h}{a} L^{\frac{1}{4}}$ then $u = 0$. (4.2.14)

Using the boundary conditions (4.2.13) and (4.2.14), we get the following analytic solution

$$u = -\frac{L}{\lambda^2}(e^{\lambda y} - 1) + \frac{L}{l\lambda^2}(e^{\lambda l} - 1)y + \frac{P_o}{2}(l - y)y \quad (4.2.15)$$

$$\lambda = -\pi L^{-\frac{1}{4}}, \quad l = \frac{h}{a} L^{\frac{1}{4}} \text{ and } P_0 = -\frac{dp}{dx}.$$

The Table-4.2.2 is drawn for $h = 3.0, P_O = 2.1, a = 2.1$.

Table-4.2.2

y	*u*	*u*	*u*
----	L=1	L=3	L=5
0	0	0	0
.1	.163055	.284952	.403227
.2	.29587	.518825	.733726
.3	.401234	.708803	1.002544
.4	.481094	.860358	1.218371
.5	.53681	.977651	1.388038
.6	.56933	1.063851	1.516921
.7	.579318	1.121369	1.609244
.8	.567236	1.15204	1.668331
.9	.533406	1.157262	1.696795
1	.478055	1.138097	1.696692
1.1	.40134	1.095354	1.669637
1.2	.303371	1.029649	1.616903
1.3	.184224	.941452	1.539488
1.4	.043953	.831118	1.438179

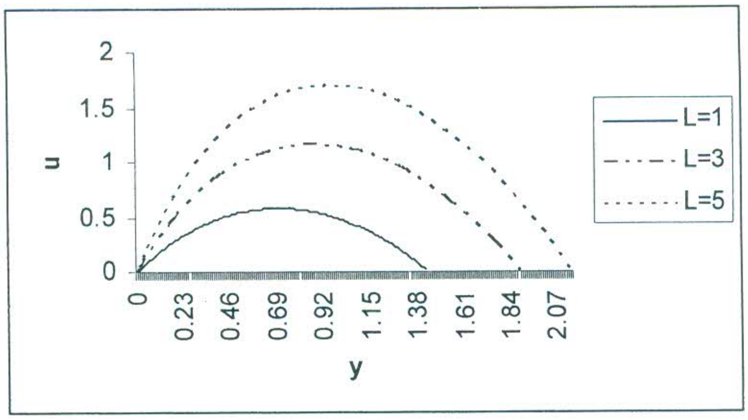


Figure-4.2.2

Velocity profile for various values of Lorentz number

4.2.5 Results and discussion

From Fig-4.2.1 we observe that the velocity of the conducting fluid increases when the distance of the fluid from the lower plate increases. From Fig-4.2.2 we observe that the velocity of the conducting fluid increases firstly and it attains a maximum value. After some distances from the lower plate the velocity of the fluid decreases in the same manner and dies out to zero velocity at the upper plate. Fig-4.2.1 and Fig-4.2.2 also indicate that the velocity increases for the increasing value of Lorentz number. For couette flow the velocity profile is linear but it is parabolic for poiseuille flow. The author believes that the results of the present work will enrich the list with the existing exact solutions of the Navier-Stokes equations and may help the investigation of flow of electrically conducting fluids like water and liquid metals.

CHAPTER V

Laminar Flow of Incompressible Fluid Between Two Parallel Porous Plates

5.1 Introduction

The motion of conducting fluid across the magnetic field generates electric currents that change the magnetic field and the action of the magnetic field on these currents gives rise to mechanical forces that modify the flow of the fluid. Since in MHD we are mainly concerned with conducting fluids in motion, it is necessary to consider first the electrodynamics of moving media. The electromagnetic field is governed by the Maxwell's equations and the motion of continuum is governed by the Navier-Stokes' equation of motion. The problem for a non-conducting fluid was considered by Sinha & Chaudury [69] for periodic momentum of the plate. Das and Sengupta [12] have discussed the unsteady flow of a conducting viscous fluid through a straight tube . Sengupta and Kumer [64] have developed MHD flow of a viscous incompressible fluid near a moving porous flat plate. Sengupta Rahman & Kandar [63] developed the flow between two parallel flat plates. Sreekanth, Nagarajan & Raman [71] have developed the transient MHD free convection flow of an incompressible viscous dissipative fluid numerically.

In this chapter the Maxwell's electromagnetic field equation and the Navier-Stokes' equation are considered as basic equations. The mutual interaction between conducting fluid and the magnetic field are considered here. In Case I the time varying pressure gradient function is also considered. Small suction and injection are imposed on the plates. The velocity of the fluid has been obtained under three different cases: (i)pressure gradient varying linearly with time,(ii) pressure gradient decreasing exponentially with time,(iii)pressure gradient varying periodically with time. In Case II of this chapter an attempt has been made to study the MHD flow of incompressible viscous fluid between an oscillating porous plate and fixed porous plate. The pressure gradient force is not taken into account. The highly conducting incompressible viscous fluid is moving under the action of body force. A small uniform suction is imposed on the fixed plate and a corresponding injection is imposed on the oscillating plate. It is assumed that the fluid enters on one side of the oscillating plate and is sucked away to the other side of the fixed plate. The numerical solution is obtained by using finite difference method. Finally the velocity distribution is shown with the help of graphs.

5.2 Mathematical formulation

A conducting viscous incompressible fluid moving in a magnetic field is governed by the following set of equations:

Firstly, the equations of electromagnetic field (i. e, Maxwell's equations):

$$\nabla \cdot \overline{B} = 0 \tag{5.1}$$

$$\nabla \cdot \overline{E} = \frac{\rho}{\varepsilon} \tag{5.2}$$

$$\nabla \times \overline{E} = -\frac{\partial \overline{\mathrm{B}}}{\partial t} \tag{5.3}$$

$$\nabla \times \overline{B} = \mu_c \overline{J} \tag{5.4}$$

where $\overline{E}$ is the electric field intensity, $\overline{B}$ the magnetic field intensity ρ_c the electric charge density, ε the electrical permittivity, μ_c the magnetic permeability of the medium, $\overline{J}$ the current density. When the frequency of the applied field is considered low, displacement current is neglected and since no charge separation takes place, ρ_c is also taken zero.

So we can write

$$\nabla.\bar{J} = 0. \tag{5.5}$$

Secondly, the mechanical equations embodying the effect of the electromagnetic forces (Navier-Stoke's equation):

$$\frac{d\bar{V}}{dt} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\bar{V} + \frac{1}{\rho}(\bar{J}\times\bar{B}). \tag{5.6}$$

Thirdly, equation of continuity:

$$\nabla\cdot\bar{V} = 0. \tag{5.7}$$

Fourthly, a conducting fluid moving with velocity $\bar{V}$, the total electric force is $\bar{E}+\bar{V}\times\bar{B}$ In this case Ohm's law gives

$$\bar{J} = \sigma(\bar{E}+\bar{V}\times\bar{B}) \tag{5.8}$$

where σ is the electrical conductivity.

When the mechanical force of electromagnetic origin is perpendicular to the magnetic field, it has no direct influence on the motion parallel to the field .When the motion is perpendicular to the field we can write

$$\bar{J}\times\bar{B} = \sigma(\bar{E}+\bar{V}\times\bar{B})\times\bar{B}. \tag{5.9}$$

If we consider the electrodes produced by highly conducting wire along the plates such that the potential at the two electrodes are the same, then the electric field E will be zero. Thus from Eq.(5.9) we have

$$\overline{J}\times\overline{B}=\sigma(\overline{V}\times\overline{B})\times\overline{B} \tag{5.10}$$

or,

$$\overline{J}\times\overline{B}=-\sigma[(\overline{B}\cdot\overline{B})\overline{V}-(\overline{B}\cdot\overline{V})\overline{B}] \ . \tag{5.11}$$

Assume $\overline{V}$ is perpendicular to $\overline{B}$. From Eq.(5.11) we have

$$\overline{J}\times\overline{B}=-\sigma B^2\overline{V} \ . \tag{5.12}$$

When the magnetic field is uniform and equal to B_0, then from Eq.(5.12)

$$\overline{J}\times\overline{B}=-\sigma B_o^{\ 2}\overline{V} \ . \tag{5.13}$$

Eq.(5.6) and Eq.(5.13) give rise to

$$\frac{d\overline{V}}{dt}=-\frac{1}{\rho}\nabla p+\nu\nabla^2\overline{V}-\frac{1}{\rho}\sigma B_o^{\ 2}\overline{V} \ . \tag{5.14}$$

5.3 Case: I

Consider the two dimensional laminar flow of viscous incompressible fluid between two fixed long parallel porous plates separated by a distance l, one plate being along the x- axis and the other plate being along $y=l$. Porous plates mean the plates with very fine holes distributed uniformly throughout the plate through which fluid can flow freely and continuously. The plate from which the fluid is entering into the flow region is called the plate with injection and the plate through which the fluid is going out of the flow region is called the plate with suction. Let the velocity of injection at $y=0$ and the velocity of suction at $y=l$ be equal, and equal to V_0; and the flow depends only on y. Then the principle of mass conservation gives that the velocity component along y -axis is constant through the flow, and so equal to

V_0. Again suppose the conducting flow along x- axis and there is no velocity component along the direction perpendicular to x -axis i.e., y- axis. For two-dimensional flow, we have

$$\frac{\partial}{\partial z} = 0 \ . \tag{5.15}$$

Furthermore the equation of continuity reduces to

$$\frac{\partial u}{\partial x} = 0 \ . \tag{5.16}$$

Hence the Eq.(5.14) reduces to

$$\frac{\partial u}{dt} + \upsilon \frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho}\sigma B_o^{\ 2} u \tag{5.17}$$

$$\frac{\partial p}{\partial y} = 0 \ . \tag{5.18}$$

Eq.(5.18) suggest that $\frac{\partial p}{\partial x}$ must be a constant or a function of time only. Hence assuming

$$\frac{1}{\nu\rho}\frac{\partial p}{\partial x} = \theta(t) \ . \tag{5.19}$$

Eq.(5.17) reduces to

$$\frac{V_O}{\nu}\frac{\partial u}{\partial y} + \frac{\partial^2 u}{\partial y^2} - \frac{1}{\nu}\frac{\partial u}{\partial t} - M^2 u = \theta(t) \tag{5.20}$$

where M represents the magnetic parameter and

$$M = B_O \sqrt{\frac{\sigma}{\rho\nu}} \ .$$

Homogeneous part of Eq.(5.20) is

$$\frac{V_O}{\nu}\frac{\partial u}{\partial y} + \frac{\partial^2 u}{\partial y^2} - \frac{1}{\nu}\frac{\partial u}{\partial t} - M^2 u = 0 \ . \tag{5.21}$$

B. When $\theta(t)$ is a transient function of time of the form

$$\theta(t) = \alpha e^{-\beta t} \ .$$

Then the general solution of Eq.(5.20) reduces to

Suppose

$$u(y,t) = \mathrm{Re}\{e^{\mathrm{int}} f(y)\} \tag{5.22}$$

be a solution of Eq.(5.21).Then from Eq.(5.21) we have

$$\frac{V_O}{\nu}\frac{df}{dy} + \frac{d^2 f}{dy^2} - (M^2 + \frac{in}{\nu})f = 0 . \tag{5.23}$$

The general solution of Eq.(5.21) is

$$f(y) = c_1 \exp[-s + (a + ib)]y + c_2 \exp[-s - (a + ib)]y \tag{5.24}$$

where c_1 and c_2 are arbitrary constants and

$$s = \frac{V_O}{2\nu}, a^2 - b^2 = \frac{V_O^{\ 2}}{4\nu^2} + M^2, ab = \frac{n}{2\nu} . \tag{5.25}$$

To determine the solution of the non-homogeneous part, we consider the following three cases:

A. When $\theta(t)$ is a linear function of time of the form

$$\theta(t) = \beta t + \beta \ . \tag{5.26}$$

Then the general solution of Eq.(5.20) reduces to

$$\begin{aligned} u \ &= c_1 \exp[-s + (a + ib)]y + c_2 \exp[-s - (a + ib)]y - \nu(\frac{\alpha t^2}{2!} + \beta t) \\ &+ M^2\nu^2(\frac{\alpha t^3}{3!} + \frac{\beta t^2}{2!}) - M^4\nu^3(\frac{\alpha t^4}{4!} + \frac{\beta t^3}{3!}) + M^6\nu^4(\frac{\alpha t^5}{5!} + \frac{\beta t^4}{4!}) + - - - - \end{aligned} \tag{5.27}$$

B. When $\theta(t)$ is a transient function of time of the form

$$\theta(t) = \alpha e^{-\beta t} .$$

Then the general solution of Eq.(5.20) reduces to

$$u = c_1 \exp[-s+(a+ib)]y + c_2 \exp[-s-(a+ib)]y + \alpha v \frac{e^{-\lambda t}}{\beta - vM^2} \tag{5.28}$$

C. When $\theta(t)$ is a periodic function of time of the form

$$\theta(t) = \alpha \sin \lambda t .$$

Then the general solution of Eq.(5.20) reduces to

$$\begin{aligned} u &= c_1 \exp[-s+(a+ib)]y + c_2 \exp[-s-(a+ib)]y \\ &+ \frac{\alpha v}{\lambda^2 + v^2 M^4}(\lambda \cos \lambda t - vM^2 \sin \lambda t). \end{aligned} \tag{5.29}$$

5.4 Case: II

Let us consider a laminar flow of conducting incompressible viscous Newtonian fluid flowing between a oscillating and a fixed plate. Again suppose that the porous plates are distributed uniformly. We consider that the suction velocity and the injection velocity are constant and fluid moves under the action of body force only.

For two-dimensional flow the equation of continuity reduces to

$$\frac{\partial u}{\partial x} = 0 . \tag{5.30}$$

Hence the4 Eq.(5.14) reduces to

$$\frac{\partial u}{dt} + \upsilon \frac{\partial u}{dy} = v \frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho} \sigma B_o^{\ 2} u . \tag{5.31}$$

Let the suction velocity and the injection velocity is same and is equal to V_0. Then the Equ.(5.31) reduces to

$$\frac{\partial^2 u}{\partial y^2} + \frac{V_0}{v}\frac{\partial u}{\partial y} - \frac{1}{v}\frac{\partial u}{\partial t} - M^2 u = 0 \tag{5.32}$$

where

$$M = B_O \sqrt{\frac{\sigma}{\rho \nu}}.$$

Let us consider the porous plate $y = 1$ is at rest and the plate $y = 0$ is oscillating harmonically in its own plate with a prescribed frequency π. We have the boundary conditions

$$\begin{aligned} &u(1,0) = 0 \\ &\frac{\partial u(0,t)}{\partial t} = 0 \\ &u(0,t) = \sin^3 \pi t. \end{aligned} \tag{5.33}$$

The Equ. (5.32) is a linear partial differential equations and are to be solved by using the boundary conditions (5.33).The equivalent finite difference scheme of equation(5.32) is as follows:

$$u_{i+1,j} = \frac{d}{a}(u_{i,j+1} - u_{i,j-1}) + \frac{c}{a}u_{i,j} + \frac{b}{a}u_{i-1,j} \tag{5.34}$$

where

$$\begin{aligned} a &= \frac{1}{h^2} + \frac{V_0}{2\nu h} \\ b &= -\frac{1}{h^2} + \frac{V_0}{2\nu h} \\ c &= \frac{2}{h^2} + M^2 \\ d &= \frac{1}{2\nu K}. \end{aligned}$$

Here the index i refers to y, j refers to time. The mesh system is divided by taking $h = 0.1$ and $k = .09$.

Let

$$u_{i,j} = u(ih, jk) . \tag{5.35}$$

So, the boundary conditions become

$$\begin{aligned} &u(10,0) = 0 \\ &u(0, jk) = \sin^3 kj\pi, \; j = 1,2,3......10. \end{aligned} \tag{5.36}$$

At first the velocity is computed along the plate $y = 0$ for various values of time. Then the velocity at the end of coordinate step viz. $u(i+1, j)(j = 1,10)$ is computed in terms of velocity at points on the earlier coordinate step . The procedure is repeated until $y = 1 (i.e., i = 10)$ .The computations were carried out for $V_0 = 22.1, \nu = 1.5, M = 1.2$. To judge the accuracy of the convergence and stability of finite difference scheme , the same program was run with smaller values of Δy and Δt and no significant change was observed . Hence we conclude that the finite difference scheme is stable and convergence.

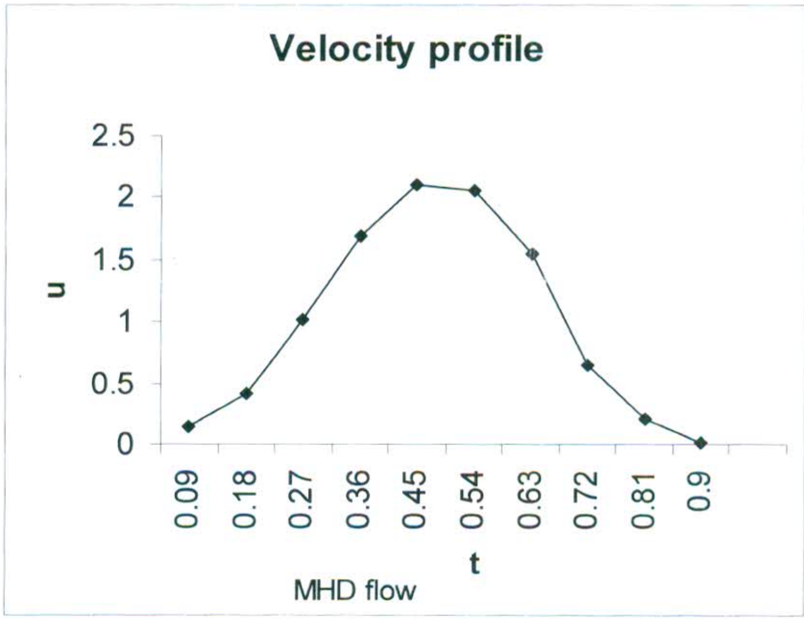


Figure-5.1

5.5 Case: III

Let us consider a steady laminar flow of a incompressible viscous fluid between two parallel porous places. Suppose the plate y = 0 is fixed and the plate $y = 1$, is in uniform motion. A small injection of the same fluid is imposed on the fixed plate and a corresponding uniform suction is imposed on the moving plate.

In absence of magnetic field, the Eq.(5.17) reduces to

$$V_0 \frac{du}{dy} = \nu \frac{d^2u}{dy^2}. \tag{5.37}$$

In this case the boundary condition will be:

$$y = 0: \qquad u = 0, \qquad \upsilon = V_0$$

$$y = 1: \qquad u = U_\infty, \qquad \upsilon = V_0 .$$

Solving Eq.(5.37), we get

$$\frac{u}{U_\infty} = \frac{e^{\lambda\eta} - 1}{e^{\lambda} - 1} \tag{5.38}$$

where $\eta = \frac{y}{h}$ and $\lambda = \frac{V_0 h}{\nu}$.

Table-5.1

$\frac{y}{h}$	$\frac{u}{U_\infty}$
0	0
.1	.032608
.2	.072837
.3	.122466
.4	.183692
.5	.259225
.6	.352409
.7	.467367
.8	.609189
.9	.784151
1	.999998

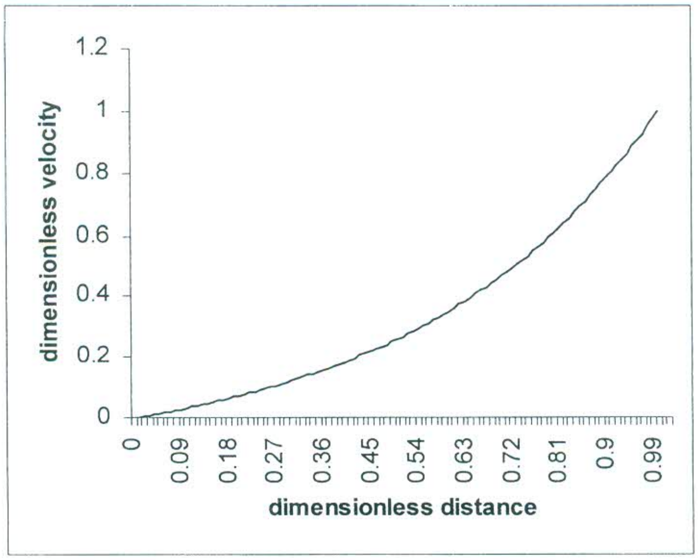


Figure 5.2

5.7 Results and discussion

Because of the motion of the fluid in the magnetic field, an associated electrical field is produced, which according to Ohm's law sets up electric currents in the fluid. The interaction between these currents and magnetic field results a body force. From Eq.(5.27), Eq.(5.28) and Eq.(5.29) we see that the velocity distribution can be drown by choosing suitable initial and boundary conditions. In Fig-5.1, the velocity distribution u is drawn against t for $y = .5$. It is shown that the velocity distribution is similar to the parabolic nature of the flow . It starts with a zero velocity and then gradually increases and attains a maximum value. After some times the velocity diminishes in the same manner as time increases and ultimately dies out to zero velocity. From Fig-5.2 we see that the velocity of the fluid increases as the distance increases and finally attains a maximum velocity.

CHAPTER VI

Laminar Flow of Incompressible Fluid Over a Suddenly Accelerated Flat Plate

6.1 Introduction

If a magnetic field is placed before a moving conducting fluid then the motion of the fluid is changed by the influence of the magnetic field. The magnetic field is also perturbed by the motion of the fluid: one affects the other and vice versa. The motion of the conducting fluid across the magnetic field generates electric-current, which changes the magnetic field and the action of magnetic field on these currents gives rise to mechanical forces which modify the flow of the fluid. In recent years, the study of MHD phenomena in liquid conductors has received considerable impetus on account of its theoretical, experimental and practical applications. Stokes (Schlichting [62]) studied the problem of an incompressible viscous fluid flow problem produced by the oscillation of a plane solid wall. This problem is also known as Stokes second problem. Panton [51] obtained the transient solution for the flow due to the oscillation plane. Von Keregek and Davis [74] performed the linear stability theory of oscillating Stokes layers. Erdogan [17] derived the analytic solutions for the flow produced by the small oscillating wall for small and large time by Laplace transformation method. Recently Poria, Mamaloukas, layek & Magumdar [52] derived the solution of laminar flow of viscous conduction fluid produced by the oscillating plane wall. They solved the problem both analytically and numerically in presence of magnetic field. In this chapter the main aim is to investigate the effects of a transverse magnetic field on the incompressible electrically conducting fluid flow produced by a moving plate. An attempt has been made to investigate the analytic solutions for the problem. The problem has also been solved numerically using well known Crank Nicholson Implicit scheme.

6.2 Mathematical formulation

Let us consider a flat plate extended to large distances in x' and z' directions. Again we consider an incompressible viscous conducting fluid over a half plane solid wall $y'=0$. Suppose the fluid is at rest at time $t' < 0$. At $t' = 0$ the plane solid wall $y' = 0$ is suddenly set in motion in x' direction at constant velocity U_o. As a result, a two dimensional parallel flow will be produced near the plate. Since the fluid flows along x' direction and there is no velocity component along the direction perpendicular to the direction of flow, so the equation of conservation of mass reduces to

$$\frac{\partial u'}{\partial x'}=0 \ . \tag{6.1}$$

As the flow is only kept in motion by the movement of the plate, one may set the pressure gradient $\frac{\partial p'}{\partial x'}=0$.

For unsteady case Eq. (4.14) reduces to

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_o^2}{\rho} u' \ . \tag{6.2}$$

Eq. (6.1) indicates that u' is a function of y' and t'.

Boundary conditions:

$u' = 0$ when $t' \leq 0$ for all y'

$u' = U_O$ at $y' = 0$ when $t' \geq 0$ (6.3)

$u' = 0$ at $y' = \infty$ when $t' \geq 0$.

We introduce the following non-dimensional quantities

$$y = \frac{y'}{L}, \quad t = \frac{t'}{T}, \quad u = \frac{u'}{U_\infty}.$$

where L and T represent the characteristic length and characteristic time respectively .

Setting these non-dimensional quantities in Eq.(6.2) ,we get

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - M^2 u \quad (6.4)$$

where

$$\rho \upsilon = \mu$$

$$M = B_0 L \sqrt{\frac{\sigma}{\mu}} .$$

Here the number M is a non-dimensional number and is called Hartmann number.

In this case the boundary conditions may be written as

$$t \leq 0 : u(y, 0) = 0 \qquad \textit{for all } y$$

$$t \geq 0 : u(0, t) = 1 , \qquad u(\infty, t) = 0 . \quad (6.5)$$

6.3 Analytic solution

We introduce the Laplace transformation and inverse Laplace trans formation as

$$L\{u(y,t)\} = U(y,s) \quad (6.6)$$

$$L^{-1}\{U(y,\, s)\}=u(y,t)\,. \tag{6.7}$$

We have

$$L\left\{\frac{\partial U}{\partial t}\right\}=s\,L\{u\}\;-\,u(y,o)\,=sU \tag{6.8}$$

and

$$L\left\{\frac{\partial^2 u}{\partial y^2}\right\}=\frac{d^2}{dy^2}[L\{u\}]=\frac{d^2 U}{dy^2}\,. \tag{6.9}$$

From (6.4), we have

$$\frac{d^2 U}{dy^2}-(s+M^2)U=0\,. \tag{6.10}$$

With the help of boundary condition (6.5),we get

$$U(o,s)\,=\,L\{u(0,t)\}\,=\frac{1}{s}\,. \tag{6.11}$$

The Solution of Eq.(6.10) subject to boundary condition (6.11) is

$$U(y,\, s)\,=\,c_1\,e^{y\sqrt{S+M^2}}\,+c_2\,e^{-y\sqrt{s+M^2}} \tag{6.12}$$

Since u is finite for $y\rightarrow\infty$ we must have $c_1 = 0$.

Eq. (6.12) reduces to

$$U(y,\, s)=c_2\,e^{-y\sqrt{s+M^2}} \tag{6.13}$$

$$\therefore\ U\ (0,\, s) = c_2 \Rightarrow c_2 = \frac{1}{s}.$$

Thus the Eq. (6.12) reduces to

$$U\,(y,s) = \frac{1}{s} e^{-y\sqrt{s+M^2}}\ . \tag{6.14}$$

Taking inverse Laplace transformation of Eq. (6.14) we have

$$u(y,\, t) = \frac{1}{2} e^{M^2 t} [e^{My} erfc(\frac{1}{2} yt^{-\frac{1}{2}} + Mt^{\frac{1}{2}}) + e^{-My} erfc(\frac{1}{2} yt^{-\frac{1}{2}} - Mt^{\frac{1}{2}}) \tag{6.15}$$

6.4 Numerical Solution

The Eq.(6.4) with initial and boundary conditions Eq.(6.5) is solved by finite difference technique. The Crank-Nicholson implicit scheme is used to solve the parabolic type of equation. In this scheme, the time derivative term is represented by forward difference formula while the space derivative term is represented by the average central difference formula. To do this the temporal first derivative can be approximated by

$$\frac{\partial u}{\partial t} \approx \frac{(u_i^{l+1} - u_i^l)}{\Delta \tau}\ . \tag{6.16}$$

The second derivative in space can be determined at the midpoint by the averaging the difference approximations at the beginning (t^l) and at the end

(t^{l+1}) of the time increment.

$$\frac{\partial^2 u}{\partial y^2} \cong \frac{1}{2}\left[\frac{u_{i+1}^{l} - 2u_i^l + u_{i-1}^l}{(\Delta\eta)^2} + \frac{u_{i+1}^{l+1} - 2u_i^{l+1} + u_{i-1}^{l+1}}{(\Delta\eta)^2}\right]. \tag{6.17}$$

Substituting Eq. (6.16) and Eq. (6.17) into Eq.(6.4), we get

$$\frac{u_i^{l+1} - u_i^l}{\Delta\tau} = \frac{1}{2}\left[\frac{u_{i+1}^{l} - 2u_i^l + u_{i-1}^l}{(\Delta\eta)^2} + \frac{u_{i+1}^{l+1} - 2u_i^{l+1} + u_{i-1}^{l+1}}{(\Delta\eta)^2}\right] - \frac{M^2}{2}(u_i^{l+1} + u_i^l) \tag{6.18}$$

or

$$ru_{i-1}^{l+1} - (2r + s + 1)u_i^{l+1} + ru_{i-1}^{l+1} = (2r - 1 + s)u_i^l - r(u_{i-1}^l + u_{i+1}^l) \tag{6.19}$$

where

$$r = \frac{\Delta\tau}{2(\Delta\eta)^2} \quad , \quad s = \frac{\Delta\tau M^2}{2}.$$

The Eq.(6.19) may be written as

$$-ru_{i-1}^{l+1} + k_1 u_i^{l+1} - ru_{i-1}^{l+1} = r(u_{i-1}^l + u_{i+1}^l) + k_2 u_i^l \tag{6.20}$$

where

$$k_1 = 1 + 2r + s \qquad and \qquad k_2 = 1 - 2r - s.$$

The system of algebraic equations in tri-diagonal form that follows from (6.20) is solved by Thomas algorithm for each time level. In this problem some grid points have been considered for numerical computation. u is obtained at each grid points at each time interval.

The Fig.-6.1 is drawn for various values of y when t=.5.

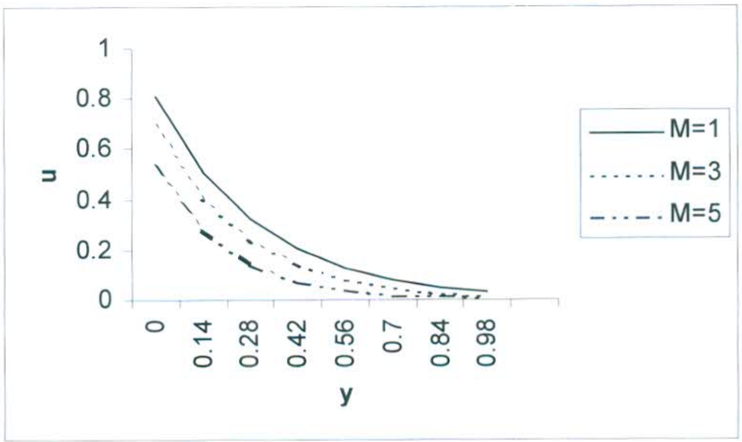


Figure-6.1

The effect of the magnetic field on velocity of fluid for different space

6.5 Results and discussion

Numerical results are displayed by the Fig-6.1. These figure shows that the velocity of the fluid decreases as the magnetic field increases. The velocity decreases gradually and attains almost zero velocity at a sufficient large distance from the plate. The Fig-6.1 indicates that for a small value of Hartmann number the velocity profile is almost linear. When the Hartmann number increases then the shape of velocity profiles becomes parabolic type. It is observed from Fig-6.1 that the velocity decreases in the direction perpendicular to the flow direction.

CHAPTER VII

Part: A

Temperature Distribution of Laminar Flow of Incompressible Fluid

7.1.1 Introduction:

The transfer of heat from solid body to liquid or liquid to solid body is a problem whose consideration involves the science of fluid motion. In order to determine the temperature distribution it is necessary to combine the equation of motion with those of heat conduction. If a solid body is placed in a fluid and is heated so that its temperature is maintained above that of the surroundings then it is clear that the temperature of the stream will increase only over a thin layer in the immediate neighborhood of the body. In analogy with flow phenomena this thin layer is called thermal of boundary layer.

In general thermal conductivity of liquid is small. In this case there is a very steep temperature gradient at right angles to the wall and heat flux due to conduction of same order of magnitude as that is due to convection only a thin layer across near the wall. When a temperature difference is established between the vertical plate and stationery fluid, the fluid adjacent to the wall will move upward if the wall temperature is higher than of fluid, and downward if the wall temperature is lower. The cause of the moment is the temperature gradient itself. This sets up density gradients in the fluid resulting in buoyancy forces and free convection currents.

The Prandtl number of some fluids is very small. On the other hand Prandtl number of some fluid is very large. As for example the Prandtl number of mercury is 0.044 and the Prandtl number of lubricating oil is 7250.

Thermal boundary layers in laminar flow have been presented in many standard books of. Arora & Domkundwar [5], Bansal [7], Rogers & Mayhew [58], & Schlichting [62]. Sparraw, Eichhorn and Gregg. [70] studied the combined force and free convection in a boundary layer theory. Lahiri, Chakraborty & Mazumdar [37] described the effects of temperature dependent viscosity on an incompressible fluid over a stretching sheet. Basic equations in this chapter are the equation of continuity, the equation of motion and the equation of energy. The equation of continuity and the equation of motion can be solved for velocity components and the pressure and the result so obtained can be used to solve the equation of energy to determine the desired temperature field.

7.1.2 Plane couette flow

Let us consider a two dimensional steady laminar flow of incompressible viscous fluid between two parallel plates separated by a distance h. Again suppose that x be the direction of the flow, y the direction perpendicular to the flow. The fluid flows in such a way that all the flow parameters depend only y-axis. If one plate $y = 0$ is kept at rest and the other plate $y = h$ is allowed to move with velocity U_∞ then the velocity distribution as mentioned in Chapter-III, is given by

$$u = \frac{U_\infty y}{h} \quad . \tag{7.1.1}$$

Without heat addition the energy equation for the steady two dimensional flow of incompressible viscous fluid is

$$\rho c_p u \frac{\partial T}{\partial x} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{7.1.2}$$

where T is the temperature of the fluid , ρ the density of the fluid, μ the coefficient of viscosity, c_p the specific heat at constant pressure.

Since all the flow parameters depend only on y, we have

$$\frac{\partial T}{\partial x} = 0 \tag{7.1.3}$$

$$\frac{\partial^2 T}{\partial x^2} = 0\,. \tag{7.1.4}$$

Thus the Eq.(7.1.2) reduces to

$$\frac{d^2 T}{dy^2} = -\frac{\mu}{k}\left(\frac{U_\infty{}^2}{h^2}\right). \tag{7.1.5}$$

Solving Eq.(7.1.5) under the boundary conditions

$$T = T_0 \qquad when \quad y = 0$$
$$T = T_1 \qquad when \quad y = h$$

we have

$$\frac{T - T_0}{T_1 - T_0} = \frac{y}{h} + \frac{1}{2} E_c \Pr \frac{y}{h}\left(1 - \frac{y}{h}\right) \tag{7.1.6}$$

where

$$E_c = \frac{U_\infty{}^2}{c_p (T_1 - T_0)}$$

is Eckert number.

and

$$P_r = \frac{\mu c_p}{k}$$

is Prandtl number.

Let $d = \frac{T - T_0}{T_1 - T_0}$, then the temperature distribution is given by the Fig-7.1.1.

Table-7.1.1

$\frac{y}{h}$	$d = 0$	$d = 2$	$d = 4$	$d = 6$	$d = 8$
0	0	0	0	0	0
.1	.1	.19	.28	.37	.46
.2	.2	.36	.52	.68	.84
.3	.3	.51	.72	.93	1.14
.4	.4	.64	.88	1.12	1.36
.5	.5	.75	1	1.25	1.5
.6	.6	.86	1.08	1.32	1.56
.7	.7	.91	1.12	1.33	1.54
.8	.8	.96	1.12	1.28	1.44
.9	.9	.99	1.08	1.17	1.26
1	1	1	1	1	1

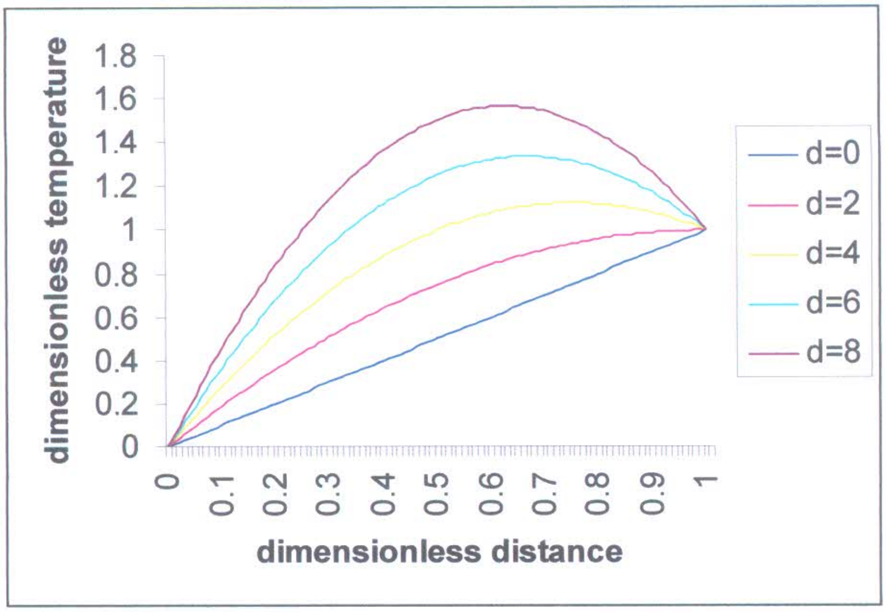


Figure-7.1.1

Temperature distribution for couette flow

From the Fig-7.1.1, we see that the dimensionless temperature varies with dimensionless distance for different values of $E_c, \Pr$.

Differentiation Eq.(7.1.6) w. r. to y and putting $y = h$, we get.

$$-\frac{h}{T_1 - T_0}\left(\frac{dT}{dy}\right)_{y=h} = \frac{1}{2} E_c \Pr - 1$$

or, $$Nu = \frac{1}{2} E_c \Pr - 1 \tag{7.1.7}$$

where

$$Nu = -\frac{h}{T_1 - T_0}\left(\frac{\partial T}{dy}\right)_{y=h}$$

is Nusselt number .

From Eq.(7.1.7), we draw the following conclusions:

(i) when $E_c \Pr > 2$, *then* $Nu > 0$.

In this case the heat will be transferred from fluid to the upper plate.

(ii) When $E_c \Pr < 2$, then Nu <0.

In this case the heat will be transferred from upper plate to the fluid.

(iii) When $E_c \Pr = 2$, then $Nu = 0$.

In this case there will be no transfer of heat between the fluid and the upper plate.

When both the plates are kept at constant temperature, then the value Nu for both the plates will be 4. In this case the temperature distribution will be parabolic.

7.1.3 Plane Poiseuille flow

We now consider the steady laminar flow of incompressible viscous fluid between two fixed infinite plates $y = \pm h$. If all the flow parameters are same as mentioned in the case of couette flow, then the velocity distribution is given by

$$u = \frac{\beta}{2\mu}(h^2 - y^2) \tag{7.1.8}$$

where $\frac{\partial p}{\partial x} = -\beta$.

In this case the Eq. (7.1.2) reduces to

$$\frac{d^2T}{dy^2} = -\frac{\beta^2}{k\mu} y^2 . \tag{7.1.9}$$

If both the plates keep at constant temperature $T = T_0$ then

$$T - T_0 = \frac{\beta^2 y^4}{12k\mu} - \frac{\beta^2 h^4}{12k\mu} \ . \tag{7.1.10}$$

The maximum temperature T_m exists in the middle of the cannel. In this case the dimensionless temperature will be given by

$$\frac{T - T_0}{T_m - T_0} = 1 - (\frac{y}{h})^4 \ . \tag{7.1.11}$$

Table-7.1.2

$\frac{y}{h}$	$\frac{T - T_0}{T_m - T_0}$
0	1
.1	.9999
.2	.9984
.3	.9919
.4	.9744
.5	.9375
.6	.8704
.7	.7599
.8	.5904
.9	.3439
1	0

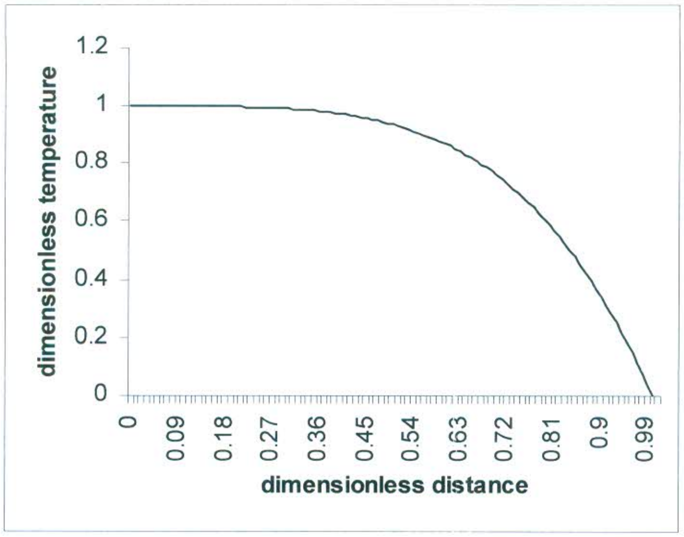


Figure-7.1.2

7.1.4 Temperature distribution near a heated flat plate

Consider the steady laminar flow of a incompressible viscous fluid over a flat plate placed along the direction of a uniform stream of velocity U_∞ and temperature T_∞. Let the origin of coordinates be at the leading edge of the plate, x axis along the plate and y axis normal to it. The basic equations are:

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0 \tag{7.1.12}$$

$$\rho(u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=\mu\frac{\partial^2 u}{\partial y^2}-\frac{dp}{dx}+\rho g_x\beta(T-T_\infty) \tag{7.1.13}$$

$$\rho c_p(u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y})=k\frac{\partial^2 T}{\partial y^2}+\mu(\frac{\partial u}{\partial y})^2 \tag{7.1.14}$$

where β is the coefficient of thermal expansion.

According to boundary layer theory the pressure gradient $\frac{dp}{dx}$ can be evaluated from the free stream velocity. i.e.

$$-\frac{dp}{dx} = \rho_\infty U_\infty \frac{dU_\infty}{dx} \; . \qquad (7.1.15)$$

If we consider the free stream velocity to be constant, then the above sets of equations reduces to

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0 \qquad (7.1.16)$$

$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g_x\beta(T - T_\infty) \qquad (7.1.17)$$

$$u\frac{\partial T}{\partial x} + \upsilon\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p}(\frac{\partial u}{\partial y})^2 \qquad (7.1.18)$$

where $\nu = \frac{\mu}{\rho}$ and $\alpha = \frac{k}{\rho c_p}$; and they are called the kinematics viscosity and thermal diffusivity respectively.

Suppose that the buoyancy force is neglected and fluid flows in such a way that the velocity is large but temperature difference between the wall of the plate and T_∞ are small. Then the basic equations for boundary layer reduce to

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0 \qquad (7.1.19)$$

$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial v}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \qquad (7.1.20)$$

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2. \tag{7.1.21}$$

We consider the following boundary conditions:

$$y=0: \quad u=\upsilon=0; T=T_w \ or, \frac{\partial T}{\partial y}=0$$

$$y=\infty: \qquad u=U_\infty; T=T_\infty.$$

In general, the velocity field is independent of temperature field so that the two flow equations (7.1.19) and (7.1.20) can be solved first and the result can be employed to evaluate the temperature field. If the generation of heat due to friction is neglected and u is replaced by T then the Eq.(7.1.20) and Eq.(7.1.21) are same provided that $\nu=\alpha$. The relation $\nu=\alpha$ indicates that the Prandtl number $\Pr=1$. If the generation of heat due to friction is neglected, then the temperature field exists only if there is a difference in temperature between the wall and external flow e.g. if $T_w-T_\infty>0$ (cooling). Hence for small velocities the temperature and velocity distributions are identical provided that the Prandtl number is equal to unity:

$$\frac{T-T_w}{T_\infty-T_w}=\frac{u}{U_\infty} \tag{7.1.22}$$

or,
$$\frac{T-T_\infty}{T_w-T_\infty}=1-\frac{u}{U_\infty}. \tag{7.1.23}$$

The local Nusselt number for heat transfer is given by

$$
\begin{aligned}
Nu(x) &= \frac{-x(\frac{\partial T}{\partial y})_{y=0}}{T_w - T_\infty} \\
&= \frac{x}{U_\infty}(\frac{\partial u}{\partial y})_{y=0} \\
&= \frac{x}{\mu U_\infty} T_{w.} \\
&= \frac{1}{2} R_e C_f
\end{aligned} \tag{7.1.24}
$$

where

$$R_e = \frac{U_\infty x}{\nu} \qquad and \qquad C_f = \frac{T_w}{\frac{1}{2}\rho U_\infty^{\ 2}}.$$

Eq.(7.1.24) leads us to the formulation of the important Reynold$_{s}$ analogy between heat transfer and skin frictions.

Now we introduce stream function ψ such that

$$
\begin{aligned}
u &= \frac{\partial \psi}{\partial y}, \\
\upsilon &= -\frac{\partial \psi}{\partial x}.
\end{aligned} \tag{7.1.25}
$$

Again we take the new dimensionless distance parameter $\eta = \frac{y}{\delta}$ so that

$$\eta = y\sqrt{\frac{U_\infty}{\nu x}}. \tag{7.1.26}$$

Hence we have

$$\psi. = \sqrt{\nu x U_\infty}\, f(\eta) \tag{7.1.27}$$

$$u = U_\infty f'(\eta) \tag{7.1.28}$$

$$\upsilon = \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\,(\eta f' - f). \qquad (7.1.29)$$

The above transformations reduce Eq.(7.1.20) and Eq.(7.1.21) to

$$ff' + 2f''' = 0 \qquad (7.1.30)$$

$$\frac{d^2T}{d\eta^2} + \frac{\Pr}{2} f \frac{dT}{d\eta} = -\Pr\frac{U_\infty{}^2}{2c_p} f''^2 \qquad (7.1.31)$$

with boundary conditions

$$\begin{aligned} &\eta = 0: \quad f = f' = 0; \qquad T = T_w \\ &\eta = \infty: \quad f' = 1.; \qquad T = T_\infty. \end{aligned}$$

The solution of Eq (7.1.30) under the boundary conditions $\eta = 0: f = f' = 0,\ \eta = \infty: f' = 1$ is given in Chapter-II.

It is convenient to represent the general solution of Eq. (7.1.31) by the superposition of two solutions of the form

$$T(\eta) - T_\infty = C\theta_1(\eta) + \frac{U_\infty^2}{2c_p}\theta_2(\eta) \qquad (7.1.32)$$

where C is an arbitrary constant to be determined. Here $\theta_1(\eta)$ is the general solution of homogeneous part of Eq. (7.1.32) and θ_2 is the particular solution of non homogeneous part of Eq. (7.1.32) . As $\theta_1(\eta)$ and $\theta_2(\eta)$ are solutions of homogeneous and non homogeneous part, from Eq. (7.1.31) we have

$$\theta_1'' + \frac{1}{2}\Pr f\theta_1' = 0 \qquad (7.1.33)$$

$$\theta_2'' + \frac{1}{2}\Pr f\theta_2' = -2\Pr f''^2 . \tag{7.1.34}$$

Eq. (7.1.33) is also obtained by putting $\theta_1 = \dfrac{T - T_\infty}{T_w - T_\infty}$ in Eq (7.1.21) when dissipation term is neglected. In this case the boundary conditions will be

$$\eta = 0: \ \theta_1 = 1; \quad \eta = \infty: \ \theta_1 = 0.$$

The problem under the above boundary condition is known as cooling problem. The solution of Eq.(7.1.33) was first given by E. Pohlhausen as

$$\theta_1(\eta, \Pr) = \frac{\int_{\xi=\eta}^{\infty} [f''(\xi)]^{\Pr} d\xi}{\int_{\xi=0}^{\infty} [f''(\xi)]^{\Pr} d\xi} . \tag{7.1.35}$$

The temperature gradient at the wall is given by

$$-\left(\frac{d\theta_1}{d\eta}\right)_{\eta=0} = \alpha_1(\Pr) = \frac{(0.332)^{\Pr_r}}{\int_0^{\infty} [f''(\xi)]^{\Pr} d\xi} \tag{7.1.36}$$

where $f''(0) = 0.332$.

E. Pohlhausen calculated $\alpha_1 \Pr$ for a wide range of values of $\Pr$ given in Table -7.1.3.

Table-7.1.3

p_r	αp_r	$0.332 p_r^{\frac{1}{3}}$
0.6	0.276	0.280
0.7	0.293	0.294
0.8	0.307	0.308
1.0	0.332	0.332
1.1	0.344	0.342
7.0	0.645	0.630
10.0	0.730	0.715
15.0	0.835	0.820

It can be seen from the table that $\alpha_1(\Pr)$ may be approximated with good accuracy by the formula

$$\alpha_1(\Pr)=0.332\Pr^{1/3} \quad \text{for } 0.6<\Pr<10.$$

Limiting cases:

(a) $\Pr \to 0$ (e.g. for mercury , $\Pr = 0.044$).

When u is replaced by U_∞ then $f'(\eta)=1$ and $f(\eta)=1$.

Thus Eq. (7.1.33) reduces to

$$\theta_1'' + \frac{1}{2}\Pr\ f'(\eta)=0. \qquad (7.1.37)$$

Solution of Eq.(7.1.37) is

$$\theta_1 = \frac{\int_\eta^\alpha \exp(-\frac{\Pr}{4}\eta^2)d\eta}{\int_0^\alpha \exp(-\frac{\Pr}{4}\eta^2)d\eta} . \tag{7.1.38}$$

Therefore,

$$\alpha_1(\Pr) = -(\frac{d\,\theta_1}{d\eta})_{\eta=0}$$

$$= \frac{1}{\int_0^\alpha exp(-\frac{\Pr}{4}\eta^2)d\eta}$$

$$= \frac{1}{\sqrt{\pi}}\Pr^{\frac{1}{2}}$$

$$= 0.564\Pr^{\frac{1}{2}} . \tag{7.1.39}$$

(b) $\Pr \to \infty$

When Prandtl number is large then the thermal boundary layer is much thinner than the velocity boundary layer. Using Blasius method, the solution of Eq.(7.1.30) for small value of η may be written as

$$f(\eta) = \frac{\varsigma}{2!}\eta^2 - \frac{\varsigma^2}{2.5!}\eta^5 + \frac{11\varsigma^8}{2^2 8!}\eta^8 - \frac{375\varsigma^4}{2^3.11!}\eta^{11} + \cdots\cdots\cdots\cdot$$

where ς is a unknown constant.

From the above expansion of $f(\eta)$ near $\eta = 0$ it is reasonable to replace $f(\eta)$ by its first term $\frac{\varsigma\eta^2}{2}$ where $\varsigma = 0.332$. Hence the Eq.(7.1.33) reduces to

$$\theta_1''^2 + \frac{1}{4}\Pr\varsigma\eta^2\theta'_1 = 0 . \tag{7.1.40}$$

Solution of Eq.(7.1.40) is

$$\theta_1 = \frac{\int_{\eta}^{\infty} \exp(-\frac{\Pr \varsigma}{12}\eta^3)d\eta}{\int_{0}^{\infty} \exp(-\frac{\Pr \varsigma}{12}\eta^3)d\eta} . \tag{7.1.41}$$

Therefore,

$$\alpha_1(\Pr) = 0.339 \Pr^{1/3} . \tag{7.1.42}$$

The Nusselt number for heat transfer is denoted as

$$\begin{aligned} Nu &= \frac{-x(\frac{\partial T}{\partial y})_{y=0}}{T_w - T_\infty} \\ &= -(\frac{\partial \theta_1}{\partial \eta})_{\eta=0}\sqrt{\frac{U_\infty x}{\nu}} \\ &= \alpha_1(\Pr)\operatorname{Re}^{\frac{1}{2}} \end{aligned} \tag{7.1.43}$$

where $\operatorname{Re} = \frac{U_\infty x}{\nu}$.

For adiabatic wall if we put $\theta_2 = \frac{T - T_\infty}{U_\infty^2 c_p / 2}$ in Eq.(7.1.21), we get Eq. (7.1.34).

So the boundary conditions are:

$$\eta = 0:\ \theta_2' = 0;\ \eta = \infty:\ \theta_2 = 0 .$$

The solution of Eq. (7.1.34) can be obtained by the method of variation of the parameters as

$$\theta_2(\eta, \Pr) = 2\Pr \int_{\xi=\eta}^{\infty} [f''(\xi)]^{\Pr} (\int_0^{\xi} f''(\tau)^{2-\Pr} d\tau)d\xi . \tag{7.1.44}$$

From Eq. (7.1.32) we have

$$C = T_w - T_\infty - \frac{U^2{}_\infty}{2c_p}\theta_2(0). \tag{7.1.45}$$

If the adiabatic wall temperature is T_a then we have

$$\theta_2(\eta_1, \mathrm{Pr}) = \frac{T_a - T_\infty}{\frac{U_\infty{}^2}{2c_p}} \tag{7.1.46}$$

or, $$T_\alpha - T_\infty = \frac{U_\infty{}^2}{2c_p}\theta_2(0, \mathrm{Pr}). \tag{7.1.47}$$

Thus

$$C = (T_w - T_\infty) - (T_a - T_\infty) \tag{7.1.48}$$

The general solution for a prescribed temperature difference between the wall and free stream velocity is thus

$$T - T_\infty = \{(T_w - T_\infty) - (T_a - T_\infty\}\theta_1(\eta, \mathrm{Pr}) + \frac{U_\infty^2}{2c_p}\theta_2(\theta_1, \mathrm{Pr}). \tag{7.1.49}$$

There the dimensionless temperature distribution becomes

$$\frac{T - T_\infty}{T_w - T_\infty} = \left[1 - \frac{1}{2}E_c\theta_2(0, \mathrm{Pr})\right]\theta_1(\eta, \mathrm{Pr}) + \frac{1}{2}E_c\theta_2(\eta, \mathrm{Pr}) \tag{7.1.50}$$

where E_c is Eckert number.

We consider a heated flat plate of temperature to T_w placed vertically under gravity in large body of fluid which is otherwise rest and has temperature T_∞ and density ρ_∞. Suppose that the viscosity and the conductivity of the fluid are small and the motions are caused solely by the density gradients created by temperature difference. In the case of a vertical plate, the pressure in each horizontal place is equal to the gravitational pressure and is thus constant.

The only cause of motion is furnished by the difference between weight and buoyancy in the gravitational field of earth. Let the origin is at the lower edge of the plate, x axis along the plate and y axis normal to the plate. Again suppose that the pressure in the boundary layer is same as at the outer edge.

Therefore the governing equations of motion are:

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0 \tag{7.1.51}$$

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\frac{T_w-T_\infty}{T_\infty}\theta \tag{7.1.52}$$

$$u\frac{\partial \theta}{\partial x}+\upsilon\frac{\partial \theta}{\partial y}=\alpha\frac{\partial^2 \theta}{\partial y^2} \tag{7.1.53}$$

where

$$\theta=\frac{T-T_\infty}{T_w-T_\infty}.$$

Boundary conditions:

$$y=0:\ u=0,\quad \upsilon=0;\quad \theta=1$$
$$y=\infty\quad u=0;\quad \theta=0.$$

E. Pohlhavsen first obtained the solution of these equations. In order to convert partial differential into ordinary differential equation he used the following substitutions:

$$\eta=\left\{\frac{g(T_w-T_\infty}{4\nu^2 T_\infty}\right\}^{1/4}\frac{y}{x^{1/4}}=c\frac{y}{x^4} \tag{7.1.54}$$

$$\psi(x,y)=4\nu c x^{3/4}f(\eta) \tag{7.1.55}$$

where c is a constant.

We introduce a stream function ψ such that

$$u = \frac{\partial\psi}{\partial y},$$
$$\upsilon = -\frac{\partial\psi}{\partial y}.$$

So that

$$u = 4\nu c^2 x^{1/2} f(\eta) \ . \qquad (7.1.56)$$

Eq. (7.1.52) and Eq. (7.1.53) lead to the following differential equations

$$f''' + 3ff'' - 2f'^2 + \theta = 0 \qquad (7.1.57)$$

$$\theta'' + 3\Pr f\theta' = 0 \qquad (7.1.58)$$

with boundary conditions

$$\eta = 0: \ f = f' = 0; \theta = 1$$
$$\eta = \infty: \qquad f' = 0; \qquad \theta = 0.$$

E. Prhlhqusen solved these equation by series taking $\Pr = 0.733$. From his result, we find

$$\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0} = -0.508. \qquad (7.1.59)$$

The local Nusselt number for heat transfer is given by

$$Nu(x) = \frac{-\left(\frac{\partial T}{\partial y}\right)_{y-0} x}{(T_w - T_\infty)}$$
$$= -\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0} \quad cx^{3/4} \qquad (7.1.60)$$
$$= 0.508\left\{\frac{g(T_w - T_\infty}{4\nu^2 T_\infty}\right\}^{1/4} x^{3/4}.$$

7.1.5 Results and discussion:

From Fig-7.1.1 we see that the change of temperature with respect to displacement is linear when the product of the Eckert number and Prandtl number is zero. Other wise temperature profile is not linear. From Fig-7.1.2 we see that the maximum temperature occurs in the middle of the channel. When the velocity of the fluid is large, the buoyancy force is neglected and the temperature difference $T_w - T_\infty$ is small then the velocity distributions coincide with temperature distributions provided that $\Pr = 1$. From Eq. (7.1.24) we see that there is a relationship among Reynolds number, heat transfer and skin friction. The rate of heat transfer can be calculated from the relation (7.1.60).

CHAPTER VII

Part: B

Unsteady MHD Flow of Incompressible Viscous Fluid Past an Infinite Vertical Plate

7.2.1 Introduction

Raptis & Massalas [56] studied the steady free convective and mass transfer flow of a viscous incompressible fluid through a porous medium bounded by an infinite vertical porous plate, with constant heat flux at the plate. Fouzia [19] studied the steady MHD free convective flow through porous medium bounded by an infinite vertical porous plate. Sreekanth, Nagarajan & Ramana [71] described the transient MHD free convective flow of an incompressible viscous dissipative fluid.

In this chapter, we consider the unsteady MHD flow of an incompressible viscous fluid past an infinite vertical plate. The uniform flow is subject to a transverse applied magnetic field. We also consider small magnetic Reynolds number so that induced magnetic field is neglected. Numerical solutions to the coupled non-linear equations are derived for the velocity and temperature fields. Effects of the various parameters occurring in the problem have been discussed with the help o graphs.

7.2.2 Mathematical formulation

A uniform magnetic field of strength H_0 is acting transversely to the plate. The pressure gradient force is taken to be zero.

The two dimensional boundary layer equations that govern the unsteady free convective flow are

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T_\infty) - \frac{\sigma \mu_e^{\ 2} H_0^{\ 2} u'}{\rho} \qquad (7.2.1)$$

$$\rho c_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} + \mu(\frac{\partial u'}{\partial y'})^2 \qquad (7.2.2)$$

where u' = velocity of the fluid in x' direction

$T =$ temperature of the fluid in the boundary layer

$T_\infty =$ temperature of the fluid outside the boundary layer

$k =$ thermal conductivity of the fluid

$c_p =$ specific heat at constant pressure

$\beta =$ coefficient of thermal expansion

$\rho =$ density of the fluid

$\nu =$ kinematic viscosity

$\mu =$ viscosity of the fluid

$g =$ acceleration due to gravity

$\mu_e =$ magnetic permeability

$\sigma =$ is the electrical conductivity of the fluid

The initial and boundary conditions are

$$t' \leq 0, u' = 0 \text{ when } T' = T'_\infty \text{ for all } y \qquad (7.2.3)$$

$$t' \geq 0, u = 0 \text{ when } T' = T'_w \text{ for } y' = 0 \qquad (7.2.4)$$

$$t' \geq 0, \frac{\partial T'}{\partial y'} \text{ at } y' = \delta' \qquad (7.2.5)$$

We introduce the following non-dimensional number:

$$t = \frac{t'}{T}, y = \frac{y'}{\delta}, u = \frac{u'}{U_0} \tag{7.2.6}$$

$$\Delta T = T'_w - T'_\infty, \theta = \frac{T' - T'_\infty}{\Delta T} \tag{7.2.7}$$

Substitute Eq.(7.2.6) and Eq.(7.2.7) in Eq.(7.2.1), we obtain

$$\frac{\partial u}{\partial t} = \frac{\nu T}{\delta^2}\frac{\partial^2 u}{\partial y^2} + \frac{T\Delta T g\beta\theta}{U_0} - Mu \tag{7.2.8}$$

where M is the magnetic parameter and

$$M = \frac{\sigma\mu_{e^2} H_0{}^2 T}{\rho}$$

Choosing $\frac{\nu^2 \rho c_p}{k\delta U_0} = 1$, we obtain from Eq.(7.2.8)

$$\frac{\partial u}{\partial t} = \Pr\frac{\partial^2 u}{\partial y^2} + Gr\theta - Mu \tag{7.2.9}$$

where

$$\Pr = \frac{\mu c_p}{k} \text{ and } Gr = \frac{\beta g \Delta T \delta^3}{\nu^2}$$

Substitute Eq.(7.2.6) and Eq.(7.2.7) into Eq.(7.2.2), we obtain

$$\frac{\partial \theta}{\partial t} = \frac{kT}{\delta^2 \rho c_p}\frac{\partial^2 \theta}{\partial y^2} + \frac{T}{\rho \Delta T c_p}\frac{\mu U_0{}^2}{\delta^2}\left(\frac{\partial u}{\partial y}\right)^2 \tag{7.2.10}$$

Choosing $\frac{Tk}{\delta^2 \rho c_p} = 1$, we have from Eq.(7.2.10)

$$\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial y^2}+\Pr E_c(\frac{\partial u}{\partial y})^2 \qquad (7.2.11)$$

where

$$E_c=\frac{U_0{}^2}{\Delta T c_p}$$

The initial and the boundary conditions (7.2.3), (7.2.4), and (7.2.5) reduce to

$$t\le 0, u=0, \theta=0 \quad \text{for all } y=0 \qquad (7.2.12)$$

$$t\ge 0, u=0, \theta=1 \quad \text{at } y=0 \qquad (7.2.13)$$

$$t\ge 0, \frac{\partial\theta}{\partial y}=0 \quad \text{at } y=1 \qquad (7.2.14)$$

The exact solutions of Eq.(7.2.9) and Eq.(7.2.11) are not possible. So we employ explicit finite difference method for its solutions. The equivalent finite difference scheme of Eq.(7.2.9) and Eq.(7.2.11) are as follows:

$$\frac{u_{i,j+1}-u_{i,j}}{\Delta t}=\Pr\frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{(\Delta y)^2}+Gr\theta_{i,j}-Mu_{i,j} \qquad (7.2.15)$$

$$\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta t}=\frac{\theta_{i+1,j}-2\theta_{i,j}+\theta_{i-1,j}}{(\Delta y)^2}+E_c\Pr(\frac{u_{i+1,j}-u_{i,j}}{\Delta y})^2 \qquad (7.2.16)$$

where the index i refers to y and j refers to t. The mesh system is divided by taking $\Delta y=0.1$. From the initial condition, we have the following equivalent:

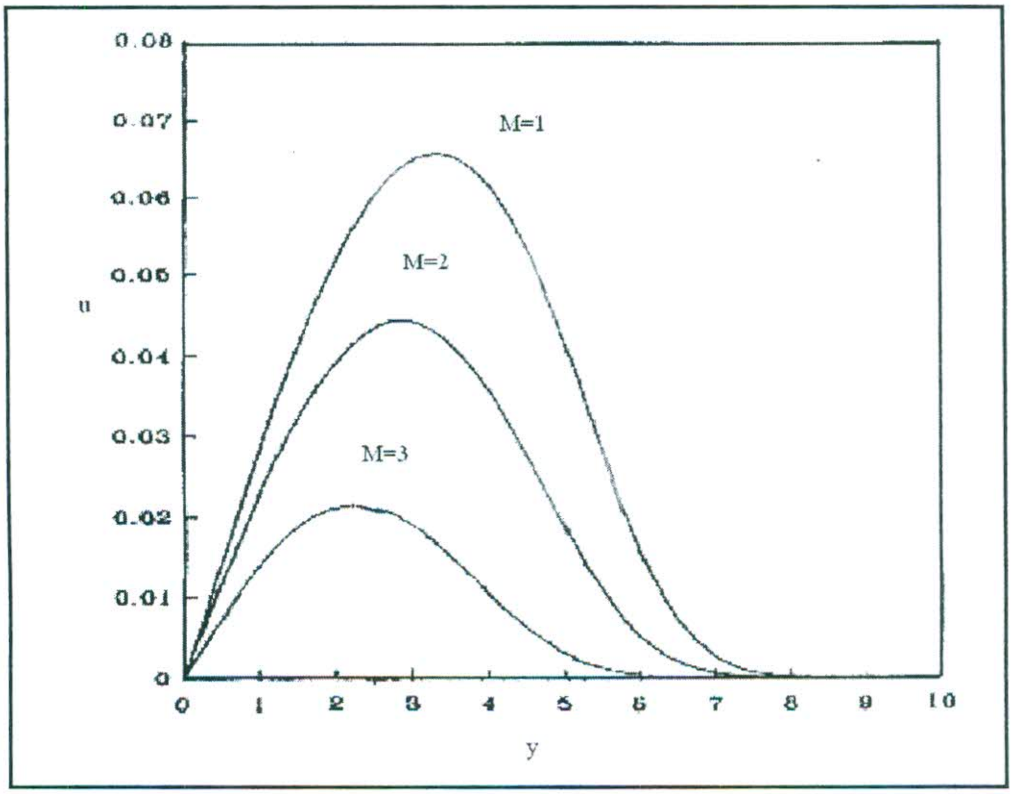
$u(0,j)=0, \theta(0,j)=1, u(i,0)=0, \theta(i,0)=0$ for all i except $i=0$.

The boundary conditions are expressed in finite difference form as follows

$u(0,j)=0 \quad \theta(0,j)=1$ for all j

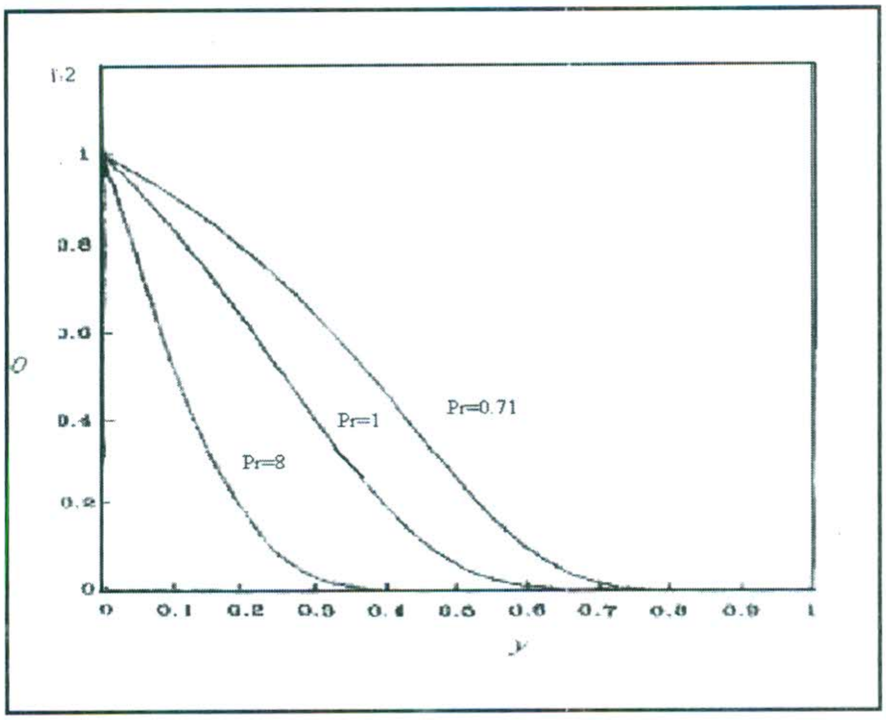
$u(i,0)=0 \qquad \theta(i,0)=0$ for all i

Firstly, the velocity at the end of time step viz., $u(i,j+1)(i=1,20)$ is computed from Eq.(7.2.15) in terms of velocities and temperatures at points of earlier time step. Them $\theta(i,j+1)$ is computed from Eq.(7.2.16). The solutions for u and θ thus obtained are plotted respectively in Fig-7.2.1 and Fig-7.2.2. Fig-7.1.1 is drawn for $\Pr = 0.71$ and $E_c = 0.3$. Fig-7.1.2 is drawn for $M=1$ and $E_c = 0.3$.



Velocity profile

Fig-7.2.1



Temperature profile

Fig-7.2.2

7.2.3 Results and discussion:

From Fig-7.2.1 and Fig-7.2.2 the velocity profile u is drown against y for different values of magnetic parameter and time. We observe that the velocity of the fluid increases with the increasing value of magnetic parameter. Again we notice that the velocity of the fluid starts with zero velocity and then gradually increases and attains a maximum value there after the velocity diminishes in the same manner and ultimately dies out to zero velocity. From Fig-7.2.2 the temperature profile θ is drawn against y for different values of Prandtl number and time. Here we observe that temperature increases with the increase in Prandtl number.

CHAPTER VIII

Steady Laminar Flow of Incompressible Fluid Through a Circular Tube

8.1 Introduction

A variety of fluid flows in a closed conduit is investigated owing to their applications in physiological and engineering problems. A closed conduit is a tube or duct through which the fluid flows while completely filling the cross section. As the fluid flows over the solid boundary a shear stress will develop at the surface of contact which will oppose the motion. A flow in a tube is more likely to be laminar if the fluid velocity is low; the diameter of the tube is small; the density of the fluid is low; and the viscosity of the fluid is high. The four variables are grouped in the form of a non dimensional parameter called Reynolds number. The flow of fluid is more likely to be laminar at law Reynold number and is more likely to be turbulent at high Reynolds number. Flow through a tube is laminar at Reynolds number less than 2000 and it is turbulent at Reynolds number more than 3000. The flow is said to be in transition stage at Reynolds number between 2000 to 3000. In a fully developed tube flow, the pressure drops linearly along the length of the tube line. In other words, the pressure gradient along the flow remains constant. The laminar flow through circular tube is discussed in many standard books of Chorlton [10], Douglas Gasiorek & Swaffield [15], Raisighania [55], Raptis [56] & Schlichting [62]. Based on Navier-Stokes equation many researchers have been developed mathematical models for transportation of blood through arteries . McMichael & Deutsch [44] studied the magneto–hydrodynamics of laminar flow in slowly varying tubes in an axial magnetic field. Desikachar & Rao [14] calculated the influence of a magnetic field on the blood oxygenation process. Shah [66] reviewed the fully developed and developing solution for blood flowing in a conduit. Krishna & Rao [34] investigated the motion of a viscous incompressible flow through a non uniform channel under a transverse magnetic field. Mazumdar, Gunguly & Venkatesan [43] investigated the solution of Newtonian fluid flow through a circular tube .They solved the problem numerically by shooting method . In this chapter an attempt has been made to study the laminar flow of incompressible viscous fluid through a circular tube in presence of magnetic field.

Case I: Steady state

8.2 Mathematical formulation

We consider a steady laminar flow of viscous incompressible fluid through an infinite circular tube of radius a. Again suppose that the fluid flows in the direction of z-axis and depends only on the radial distance and also flows symmetrically about the axis of the circular tube. If we consider cylindrical polar co-ordinates (r, θ, z), the velocity vector can be taker as $\overline{V} = [0,0,u]$. In the absence of body force, the mass conservation equation and the momentum equation reduce to

$$\frac{\partial u}{\partial z} = 0 \tag{8.1}$$

$$\frac{\partial p}{\partial r} = 0 \tag{8.2}$$

$$-\frac{1}{r}\frac{\partial p}{\partial \theta} = 0 \tag{8.3}$$

$$0 = -\frac{\partial p}{\partial z} + \mu[\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r})] . \tag{8.4}$$

Equ. (8.1) shows that u is a function of r alone and the Eq.(8.2) and the Eq. (8.3) show that p is a function of z alone. If we choose $\frac{\partial p}{\partial z} = -P$ (a constant) then Eq. (8.4) becomes

$$\mu[\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r})] = -P \ . \tag{8.5}$$

Boundary condition:

$u = U_O$ at $r = 0$

$u = 0$ at $r = a$.

Solving Eq. (8.5), we get

$$u = \frac{-r^2 P}{4\mu} + c_1 \ln r + c_2 \tag{8.6}$$

where c_1 and c_2 are arbitrary constants to be determined.

Since at the axis of the cylinder the velocity should be finite, we must have $c_1 = 0$. Again if the circular tube is fixed, $u = 0$ at $r = a$. Thus from Eq.(8.6), we get

$$u = -\frac{Pa^2}{4\mu}[1-(\frac{r}{a})^2] \ . \tag{8.7}$$

From Eq. (8.7) the maximum velocity occurs at the axis of the tube and is equal to $-\frac{Pa^2}{4\mu}$.

The skin friction in this case will be $-\frac{Pa}{2}$.The average velocity can be measured by the formula

$$u_{ave} = \frac{1}{\pi a^2}\int_0^{2\pi}\int_0^{a} urdrd\theta \tag{8.8}$$

and is equal to $-\dfrac{Pa^2}{8\mu}$.

The coefficient of skin friction is

$$c_f = \frac{\tau}{\frac{1}{2}\rho u_{ave}{}^2} = \frac{16}{\text{Re}} \tag{8.9}$$

where Re is Reynold number.

The total volume of the fluid crossing any section per unit time is given by

$$Q = \int_0^a 2\pi r u_{ave} dr = -\frac{P\pi a^4}{8\mu}. \tag{8.10}$$

Eq.(8.10) shows that the total flux is proportional to the pressure gradient and to the fourth power of the radius of the tube.

8.3 Circular tube flow past across a transverse magnetic field

Let us consider a conducting fluid passing across a transverse magnetic field. Then the new form of Eq.(8.4) will be

$$\frac{1}{r'}\frac{d}{dr'}(\mu r'\frac{du'}{dr'}) - \frac{dp'}{dz'} + \sigma B_0^2 u' = 0 \tag{8.11}$$

where B_o is uniform magnetic field and σ is electrical conductivity .

We introduce the following non dimensional quantities

$$u = \frac{u'}{U_o}, r = \frac{r'}{a}, P = \frac{P'}{\rho U_o{}^2}, x = \frac{x'}{L} \tag{8.12}$$

where L is the length of the pipe.

Then the new form of Eq.(8.11) will be

$$\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}-M^2u=-P_O \tag{8.13}$$

where

$$M=B_O L\sqrt{\frac{\sigma}{\mu}} and \frac{dp}{dz}=-P_O . \tag{8.14}$$

Boundary condition:

$u=1$ at $r=0$

$u=0$ at $r=1$.

Homogeneous part of Eq.(8.13)is

$$\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}-M^2u=0. \tag{8.15}$$

The general solution of (8.15) i.e., the complementary function of Eq.(8.13) is

$$u_c=a_o J_O(iMr) \tag{8.16}$$

where

$$J_O(iMr)=1+\frac{r^2M^2}{4}+\frac{r^4M^4}{64}+\frac{r^6M^6}{2304}+------ \tag{8.17}$$

is Bessel's function of order zero and a_o is an arbitrary constant.

The particular solution of (8.13) is

$$u_p=\frac{P_O}{M^2}. \tag{8.18}$$

The general solution of (8.13) is

$$u(r:M) = a_o J_O(iMr) + \frac{P_O}{M^2}. \tag{8.19}$$

Again applying the boundary conditions, we get

$$u(r:M) = \frac{J_O(iM) - J_O(iMr)}{J_O(iM) - 1}. \tag{8.20}$$

Table-8.1

r	u M=2	u M=4	u M=6

0.1	.992154	.995868	.998141
0.2	.968381	.98297	.992047
0.3	.927965	.959736	.980025
0.4	.869685	.923328	.958761
0.5	.79178	.869318	.922585
0.6	.69189	.791226	.862441
0.7	.566989	.679932	.764558
0.8	.413293	.522956	.608833
0.9	.226168	.303608	.366917
1	.000001	.000002	.000003

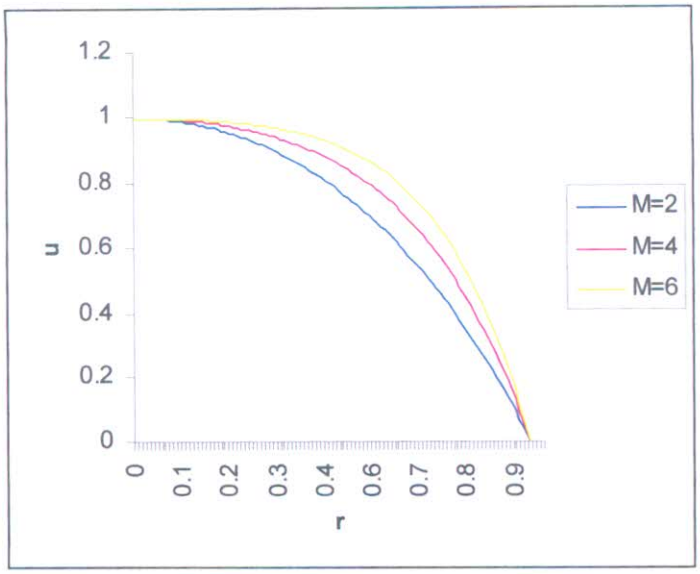


Figure-8.1

Velocity profile for different values of Hartmann number M

8.4 Circular tube flow past a narrow obstacle

Suppose that the steady flow past a narrow obstacle whose surface is give by

$$\frac{r}{a} = 1 - \frac{\delta}{2a}(1 + \cos\frac{\pi z}{z_o}) \tag{8.21}$$

where δ is the thickness of the obstacle along the normal direction to the flow , a the radius of the tube without obstacle, r the radius the tube with obstacle $r = a - \delta$, z_o the constant $(-z_o \le z \le z_o)$ .Assuming that $\frac{\delta}{a} << 1$ .Since at the surface of the obstacle the fluid velocity is zero so the Eq. (8.7) and Eq.(8.10) reduce to

$$u = -\frac{P(z)}{4\mu}[a^2 - r^{2]} \tag{8.22}$$

$$Q = \frac{\pi P(z)}{8\mu} r(z)^4 . \qquad (8.23)$$

The pressure drop across the length of the obstacle is

$$\Delta p = \frac{16\mu Q z_0}{\pi a^4} \int_{-z_0}^{z_0} \frac{1}{r^4} dz$$

$$= \frac{16\mu Q z_0}{\pi a^4} (1 - \frac{\eta}{2})(1 - \eta + \frac{5}{8}\eta^2)(1-\eta)^{-\frac{7}{2}} \qquad (8.24)$$

where

$$\eta = \frac{\delta}{a} .$$

If ξ represent the ratio of pressure drop across the length of the obstacle to the pressure without the obstacle then

$$\xi = (1 - \frac{\eta}{2})(1 - \eta + \frac{5}{8}\eta^2)(1-\eta)^{-\frac{7}{2}} . \qquad (8.25)$$

Table-8.2

$a = 10, \delta < 1$		$a = 10, \delta < 3$	
η	ξ	η	ξ
.01	1.002004	.01	1.002004
.02	1.022462	.05	1.088764
.03	1.043714	.1	1.218519
.04	1.065799	.15	1.378809

.05	1.088764	.2	1.5798825
.06	1.112653	.25	1.836208
.07	1.137518	.3	2.169521
.08	1.163412		
.09	1.190392		
.1	1.218519		

Figure-8.2 is drown for $a = 10, \delta < 1$ and Figure-8.3 is drawn for $a = 10, \delta < 3$

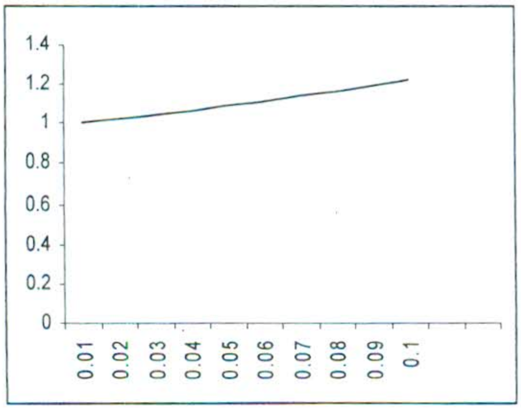


Figure-8.2

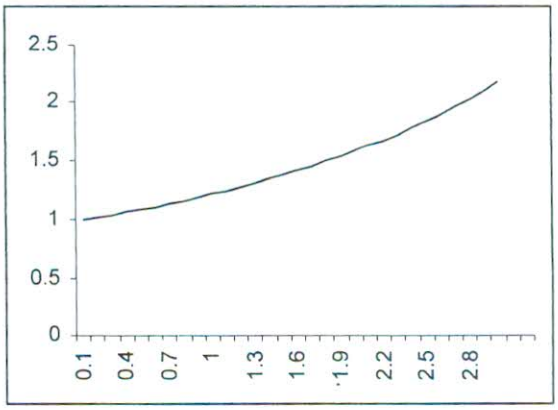


Figure-8.3

Case II: Unsteady state

8.5 Formulation of the problem

For unsteady state, the modified form of Eq.(8.1,8.2,8.3,8.4) are:

$$\frac{\partial u(r,z,t)}{\partial z} = 0 \tag{8.26}$$

$$\partial p(r,z,t)/\partial r = 0 \tag{8.27}$$

$$\partial u(r,z,t)/\partial t+u(r,z,t)\partial u(r,z,t)/\partial z=-\frac{1}{\rho}\partial p(r,z,t)/\partial z$$
$$+v\left(\partial^2 u(r,z,t)/\partial r^2+\frac{1}{r}\partial u(r,z,t)/\partial r+\partial^2 u(r,z,t)/\partial z^2\right) \tag{8.28}$$

From Eq. (8.26) and Eq. (8.27) we conclude that u is a function of r and t only and p is a function of z and t only .Eq. (8.26) and Eq. (8.27) reduce the Eq. (8.28) to

$$\frac{\partial u}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{v}{r}\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r}). \tag{8.29}$$

Suppose that

$$\frac{\partial p}{\partial z}=-Pe^{iwt} \tag{8.30}$$

$$u(r,\ t)=V(r)e^{iwt} \tag{8.31}$$

where $i=\sqrt{-1}$.

Using Eq. (8.30) and Eq. (8.31) in Eq. (8.29), we get

$$\frac{d^2V}{dr^2}+\frac{1}{r}\frac{dV}{dr}-\frac{iw}{\mu}\rho V=-\frac{P}{\mu}. \tag{8.32}$$

The general Solution of the Eq. (8.32) is

$$V=c_1 J_0\left[i^{3/2}\sqrt{w\rho/(\mu r)}\right.+c_2 Y_0\left[i^{3/2}\sqrt{w\rho/\mu r}\right]+\frac{P}{w\rho i} \tag{8.33}$$

where c_1 and c_2 are arbitrary constants to be determined. In Eq. (8.33) J_0 and Y_0 are Bessel functions of zero order and are of the first and second kind. When $r=0$ then both u and V are finite but Y_0 is not finite, so we must have c_2=0. Again on the surface of the tube ($r=a$) the velocity $u=0$, we have

$$c_1 = -\frac{P}{w\rho} i . \frac{1}{J_0\left[i^{3/2} \sqrt{wp/(\mu a)} \right]} .$$

Substituting the value of A in Eq. (8.33), we get

$$V(r) = -\frac{P}{w\rho} i \frac{1}{J_0\left[i^{3/2} \sqrt{w\rho/(\mu R)} \right]} \times J_0\left[i^{3/2} \sqrt{w\rho/\mu r)} \right] . \tag{8.34}$$

Finally

$$u(r, t) = -\frac{iP}{w\rho} \frac{J_0\left[i^{3/2} \sqrt{w\rho/(\mu r)} \right]}{J_0\left[i^{3/2} \sqrt{w\rho/(\mu a)} \right]} e^{iwt} . \tag{8.35}$$

The total value of the fluid crossing any section per unit time is given by

$$Q = \int_0^{\alpha} v \, 2\pi r dr$$

$$= -\frac{\pi a^4}{\mu \xi^2} i P e^{iwt} \left[1 - \frac{2}{j_o(i^{\frac{3}{2}} \xi)} \int_0^{i^{\frac{3}{2}\xi}} \left[\frac{x J_0(x)}{i^{3/2} \xi^2}) dx \right] \right] \tag{8.36}$$

where

$$\xi^2 = w\rho/(\mu a) .$$

Applying the formula

$$\int x J_0(x) dx = x J_1(x) ,$$

we get

$$Q = -\frac{\pi a^4}{i \mu \xi^2} P e^{iwt} \chi(\xi)$$

where

$$\chi(\xi) = 1 - \frac{2j_1(i^{\frac{3}{2}}\xi)}{i^{\frac{3}{2}}\xi j_o(i^{\frac{3}{2}}\xi)} \tag{8.37}$$

$$Q = \frac{\pi a^4}{\mu \xi^2}\{[\chi_2(\xi)\cos wt + \chi_1(\xi)\sin wt] - i[\chi_1(\xi)\cos wt - \chi_2(\xi)\sin wt]\} \tag{8.38}$$

where

$$\chi(\xi) = \chi_1(\xi) + i\chi_2(\xi).$$

8.6 Results and discussion

From Eq.(8.10) if we consider that Q is constant for all sections of the tube, then the pressure gradient varies inversely as the fourth power of the surface distance from the axis of the tube so that the pressure gradient is minimum at the middle of the obstacle and is maximum at the ends. The Eq.(8.9) shows that the skin friction can be obtained from the knowledge of Re. The Eq.(8.9) is used to determine energy losses in tube flows . From Fig-8.1 we observe that the maximum velocity occurs at the middle of the tube and the velocity of the fluid increases as the radius of the tube increases. The Fig-8.1 also indicates that the velocity of the fluid increases when the magnetic field increases. It is also noticed that the typical laminar velocity profile is parabolic. Fig.-8.2 shows that the pressure drops linearly for small value of the thickness of the obstacle. Fig.-8.3 shows that the pressure drops for large value of the thickness of the obstacle. The Eq.(8.38) shows that the real part gives the flux when the pressure gradient is $P\cos wt$ and the imaginary part gives the flux when it is $P\cos wt$.

CHAPTER IX
Conclusion

In this thesis we have considered the laminar flow of incompressible viscous Newtonian fluid. The nature of the flow of fluid is very complex since the basic laws describing the complete motion of fluid are not easily formulated and handled mathematically. Several fluid problems, which are governed by ordinary differential equations, are difficult to solve analytically. In each such problem, the equations have been put into proper form for numerical solution. In this thesis, the differential equations related to laminar flow problem have been solved by either one of the two methods or both methods. In certain instances, the numerical results from a computer program have been taken. The general problem of evaluation of velocity boundary layer or thermal boundary layer for a body of arbitrary shape provides to be extremely difficult. Here we have considered only simple flat plate at zero incidences. Both velocity boundary layer and thermal boundary layer have been considered in this thesis.

In Chapter III, some numerical methods of boundary layer flow problem have been discussed. A suitable example is given using shooting method. The effects of pressure gradient force have been shown both analytically and graphically in part: A of Chapter IV. Various types of laminar flows between two parallel plates in presence of magnetic field have been considered in part: B of Chapter IV and in Chapter V. A flow problem related to porous plates has been investigated in Chapter V. In chapter VI we have considered a flow problem over a suddenly accelerated flat plate which has been tackled by means of Crank-Nicholson implicit scheme.

In many cases the temperature field around a hot body in a fluid stream is of boundary layer type. This layer extends only over a narrow zone in the immediate neighborhood of the surface, whereas the higher body temperature does not affect the regions at a large distance from it. The study of heat transfer by convection is concerned with the calculation rates of heat exchange between fluids and solid boundaries. Heat generation due to friction is a common phenomenon of fluid dynamics. In chapter VII various problems related to velocity and temperature have been discussed.

Some problems of fluid flows in circular tube have been investigated owing to their applications in physiological and engineering problems. The flow through circular tube in presence of magnetic field and the flow past a narrow obstacle have been investigated in Chapter VIII.

We believe that the results of the present work will enrich the list of exact and numerical solutions and may help the investigation of the laminar flow of incompressible viscous Newtonian fluid like air, water, liquid metals etc. . We also hope that the various methods discussed in this thesis will help the researchers in working in the field of Fluid Dynamics.

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